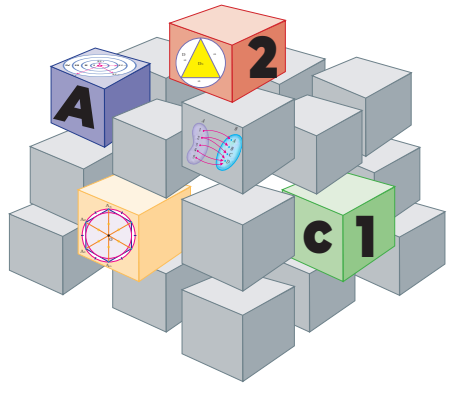
ALGEBRA
VA ANALIZ ASOSLARI 10



Umumiy o‘rta ta’lim maktablarining
10-sinfi uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi vazirligi
nashrga tavsiya etgan

Yangi nashr

TOSHKENT 2022

www.idum.uz saytidan yuklab olindi

UO‘K 51(075.3)
51(075.3)
M31

Tuzuvchilar:

Adilbek Zaitov, Ra’no Hamrayeva, Kalmurza Sagidullayev, Umid Rahmonov, Baxtiyor Abdiyev, Baljan Urinbayeva

Xalqaro ekspert:

Marcelo Staricoff

Taqrizchilar:

D. B. Eshmamatova – Toshkent shahar Sergeli tumanidagi 6-ixtisoslashtirilgan davlat umumiy o‘rta ta’lim maktabi matematika fani o‘qituvchisi.

J. A. Qo‘yjonov – Navoiy viloyati Xatirchi tumanidagi 5-umumiy o‘rta ta’lim maktabi matematika fani o‘qituvchisi.

A. Q. Qodirov – Toshkent viloyati Oqqo‘rg‘on tumanidagi 6-umumiy o‘rta ta’lim maktabi matematika fani o‘qituvchisi.

10-sinf Algebra va analiz asoslari [Matn]: darslik / A. Zaitov [va boshq.] – Toshkent: Respublika ta’lim markazi, 2022. – 192 b.

UNICEFning O‘zbekistondagi vakolatxonasi
bilan hamkorlikda tayyorlandi.

O‘zbekiston Respublikasi Fanlar akademiyasi V. I. Romanovskiy nomidagi
matematika instituti xulosasi asosida takomillashtirildi.

Original maket va dizayn konsepsiyasi
Respublika ta’lim markazi tomonidan ishlandi.

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

SHARTLI BELGILAR

murakkablik darajasi I

murakkablik darajasi II

murakkablik darajasi III

kichik mavzular

ISBN 978-9943-0000-0-0

© Respublika ta’lim markazi, 2022

www.idum.uz saytidan yuklab olindi

MUNDARIJA

TAKRORLASH

KVADRAT FUNKSIYA VA UNING GRAFIGI....6

KVADRAT TENGSIZLIK VA UNING YECHIMI....9

TRIGONOMETRIK AYNIYATLAR....14

ARIFMETIK VA GEOMETRIK PROGRESSIYALAR....20

1-BOB ELEMENTAR FUNKSIYALAR

FUNKSIYA. FUNKSIYANING BERILISH USULLARI....24

FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO‘PLAMI....28

FUNKSIYALAR USTIDA ARIFMETIK AMALLAR....33

MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR....36

FUNKSIYA XOSSALARI....43

FUNKSIYA GRAFIKLARI USTIDA SODDA ALMASHTIRISHLAR....48

CHIZIQLI VA KVADRATIK MODELLASHTIRISHLAR....57

LOYIHA ISHI....61

2-BOB RATSIONAL TENGLAMALAR VA TENGSIZLIKLAR. IRRATSIONAL TENGLAMALAR

RATSIONAL TENGLAMALAR....64

RATSIONAL TENGLAMALAR SISTEMASI....75

RATSIONAL TENGSIZLIKLAR....79

RATSIONAL TENGSIZLIKLAR SISTEMASI....83

IRRATSIONAL TENGLAMALAR....86

IRRATSIONAL TENGLAMALAR SISTEMASI....100

www.idum.uz saytidan yuklab olindi

3-BOB KO‘RSATKICHLI VA LOGARIFMIK FUNKSIYALAR

KO‘RSATKICHLI FUNKSIYA VA UNING XOSSALARI. GRAFIGI.....108
LOGARIFM HAQIDA TUSHUNCHA. LOGARIFMIK FUNKSIYA VA UNING XOSSALARI, GRAFIGI.....111
LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH.....113
KO‘RSATKICHLI TENGLAMALAR.....118
SODDA LOGARIFMIK TENGLAMALAR.....121
KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI.....124
KO‘RSATKICHLI TENGSIZLIKLAR.....128
LOGARIFMIK TENGSIZLIKLAR.....131
KO‘RSATKICHLI MODELLASHTIRISH. MURAKKAB FOIZ FORMULASI VA UNING TATBIQLARI. O‘SISH HODISALARI. RADIOAKTIV YEMIRILISH.....133

4-BOB TRIGONOMETRIK FUNKSIYALAR

TRIGONOMETRIK FUNKSIYALAR VA ULARNING XOSSALARI, GRAFIGI. DAVRIY JARAYONLAR.....139
TESKARI TRIGONOMETRIK FUNKSIYALAR VA ULARNING XOSSALARI, GRAFIGI.....144
LOYIHA ISHI.....150

5-BOB TRIGONOMETRIK TENGLAMALAR

TRIGONOMETRIK TENGLAMALAR.....153
BA’ZI TRIGONOMETRIK TENGLAMALARNI YECHISH USULLARI.....158
TRIGONOMETRIK TENGSIZLIKLAR.....161

6-BOB EHTIMOLLAR NAZARIYASI

TASODIFIY HODISALAR.....167
EHTIMOLLIK TA'RIFLARI.....170

TAKRORLASH

10-SINF ALGEBRA VA ANALIZ ASOSLARI DARSLIGI UCHUN TA’LIMIY ILOVA

10-SINF ALGEBRA VA ANALIZ ASOSLARI DARSLIGI UCHUN VIDEODARSLAR



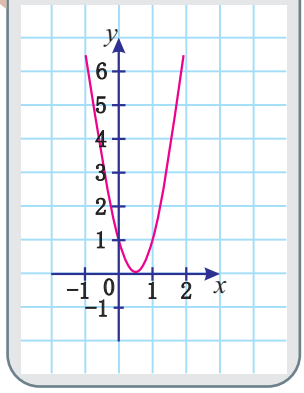
TAKRORLASH

- KVADRAT FUNKSIYA VA UNING GRAFIGI
- KVADRAT TENGSIZLIK VA UNING YECHIMI
- TRIGONOMETRIK AYNIYATLAR
- ARIFMETIK VA GEOMETRIK PROGRESSIYALAR

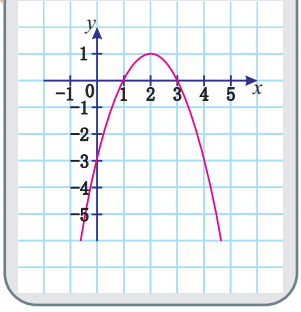
www.idum.uz saytidan yuklab olindi

KVADRAT FUNKSIYA VA UNING GRAFIGI

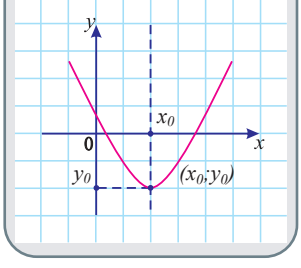
1-rasm



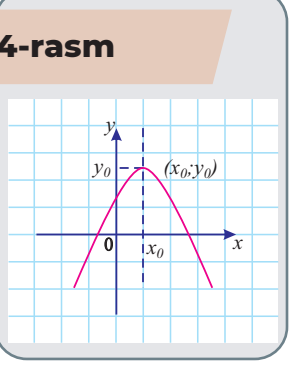
2-rasm



3-rasm



4-rasm



Kvadrat funksiya ta'rifi

Ta'rif.

$y = ax^2 + bx + c$ funksiya *kvadrat funksiya* deyiladi, bunda a, b, c – berilgan haqiqiy sonlar, $a \neq 0$, x – haqiqiy o'zgaruvchi.

Masalan, quyidagi funksiyalar kvadrat funksiyalardir:
$y = 3x^2 + 2x - 1$, $y = -4x^2 - 5x$, $y = 6x^2 - 3$, $y = 4x^2$, $y = 2 - x^2$.

Kvadrat funksiyaning grafigi

1. $y = ax^2 + bx + c$ kvadrat funksiyaning grafigi parabola deb ataladigan egri chiziqdan iborat bo'ladi. 1-rasmda $y = 4x^2 - 4x + 1$ va 2-rasmda $y = -x^2 + 4x - 3$ funksiyalar grafiklari tasvirlangan.
2. $y = ax^2 + bx + c$ parabola tarmoqlari $a > 0$ bo'lganda (3-rasm) ordinata o'qi bo'yicha yuqoriga yo'nalgan, $a < 0$ bo'lganda (4-rasm) esa pastga yo'nalgan bo'ladi.
3. $y = ax^2 + bx + c$ parabola uchining koordinatalari $(x_0; y_0)$ quyidagi $x_0 = -\frac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ yoki $y_0 = -\frac{b^2 - 4ac}{4a}$ formulalar bilan hisoblanadi.
4. $y = ax^2 + bx + c$ parabola o'zining uchi orqali ordinata o'qiga parallel qilib o'tkazilgan to'g'ri chiziqqa nisbatan simmetrik bo'ladi.
5. $y = ax^2 + bx + c$ parabolaning Ox o'qi bilan kesishish nuqtalarining abssissalari kvadrat funksiyaning nollari bo'ladi. Kvadrat funksiya nollarini topish uchun $ax^2 + bx + c = 0$ tenglamani yechish kerak.

$y = ax^2 + bx + c$ kvadrat funksiya grafigini yasash

1. Parabola tarmoqlari yo'nalishini aniqlash.
2. Parabola uchining koordinatalarini $x_0 = -\frac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ formulalar yordamida topish va koordinata tekisligida belgilash.
3. Parabolaning abssissa o'qi bilan kesishish nuqtalarini (nollarini) topish. Agar funksiya nollari mavjud bo'lmasa, u holda odatda parabolaning simmetriya o'qiga nisbatan simmetrik bo'lgan qandaydir ikkita nuqtasini topish mumkin. Masalan, parabolaning abssissalari $x = -2x_0$ va $x = 2x_0$ bo'lgan nuqtalarini yasash mumkin.
4. Parabolaning Oy o'qi bilan kesishish nuqtasi ordinatasini topish va bu nuqtani ordinata o'qida belgilash.
5. Yasalgan nuqtalar uzluksiz silliq egri chiziq bilan tutashtiriladi (agar lozim bo'lsa, parabolaning yana bir nechta nuqtasini yasash mumkin).

Kvadrat funksiya xossalari

1. Aniqlanish sohasi.

$D(y)=(-\infty;\infty)$.

2. Qiymatlar to‘plami.

a) $a>0$ bo‘lsa, $E(y)=[y_0;\infty)$;

b) $a<0$ bo‘lsa, $E(y)=(-\infty;y_0]$.

3. Eng katta va eng kichik qiymatlari.

a) $a>0$ bo‘lsa, $x=x_0$ nuqtada eng kichik qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng katta qiymatga esa erishmaydi;

b) $a<0$ bo‘lsa, $x=x_0$ nuqtada eng katta qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng kichik qiymatga esa erishmaydi.

4. Funksiya nollari.

a) $D>0$ bo‘lsa, ikkita nollarga ega: $x_1=\dfrac{-b+\sqrt{D}}{2a}$ va $x_2=\dfrac{-b-\sqrt{D}}{2a}$;

b) $D=0$ bo‘lsa, funksiya bitta (o‘zaro teng ikkita) nolga ega: $x=\dfrac{-b}{2a}$;

c) $D<0$ bo‘lsa, funksiya nollarga ega emas.

5. Monotonlik oraliqlari.

a) $a>0$ bo‘lsa, $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da kamayuvchi, $[x_0;\infty)$ da o‘suvchi bo‘ladi;

b) $a<0$ da $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da o‘suvchi, $[x_0;\infty)$ da kamayuvchi bo‘ladi, bu yerda $\left(x_0=-\dfrac{b}{2a}\right)$.

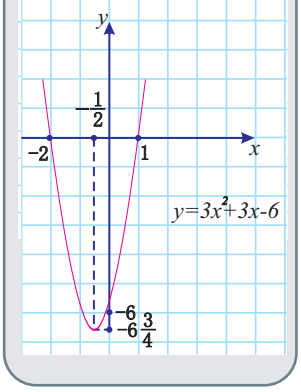
1-misol. $y=3x^2+3x-6$ kvadrat funksiya berilgan bo‘lsin. Uning xossalarini yozing va grafigini chizib ko‘rsating.

Yechish:

1. Aniqlanish sohasi: $D(y)=(-\infty;\infty)$;
2. $a=3>0$ va $x_0=-\dfrac{1}{2}$, $y_0=-6,75$, $E(y)=[-6,75;\infty)$;
3. $x=-\dfrac{1}{2}$ bo‘lganda, eng kichik qiymati $y=-6,75$ ga teng, eng katta qiymatga erishmaydi;
4. $D=81>0$, demak, nollari ikkita: $x_1=1, x_2=-2$;
5. $x\in(-\infty;-2)\cup(1;\infty)$ da $y>0$ va $x\in(-2;1)$ da $y<0$ bo‘ladi;
6. Funksiya juft ham toq ham emas;
7. Funksiya $\left(-\infty;-\dfrac{1}{2}\right]$ da kamayuvchi, $\left[-\dfrac{1}{2};\infty\right)$ da o‘suvchi bo‘ladi;

Funksiya grafigi 5-rasmda ko‘rsatilgan.

5-rasm



MISOLLAR

1. Qaysi funksiyalar kvadrat funksiya bo‘ladi?

a) $y=\frac{1}{3}x+2$ b) $y=-x^2+5x+1$ c) $y=x^2-x^3$ d) $y=x^2$

2. $x=-3$ bo‘lganda, $y=4x^2+7x-5$ funksiyaning qiymati nechaga teng bo‘ladi?

3. x ning qanday qiymatlarida $y=-3x^2+x+1$ funksiyaning qiymati -1 ga teng bo‘ladi?

4. $y=-5x^2+x+\sqrt{7}$ funksiya x ning qanday qiymatlarida aniqlangan?

5. -5 soni $y=x^2-5x$ funksiyaning noli bo‘ladimi?

6. Funksiya grafigini yasang.

a) $y=x^2$ b) $y=-x^2$ c) $y=3x^2$

d) $y=-3x^2-5$ e) $y=x^2-2x$ f) $y=-2x^2+5x$

7. Funksiya nollarini toping.

a) $y=2x^2+5x+2$ b) $y=3x^2+10x+3$ c) $y=-2x^2+x-5$

8. Funksiyaning qiymatlar to‘plamini toping.

a) $y=2x^2-8x+19$ b) $y=-3x^2-12x+1$ c) $y=x^2+2$ d) $y=3-4x^2$

e) $y=3x-x^2$ f) $y=(x-5)^2+3$ g) $y=(x-4)^2-1$ h) $y=3x^2+2x$

9. x ning qanday qiymatlarida funksiya eng katta (yoki eng kichik) qiymat qabul qilishini aniqlang va uni toping.

a) $y=x^2+9x+34$ b) $y=-9x^2-3x+7$

10. t ning qanday qiymatlarida $y=2x^2-tx+8$ funksiyaning nollari yo‘q bo‘ladi?

11. x ning qanday qiymatlarida $y=5x^2-4x-1$ funksiyaning qiymatlari manfiy bo‘ladi?

12. $y=x^2+6x+13$ funksiya manfiy qiymatlarni qabul qiladimi?

13. $y=-x^2-4x-5$ funksiya musbat qiymatlarni qabul qiladimi?

14. $y=6x^2+7x+1$ funksiya grafigini yasang va grafik bo‘yicha x ning funksiyaning qiymatlari musbat, manfiy bo‘ladigan qiymatlarini toping.

15. $y=-x^2+4x-3$ funksiya grafigini yasang. Grafik yordamida funksiyaning o‘sish va kamayish oraliqlarini toping.

16. x ning qanday qiymatlarida $y=x^2-22x+27$ va $y=2x^2-20x+3$ funksiyalarning qiymatlari teng bo‘ladi?

17. Agar parabolaning $(-1;\ 6)$ nuqta orqali o‘tishi va uning uchi $(1;\ 2)$ nuqta ekani ma’lum bo‘lsa, parabolaning tenglamasini toping.

18. $y=x^2+px+q$ parabolaning uchi $A(1;\ -2)$ bo‘lsa, p va q ni toping.

19. Agar $ax^2+bx+c=0$ parabolaning uchi $M(-1;\ -7)$ va parabola ordinatalar o‘qi bilan $N(0;\ -4)$ nuqtada kesishsa, a, b, c larni toping.

20. Agar parabola $A(1;\ 4)$, $B(-1;\ 10)$, $C(2;\ 7)$ nuqtalardan o‘tsa, $y=ax^2+bx+c$ funksiyani toping.

KVADRAT TENGSIZLIK VA UNING YECHIMI

Ta'rif.

Agar tengsizlikning chap qismida kvadrat funksiya, o'ng qismida esa nol turgan bo'lsa, bunday tengsizlik *kvadrat* (bir noma'lumli ikkinchi darajali) *tengsizlik* deyiladi.

$ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c\leq 0$ $(a\neq 0)$ tengsizliklar kvadrat tengsizliklardir, bunda a,b,c – berilgan sonlar, x esa noma'lum son.

Tengsizlikning yechimi deb, noma'lumning shu tengsizlikni to'g'ri sonli tengsizlikka aylantiruvchi barcha qiymatlariga aytiladi.

Tengsizlikni yechish – uning barcha yechimlarini topish yoki ularning yo'qligini ko'rsatish demak,dir.

Kvadrat tengsizlikni quyidagi usullar bilan yechish mumkin:

1-usul. Chiziqli tengsizliklar sistemasiga keltirib yechish

Agar $ax^2+bx+c=0$ kvadrat tenglama ikkita turli ildizga ega bo'lsa, u holda $ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c\leq 0$ kvadrat tengsizliklarni yechishni, kvadrat tengsizlikning chap qismini ko'paytuvchilarga ajratib, birinchi darajali tengsizliklar sistemasini yechishga keltirish mumkin.

1-misol. $x^2-5x+6<0$ tengsizlikni yeching.

Yechish: Tengsizlikning chap tomonini ko'paytuvchilarga ajratamiz:

$$(x-2)(x-3)<0.$$

1-hol: $\begin{cases} x-2>0 \\ x-3<0 \end{cases} \Rightarrow \begin{cases} x>2 \\ x<3 \end{cases} \Rightarrow x\in(2;3).$

2-hol: $\begin{cases} x-2<0 \\ x-3>0 \end{cases} \Rightarrow \begin{cases} x<2 \\ x>3 \end{cases} \Rightarrow x\in\varnothing.$

Javob: (2; 3).

2-usul. Kvadrat tengsizlikni kvadrat funksiya grafigi yordamida yechish

$ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c\leq 0$ kvadrat tengsizliklarni, kvadrat funksiya grafigini sxematik tasvirini yasab, keyin grafik bo'yicha bu funksiya musbat yoki manfiy qiymatlarni qabul qiladigan oraliqlarni topib yechish mumkin.

Kvadrat tengsizlikni grafik usulda yechish uchun:

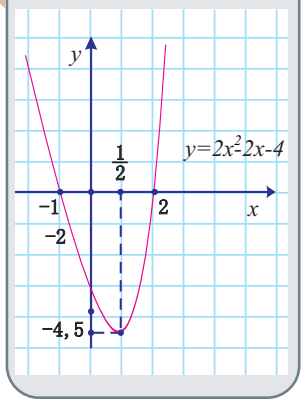
1) parabola tarmoqlari yo'nalishini aniqlash;

2) funksiya nollarini (agar ular mavjud bo'lsa,) topish yoki ular yo'qligini aniqlash;

3) $y=ax^2+bx+c$ funksiya grafigini sxematik tasvirlash;

4) grafik bo'yicha funksiya musbat yoki manfiy qiymatlar qabul qiladigan oraliqlarni aniqlash.

1-rasm



2-misol. $2x^2-2x-4\ge 0$ tengsizlikni kvadrat funksiya grafigi yordamida yeching (1-rasm).

Yechish: $2x^2-2x-4\ge 0$ funksiya grafigini yasaymiz.

Avval parabola uchini topamiz:

$$x_0=-\frac{b}{2a}=-\frac{-2}{4}=\frac{1}{2};\quad y_0=2\left(\frac{1}{2}\right)^2-2\cdot\frac{1}{2}-4=-4{,}5.$$

Parabola nollari:

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{2\pm\sqrt{4+32}}{4}=\frac{2\pm 6}{4}$$

$$x_1=2;\quad x_2=-1$$

Javob: $(-\infty;-1]\cup[2;\infty)$.

Tengsizlikni bu usulda yechishda, parabola uchining koordinatalarini topish umuman olganda shart emas, shuningdek parabolaning *Oy* o‘qi bilan kesishish nuqtalarini grafikda ko‘rsatilishi ham muhim emas. Eng muhimi, parabola yo‘nalishini va funksiya nollari bor yoki yo‘qligini bilishdir.

3-usul. Kvadrat tengsizlikni oraliqlar (intervallar) usuli bilan yechish

Agar biror *(a; b)* oraliqda $y = f(x)$ funksiya grafigini qalamni qog‘ozdan uzmasdan chizish mumkin bo‘lsa, bu funksiya *(a; b)* oraliqda uzluksiz deyiladi.

Masalan, $y = kx + b$, $y = ax^2 + bx + c$ funksiyalar o‘z aniqlanish sohasida uzluksiz funksiyalardir.

Uzluksiz funksiyalarning bir muhim xossasini isbotsiz qabul qilamiz.

Agar *f(x)* funksiya *(a; b)* oraliqda uzluksiz bo‘lsa, va nolga aylanmasa, u holda bu oraliqda funksiyaning qiymatlari bir xil ishoraga ega bo‘ladi, ya’ni shu oraliqda funksiya o‘z ishorasini saqlaydi.

Kvadrat funksiya aniqlanish sohasini chekli sondagi $(-\infty;x_1),(x_1;x_2),(x_2;\infty)$ oraliqlarga ajratish mumkin (bunda $x_1<x_2$). Bu oraliqlarning har birida kvadrat funksiya uzluksiz va nolga aylanmaydi, ya’ni o‘z ishorasini saqlaydi. Bir noma’lumli tengsizliklarni yechishning *oraliqlar usuli* deb ataladigan usul shu faktga asoslangan.

$ax^2+bx+c>0$, $ax^2+bx+c<0$ kvadrat tengsizliklarni yechishda oraliqlar usulining qo‘llanilishini qaraymiz.

1-hol. $D>0$. Bu holatda kvadrat funksiyaning nollari deb ataladigan ikkita haqiqiy x_1 va x_2 $(x_1<x_2)$ sonlar mavjud bo‘ladi. Ular kvadrat funksiyaning aniqlanish sohasini: $(-\infty;x_1),(x_1;x_2),(x_2;\infty)$ oraliqlarga ajratadi va bu oraliqlarning har birida funksiyaning qiymatlari doimiy ishoraga (“+” yoki “ –“) ega bo‘ladi.

Kvadrat funksiya qiymatlarining hosil qilingan oraliqlarning har biridagi ishorasini har xil yo‘l bilan topish mumkin:

1) $y=ax^2+bx+c$ funksiya qiymatining $(-\infty;x_1)$, $(x_2;\infty)$ oraliqlarning har biridagi ishorasi *a* koef-

fitsiyentning ishorasi bilan bir xil bo‘ladi; $(x_1;\ x_2)$ oraliqdagi ishorasi esa a koeffitsiyent ishorasiga qarama-qarshi bo‘ladi;

2) funksiya qiymatlarining ishorasini har bir oraliqdagi “qulay” nuqtada aniqlash mumkin;

3) $y = ax^2 + bx + c$ funksiyani boshqa $y = a(x - x_1)(x - x_2)$ analitik ko‘rinishga o‘tish va har bir oraliqda chiziqli ko‘paytuvchilar ishoralarini topish orqali aniqlash mumkin.

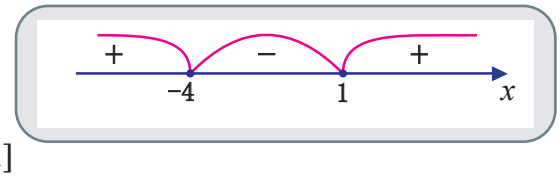
3-misol. $x^2 + 3x - 4 \le 0$ tengsizlikni oraliqlar usuli bilan yeching.

Yechish: Tengsizlikni ko‘paytuvchilarga ajratamiz.

$$(x+4)(x-1) \le 0.$$

va uning nollarini topamiz: – 4 va 1.

Topilgan nuqtalarni sonlar o‘qida belgilaymiz va sonlar o‘qini oraliqlarga ajratamiz. Har bir oraliqning ishorasini aniqlaymiz:



Javob: [–4;1]

2-hol. $D = 0$ bo‘lsin. U holda $y = ax^2 + bx + c$ funksiya faqat bitta x_0 nuqtada nolga aylanadi. x_0 nuqta koordinata o‘qini ikkita $(-\infty; x_0)$ va $(x_0; \infty)$ oraliqlarga ajratadi. Har bir $x \ne x_0$ uchun $y = ax^2 + bx + c$ kvadrat funksiya qiymatlarining ishorasi a koeffitsiyentning ishorasi bilan bir xil bo‘ladi (2, 3-rasmlar).

3-hol. $D < 0$. U holda $y = ax^2 + bx + c$ kvadrat funksiya nollarga ega bo‘lmaydi.

Bu holatda x ning ixtiyoriy qiymatlarida funksiya a koeffitsiyentning ishorasi bilan ustma-ust tushadigan bir xil ishorali qiymatlarni qabul qiladi:

1) agar $a > 0$ bo‘lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c > 0$;

2) agar $a < 0$ bo‘lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c < 0$;.

Ushbu faktlarni bilish va qo‘llay olish muhim:

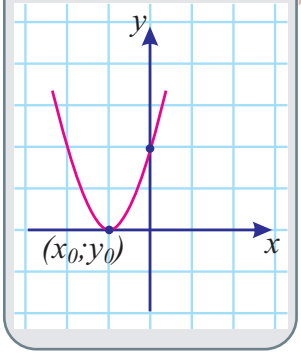
1) $a > 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari barcha haqiqiy sonlardan iborat bo‘ladi (4-rasm);

2) $a > 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsizliklarning yechimlari bo‘sh to‘plamdan iborat (4-rasm).

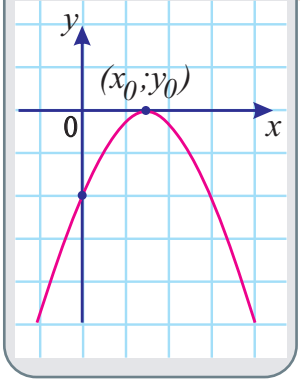
3) $a < 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari bo‘sh to‘plamdan iborat (5-rasm);

4) $a < 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsizliklarning yechimlari barcha haqiqiy sonlardan iborat bo‘ladi (5-rasm).

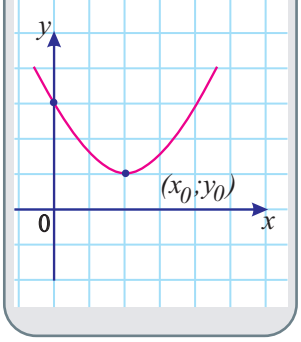
2-rasm



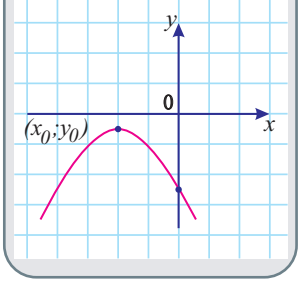
3-rasm



4-rasm



5-rasm



MISOLLAR

1. Quyidagi tengsizliklarni chiziqli tengsizliklar sistemasiga keltirib yeching.

a) $(x+4)(2x-3)>0$ b) $x^2+10x-11<0$

c) $(5x-2)(4x+3)\le 0$ d) $2x^2-5x+2\ge 0$

2. 0, –2, 3 sonlaridan qaysi biri $-4x^2+5x-5>0$ tengsizlikning yechimi bo‘ladi?

3. Tengsizliklar teng kuchlimi?

a) $5x^2>2x$ va $5x>2$ b) $3x^3<7x^2$ va $3x<7$

4. Tengsizliklarni yeching.

a) $x^2>0$ b) $4x^2\ge 0$ c) $x^2<0$

d) $-x^2\le 0$ e) $x^2+7>0$ f) $5x^2+11\le 0$

g) $-x^2-5>0$ h) $3x^2-2x<0$ i) $-4x^2+11x<0$

j) $x^2-9x+20<0$ k) $x^2-10x+25>0$ l) $-x^2+6x-8>0$

m) $3x^2-x+2\ge 0$ n) $-9x^2+24x+20>0$ o) $-7\cdot(3-x)^2>0$

5. Yechimi berilgan oraliqlar bo‘lgan biror kvadrat tengsizlik tuzing.

a) $(-\infty;-3)\cup(6;\infty)$ b)$(-\infty;\infty)$

6. Koordinata o‘qida $x^2+9x\le -14$ tengsizlikning yechimi bo‘lgan kesma uzunligini toping.

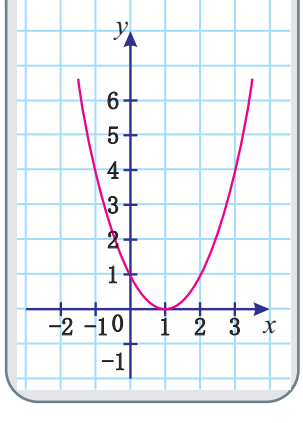
7. Nechta butun son $2x^2+7x-15<0$ tengsizlikning yechimi bo‘ladi?

8. Tengsizlikni oraliqlar usuli bilan yeching.

a) $x^2+5x-6>0$ b) $-x^2+x+2<0$ c) $x^2+3x+7>0$ d) $x^2+3x+7\le 0$

e) $-2x^2+5x+3>0$ f) $6x^2-x-2<0$ g) $2x^2+5x+9\le 0$ h) $49x^2-28x+4\le 0$

6-rasm



9. Tengsizliklarni yeching.

a) $8x^2+3x-5\ge 0$ b) $5x^2-12x+8\le 0$

c) $49x^2-70x+25>0$ d) $(2x^2+3x+4)(x+3)\ge 0$

e) $(7+6x-x^2)(3x-5)<0$

10. Tengsizlikni kvadrat funksiya grafigi yordamida yeching.

a) $2x^2+5x-3>0$ b) $4x^2-9x-90>0$

11. Tengsizlikning barcha butun yechimlari yig‘indisini toping.

a) $2x^2-9x+4<0$ b) $\frac{x-1}{4}+\frac{3-2x}{2}>\frac{3x+x^2}{8}$

c) $(5x+7)(x-2)\le 21x^2-11x+3$

12. 6-rasmda $y=ax^2+bx+c$ funksiya grafigi tasvirlangan. Quyidagi tengsizliklarning yechimini toping.

a) $ax^2+bx+c>0$ b) $ax^2+bx+c\le 0$

13. $3x(x-2)<2x(x+4)-(x-16)\le 0$ tengsizlikning $[0;9]$ kesmaga tegishli bo'lgan nechta butun yechimi bor?

14. $y=-x^2+4x-3$ funksiya grafigi yordamida, quyidagi tengsizliklarning yechimini toping.

a) $-x^2+4x-3>0$ b) $-x^2+4x-3\ge 0$ c) $-x^2+4x-3<0$ d) $-x^2+4x-3\le 0$

15. a ning qanday qiymatlarida $ax^2+2ax+4=0$ tenglama haqiqiy ildizlarga ega bo'lmaydi?

16. Tengsizlikni yeching: $(x-1)^2(x^2-2)<(x-1)^2(6-2x)$

17. $f(x)=(x-1)^4(x+1)^3x^2$ funksiya berilgan.

a) $f(x)=0$ b) $f(x)\le 0$ c) $f(x)\ne 0$ d) $f(x)\ge 0$

bo'ladigan x ning barcha qiymatlarini toping.

18. Tengsizliklarni yeching:

a) $x^2-2(b-c)x+a^2>0$, bunda a,b,c lar uchburchakning tomonlari;

b) $x^2+(a^2+b^2-c^2)x+a^2b^2>0$, bunda a,b,c lar uchburchakning tomonlari.

19. Agar $a^2+12b<0$ bo'lsa, $3x^2-b\le ax$ ni yeching.

20. Agar $b>0,05a^2$ bo'lsa, $5x^2-ax+b>0$ ni yeching.

21. Agar, b^2-4ac va $a+c>b$ bo'lsa, $ax^2+bx+c\le 0$ ni yeching.

22. c ning qanday qiymatlarida $y=cx^2-x+c$ va $y=cx+1-c$ funksiyalar grafiklari umumiy nuqtaga ega bo'lmaydi?

23. p ning qanday qiymatlarida $y=px^2-24x+1$ va $y=12x^2-2px-1$ funksiyalar kesishmaydi?

24. a ning qanday qiymatlarida $x^2+3x+a=0$ tenglamaning ildizlari $\frac{x_1}{x_2}+\frac{x_2}{x_1}+1>0$ shartni qanoatlantiradi?

25. b ning qanday qiymatlarida $x^2-2bx+b+6=0$ tenglamaning:

a) ildizlari manfiy ishorali; b) ildizlari musbat ishorali;

c) ildizlari har xil ishorali bo'ladi?

26. a ning qanday qiymatlarida barcha haqiqiy sonlar tengsizlikni qanoatlantiradi?

a) $x^2-(a+2)x+8a+1>0$ b) $\frac{1}{24}x^2+ax-a+1>0$

c) $ax^2+4x+a+3<0$ d) $ax^2-4ax-3\le 0$

27. b ning qanday qiymatlarida tengsizlik yechimga ega emas?

a) $x^2+2bx+1<0$ b) $bx^2+4bx+5<0$ c) $bx^2+(2b+3)x+b-1\ge 0$

TRIGONOMETRIK AYNIYATLAR

Asosiy trigonometrik ayniyatlar

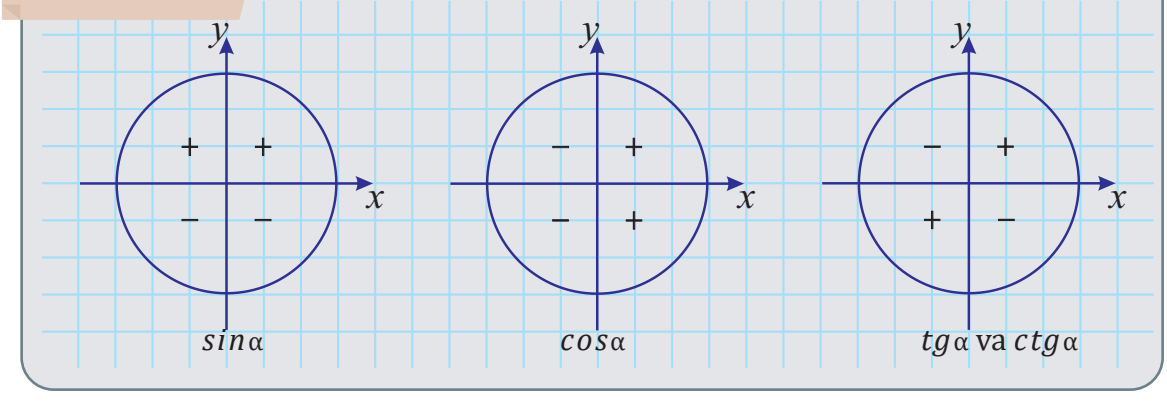
1. $\sin^2\alpha+\cos^2\alpha=1$
2. $tg\alpha=\frac{sin\alpha}{cos\alpha}$, $\cos\alpha\neq 0$
3. $ctg\alpha=\frac{cos\alpha}{sin\alpha}$, $\sin\alpha\neq 0$
4. $tg\alpha\cdot ctg\alpha=1$
5. $1+tg^2\alpha=\frac{1}{cos^2\alpha}$, $\cos\alpha\neq 0$
6. $1+ctg^2\alpha=\frac{1}{sin^2\alpha}$, $\sin\alpha\neq 0$

Ba'zi burchaklarning sinusi, kosinusi, tangensi va kotangensining qiymatlari

α	0^0 (0)	$30^\circ\left(\frac{\pi}{6}\right)$	$45^\circ\left(\frac{\pi}{4}\right)$	$60^\circ\left(\frac{\pi}{3}\right)$	$90^\circ\left(\frac{\pi}{2}\right)$	$180^\circ(\pi)$
sinα	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
cosα	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1
tgα	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Mavjud emas	0
ctgα	Mavjud emas	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Mavjud emas

Sinus, kosinus, tangens va kotangensning ishoralari

1-rasm



a va (–α) burchaklarning sinusi, kosinusi, tangensi va kotangensi

1. sin(–α) = – sinα
2. cos(–α) = cosα
3. tg(–α) = –tgα
4. ctg(–α) = –ctgα

Keltirish formulalari

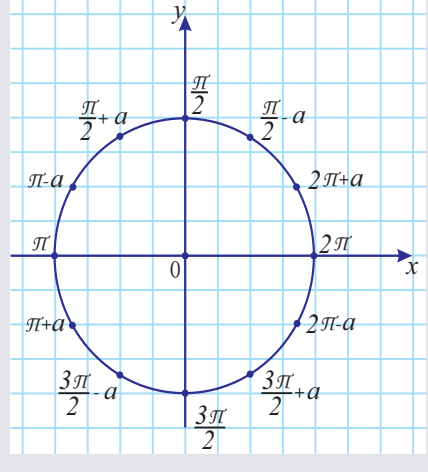
	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
$\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$
$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$	$\cos\alpha$
$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$
$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$

Keltirish formulalaridagi ushbu *(mnemonik)* qonuniyatga e'tibor qarating: agar α ni I chorakka tegishli deb olsak, $\pi\pm\alpha$, $2\pi\pm\alpha$ burchaklar uchun funksiya nomi almashmaydi, $\frac{\pi}{2}\pm\alpha$, $\frac{3\pi}{2}\pm\alpha$ burchaklar uchun esa sinus kosinusga, kosinus sinusga, tangens kotangensga, kotangens tangensga almashadi.

Masalan, $sin\left(n\cdot\frac{\pi}{2}\pm\alpha\right)$ ni qarasak $(n\in N)$, $\frac{\pi}{2}$ lar soni n – juft bo'lsa, funksiya nomi almashmaydi; $\frac{\pi}{2}$ lar soni n - toq bo'lsa, funksiya nomi almashadi. Ishorani aniqlash esa $n\cdot\frac{\pi}{2}\pm\alpha$ burchak (bunda α I chorak burchagi) qaysi chorakka tegishli ekani va bu chorakda sinusning ishorasi qandayligiga qarab aniqlanadi.

Masalan, $sin\left(\frac{3\pi}{2}-\alpha\right)=-cos\alpha$, chunki $\frac{\pi}{2}$ lar soni miqdori 3 ga teng, $\frac{3\pi}{2}-\alpha$ burchak III chorakka tegishli va III chorakda $sin\left(\frac{3\pi}{2}-\alpha\right)$ - manfiy. $\frac{\pi}{2}$ lar soni miqdori deyish o'rniga, 90^0 lar soni miqdori deb gapirish mumkin.

2-rasm



1-misol. Hisoblang.

a) $\sin855^\circ=\sin(9\cdot90^\circ+45^\circ)\cos45^\circ=\frac{\sqrt{2}}{2}$

b) $\cos2025^\circ=\cos(22\cdot90^\circ+45^\circ)=-\cos45^\circ=-\frac{\sqrt{2}}{2}$

c) $tg1680^\circ=tg(18\cdot90^\circ+60^\circ)=tg60^\circ=\sqrt{3}$

d) $ctg1200^\circ=ctg(13\cdot90^\circ+30^\circ)=-tg30^\circ=-\frac{\sqrt{3}}{3}$

Qo‘shish formulalari

1. $sin(\alpha+\beta)=sin\alpha cos\beta+cos\alpha sin\beta$
2. $sin(\alpha-\beta)=sin\alpha cos\beta-cos\alpha sin\beta$
3. $cos(\alpha+\beta)=cos\alpha cos\beta-sin\alpha sin\beta$
4. $cos(\alpha-\beta)=cos\alpha cos\beta+sin\alpha sin\beta$
5. $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$
6. $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$
7. $ctg(\alpha+\beta)=\dfrac{ctg\alpha ctg\beta-1}{ctg\alpha+ctg\beta}$
8. $ctg(\alpha-\beta)=\dfrac{ctg\alpha ctg\beta+1}{ctg\alpha-ctg\beta}$

Ikkilangan burchak formulalari

1. $\sin 2\alpha=2\sin\alpha\cdot\cos\alpha$
2. $\cos 2\alpha=1-2\sin^2\alpha$
3. $\cos 2\alpha=2\cos^2\alpha-1$
4. $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$
5. $tg2\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$
6. $ctg2\alpha=\dfrac{ctg^2\alpha-1}{2ctg\alpha}$

Trigonometrik funksiyalar yig‘indisi va ayirmasini ko‘paytmaga almashtirish formulalari

1. $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
2. $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cdot\cos\dfrac{\alpha+\beta}{2}$
3. $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
4. $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\cdot\sin\dfrac{\alpha-\beta}{2}$
5. $tg\alpha+tg\beta=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$
6. $tg\alpha-tg\beta=\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$
7. $ctg\alpha+ctg\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$
8. $ctg\alpha-ctg\beta=\dfrac{-\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$

Trigonometrik funksiyalar ko‘paytmasini yig‘indiga almashtirish formulalari

1. $\sin\alpha\cdot\cos\beta=\dfrac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$
2. $\cos\alpha\cdot\cos\beta=\dfrac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$
3. $\sin\alpha\cdot\sin\beta=\dfrac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$

Daraja pasaytirish formulalari

1. $\sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$
2. $\cos^2\alpha=\dfrac{1+\cos 2\alpha}{2}$
3. $tg^2\alpha=\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}$
4. $ctg^2\alpha=\dfrac{1+\cos 2\alpha}{1-\cos 2\alpha}$

Yarim argumentning trigonometrik funksiyalari

1. $cos^2\frac{\alpha}{2}=\frac{1+cos\alpha}{2}$
2. $sin^2\frac{\alpha}{2}=\frac{1-cos\alpha}{2}$
3. $tg^2\frac{\alpha}{2}=\frac{1-cos\alpha}{1+cos\alpha}$
4. $ctg^2\frac{\alpha}{2}=\frac{1+cos\alpha}{1-cos\alpha}$
5. $tg\frac{\alpha}{2}=\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}$
6. $ctg\frac{\alpha}{2}=\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}$

sinα, cosα va tgα ni $tg\frac{\alpha}{2}$ orqali ifodalash formulalari

1. $\sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
2. $\cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
3. $tg\alpha=\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$

Yordamchi burchak kiritish

asinx + ***bcosx*** ifodani yordamchi burchak kiritish usuli bilan, sinus yoki kosinus orqali ifodalash mumkin. Avval $\sqrt{a^2+b^2}$ qo‘shimcha ifoda kiritib:

$a\sin x+b\cos x=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cdot\sin x+\frac{b}{\sqrt{a^2+b^2}}\cdot\cos x\right)$ ko‘rinishda yozamiz

$\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2+\left(\frac{b}{\sqrt{a^2+b^2}}\right)^2=1$ bo‘lgani uchun, shunday φ burchak topiladiki,

$\frac{a}{\sqrt{a^2+b^2}}=\cos\varphi$, $\frac{b}{\sqrt{a^2+b^2}}=\sin\varphi$ bo‘ladi. Shunga ko‘ra,

$a\sin x+b\cos x=\sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cdot\sin x+\frac{b}{\sqrt{a^2+b^2}}\cdot\cos x\right)=$

$=\sqrt{a^2+b^2}\,(\cos\varphi\cdot\sin x+\sin\varphi\cdot\cos x)=\sqrt{a^2+b^2}\,\sin(x+\varphi)$

x ning ixtiyoriy qiymatida $-\sqrt{a^2+b^2}\le asinx+bcosx\le\sqrt{a^2+b^2}$ tengsizlik o‘rinli, ya’ni $asinx+bcosx$ ifodaning eng katta qiymati $\sqrt{a^2+b^2}$ ga, eng kichik qiymati esa $-\sqrt{a^2+b^2}$ ga teng.

(φ burchakni $tg\varphi=\frac{\sin\varphi}{\cos\varphi}=\frac{b}{a}$ tenglikdan topish mumkin, bunda $\varphi=arctg\frac{b}{a}$).

MISOLLAR

1. Agar $tg\alpha = \frac{3}{2}$ bo'lsa, $\frac{2sin\alpha + 5cos\alpha}{3sin\alpha - 4cos\alpha}$ ni toping.

2. Agar $tg\alpha = -\sqrt{5}$ va $\frac{\pi}{2} < \alpha < \pi$ bo'lsa, $sin\alpha$ ni toping.

3. Agar $ctg\alpha = -\frac{3}{4}$ va $\frac{3\pi}{2} < \alpha < 2\pi$ bo'lsa, $cos\alpha$ ni toping.

4. Soddalashtiring.

a) $\frac{2sin^2\alpha - 1}{2cos^2\alpha - 1}$ b) $\frac{ctg\alpha\,(2sin^2\alpha + tg\alpha)}{(sin\alpha + cos\alpha)^2}$ c) $\frac{1 - cos^2\alpha}{sin^4\alpha} - ctg^2\alpha$ d) $2 - \frac{1 - sin^4\alpha}{cos^2\alpha}$

5. Hisoblang.

a) $4cos150° - sin240° - 3tg210°$ b) $2cos135° + tg60° - ctg240°$

c) $sin300° - 3cos135° + 2cos210°$ d) $tg150° - ctg315° + 5sin135°$

6. Hisoblang.

a) $tg\frac{7\pi}{6} - 2tg\frac{5\pi}{3} + 3tg\frac{11\pi}{6}$ b) $2cos\frac{4\pi}{3} + sin\frac{5\pi}{4} - cos\frac{4\pi}{3}$

c) $20ctg\frac{3\pi}{2} - sin\frac{2\pi}{3} + cos\frac{4\pi}{3}$ d) $ctg\frac{7\pi}{4} + 2ctg\frac{4\pi}{3} - 2cos\frac{5\pi}{6}$

7. Soddalashtiring.

a) $\frac{1 - tg(360° - \alpha)tg\alpha}{tg(270° - \alpha) + tg\alpha}$ b) $\frac{\cos(90° + \alpha) + \sin(90° - \alpha)}{\cos(270° - \alpha) + \cos(180° + \alpha)}$

8. Soddalashtiring.

a) $\frac{\sin\left(\frac{\pi}{2} + \beta\right)}{tg^2\left(\frac{\pi}{2} - \frac{\beta}{2}\right) - ctg^2\left(\frac{\pi}{2} + \frac{\beta}{2}\right)}$ b) $\frac{\cos\left(\frac{3\pi}{2} - 2a\right) + \sin\left(\frac{3\pi}{2} + 2a\right)}{\cos\left(\frac{3\pi}{2} + 2a\right) + \sin\left(\frac{3\pi}{2} - 2a\right)}$

9. Ayniyatni isbotlang.

a) $\frac{sin(\pi - 2\alpha) - 2sin\left(\frac{\pi}{2} - \alpha\right)}{sin^2(\pi + \alpha) - cos\left(\frac{3\pi}{2} + \alpha\right)} = 2ctg\alpha$ b) $\frac{sin^4\left(\alpha - \frac{\pi}{2}\right) - cos^2(2\alpha + \pi)}{1 - 3\cos(2\alpha + \pi)} = \frac{sin^2\alpha}{2}$

10. Hisoblang.

a) $\sin(-43°)\cos 88° + \cos(-43°)\sin 88°$ b) $\cos 11°\cos 19° - \sin 19°\sin 11°$

11. Hisoblang.

a) $\dfrac{1+tg33°tg78°}{tg78°-tg33°}$

b) $\sin\dfrac{2\pi}{7}\cos\dfrac{3\pi}{14}+\cos\dfrac{2\pi}{7}\sin\dfrac{3\pi}{14}$

12. Hisoblang.

a) $cos\left(-\dfrac{19\pi}{36}\right)\cos\dfrac{7\pi}{9}-\sin\dfrac{7\pi}{9}\sin\left(-\dfrac{19\pi}{36}\right)$

b) $\dfrac{1-tg\dfrac{\pi}{11}tg\dfrac{5\pi}{66}}{tg\dfrac{5\pi}{66}+tg\dfrac{\pi}{11}}$

13. Soddalashtiring.

a) $\cos(\alpha-\beta)-sin\alpha sin\beta$

b) $\sin(\alpha-\beta)+\sin(\alpha+\beta)$

c) $\sin 4\alpha\cos\alpha-\cos 4\alpha\sin\alpha$

d) $\cos\alpha\cos 2\alpha+\sin 2\alpha\sin\alpha$

e) $\dfrac{1}{2}\cos\alpha+\dfrac{\sqrt{3}}{2}\sin\alpha$

f) $\dfrac{1}{\sqrt{2}}\cos\alpha-\dfrac{\sqrt{2}}{2}\sin\alpha$

14. a) $\cos\alpha=\dfrac{2\sqrt{2}}{3}$ va $\dfrac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $\cos\left(\alpha+\dfrac{\pi}{4}\right)$ ni toping.

b) $\cos\alpha=-\dfrac{1}{\sqrt{3}}$ va $\pi<\alpha<\dfrac{3\pi}{2}$ bo‘lsa, $\sin\left(\alpha+\dfrac{\pi}{6}\right)$ ni toping.

15. Hisoblang.

a) $\dfrac{6\sin10°\cos10°}{\sin20°}$

b) $\dfrac{\sin88°}{\sin22°\cos22°\cos44°}$

c) $sin\dfrac{\pi}{12}\left(2sin^2\dfrac{\pi}{24}-1\right)$

d) $\left(\left(cos\dfrac{\pi}{16}+sin\dfrac{\pi}{16}\right)^2-1\right)cos\dfrac{\pi}{8}$

16. a) Agar $cos\alpha=0,4$ bo‘lsa, $cos2\alpha$ ni toping.

b) Agar $sin\alpha=-0,7$ bo‘lsa, $cos2\alpha$ ni toping.

17. a) Agar $cos\alpha=\dfrac{2\sqrt{2}}{3}$ va $\dfrac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $sin2\alpha$ ni toping.

b) Agar $sin\alpha=\dfrac{1}{5}$ va $\dfrac{\pi}{2}<\alpha<\pi$ bo‘lsa, $sin2\alpha$ ni toping.

18. Soddalashtiring:

a) $2\cos^2\dfrac{\alpha}{2}(2cos\alpha-1)$

b) $\dfrac{sin2\alpha}{1+cos2\alpha}$

19. Agar $tg\alpha=-2$ bo‘lsa, $sin2\alpha, cos2\alpha, tg2\alpha, ctg2\alpha$ ni toping:

a) a ning qanday qiymatlarida, $tgx=\dfrac{a+1}{a-1}$ tenglik o‘rinli bo‘ladi;

ARIFMETIK VA GEOMETRIK PROGRESSIYA

Arifmetik progressiya

1. $a_{n+1} = a_n + d$, $n \in N$;
2. $a_n = a_1 + (n-1)d$, $n \in N$;
3. $a_n = a_k + (n-k)d$, $n,k \in N$, $n>k$;
4. $a_n = \frac{a_{n-1} + a_{n+1}}{2}$, $n \in N$;
5. $a_n = \frac{a_{n-k} + a_{n+k}}{2}$, $n,k \in N$, $n>k$;
6. $\{a_n\}$ – arifmetik progressiya hadlari uchun $a_n + a_m = a_k + a_l$ tenglik o‘rinli, bunda $n+m=k+l$;
7. $a_1 + a_n = a_{1+k} + a_{n-k}$, $n,k \in N$ va $n>k$;
8. $S_n = \frac{(a_1 + a_n)n}{2}$, agar $d = 0$ bo‘lsa, $S_n = a_1 \cdot n$;
9. $S_n = \frac{(2a_1 + (n-1)d)n}{2}$.

Geometrik progressiya

1. $b_{n+1} = b_1 \cdot q$, $n \in N$;
2. $b_n = b_1 \cdot q^{n-1}$, $n \in N$;
3. $b_n = b_k \cdot q^{n-k}$, $n,k \in N$ va $n>k$;
4. $b_n^2 = b_{n-1} \cdot b_{n+1}$, $n \in N$;
5. $b_n^2 = b_{n-k} \cdot b_{n+k}$, $n,k \in N$, $n>k$;
6. $\{b_n\}$ geometrik progressiya hadlari uchun $b_n \cdot b_m = b_k \cdot b_l$ tenglik o‘rinli, bunda $n+m=k+l$;
7. $b_1 \cdot b_n = b_{1+k} \cdot b_{n-k}$, $n,k \in N$ va $n>k$;
8. $S_n = \frac{b_1(1-q^n)}{1-q}$, $q \neq 1$, agar $q = 1$ bo‘lsa, $S_n = b_1 \cdot n$;
9. $S_n = \frac{b_n q - b_1}{q-1}$, $q \neq 1$;
10. Cheksiz kamayuvchi geometrik progressiya (barcha hadlari) yig‘indisi

$$S_n = \frac{b_1}{1-q}, \quad |q| < 1, q \neq 0.$$

MISOLLAR

1. Arifmetik progressiyaning n-hadining formulasini yozing.

a) $a_1 = 5,\ a_2 = -5$ b) $a_1 = -3,\ a_6 = 12$ c) $a_1 = 6,\ a_{10} = 33$

2. Arifmetik progressiyada amallarni bajaring.

a) $a_5 = 2,\ a_{40} = 142$ bo'lsa, a_{13} ni toping

b) $a_{25} - a_{20} = 10,\ a_{16} = 13$ bo'lsa, a_{10} ni toping

c) $a_7 = -5,\ a_{32} = 70$ bo'lsa, a_1 va d ni toping

d) $a_{14} = 5,\ a_{12} = 1$ bo'lsa, a_{13} ni toping

3. Arifmetik progressiyada amallarni bajaring.

a) $a_1 = -3,\ a_3 \cdot a_7 = 24$ bo'lsa, S_{12} ni toping b) $a_2 + a_9 = 20$ bo'lsa, S_{10} ni toping

c) $a_3 + a_6 = 19$ bo'lsa, S_8 ni toping d) $S_4 = -28,\ S_6 = 58$ bo'lsa, S_{16} ni toping

4. Arifmetik progressiyada $S_n = 3n^2 + n$ bo'lsa, a_1 va d ni toping.

5. Geometrik progressiyada amallarni bajaring.

a) $b_1 = \frac{1}{2},\ q = \frac{1}{2}$ bo'lsa, b_7 ni toping b) $b_9 = -1,\ q = -1$ bo'lsa, b_1 va b_{17} ni toping

c) $b_4 = 8,\ b_8 = 128$ bo'lsa, b_1 va q ni toping d) $b_1 = 18,\ q = \frac{1}{9}$ bo'lsa, b_2 ni toping

6. Geometrik progressiyada amallarni bajaring.

a) $b_1 = 24,\ b_2 = 36$ bo'lsa, q ni toping b) $b_5 = 36,\ b_7 = 144$ bo'lsa, b_6 ni toping

c) $b_6 = \frac{1}{486},\ b_8 = \frac{1}{4374}$ bo'lsa, b_7 ni toping

7. Geometrik progressiyada amallarni bajaring.

a) $b_1 = 3,\ q = 5$ bo'lsa, S_4 ni toping b) $b_2 = 8,\ b_3 = 4$ bo'lsa, S_6 ni toping

c) $b_1 = 5,\ q = 3,\ S_n = 200$ bo'lsa, n ni toping d) $b_1 = -2,\ b_6 = -486$ bo'lsa, S_6 ni toping

8. Geometrik progressiyada $q = -\frac{1}{2},\ S_8 = \frac{85}{64}$ bo'lsa, b_1 ni toping.

9. Agar $a_1 = -3$ va $d = 6$ bo'lsa, arifmetik progressiyaning saksoninchi hadini toping.

10. 2, 6, 10, 14, 18, ... ketma-ketlik arifmetik progressiya tashkil qiladi. Uning n-hadi formulasini yozing.

11. Arifmetik progressiyaning to'rtinchi va oltinchi hadlari mos ravishda 16 va 19 ga teng bo'lsa, birinchi hadini toping.

12. Dastlabki 25 ta natural sonlar yig'indisini toping.

13. (a_n) arifmetik progressiyada $a_3 + a_7 = 5$ va $a_4 = 1$ bo'lsa, uning dastlabki o'nta hadlari yig'indisini toping.

14. Arifmetik progressiyada $a_1 = -20{,}7$, $d = 1{,}8$ bo‘lsa, qaysi nomerli hadidan boshlab progressiyaning barcha hadlari musbat bo‘ladi?

15. Arifmetik progressiyada $a_{12} + a_{15} = 20$ bo‘lsa, S_{26} ni toping.

16. Arifmetik progressiyada $a_2 + a_6 = 44$, $a_5 - a_1 = 20$ bo‘lsa, a_{100} ni toping.

17. Arifmetik progressiyaning uchinchi va to‘qqizinchi hadlari yig‘indisi 8 ga teng. Shu progressiya dastlabki o‘n bitta hadlari yig‘indisini toping.

18. Geometrik progressiyaning birinchi hadi 5, oltinchi hadi 1215 ga teng. Shu progressiya maxrajini toping.

19. Agar geometrik progressiyada $b_1=3$ va $q=-2$ bo‘lsa, b_8 ni toping.

20. Agar geometrik progressiyada $b_2=4$ va $b_3=6$ bo‘lsa, b_7 ni toping.

21. Geometrik progressiyada $b_5 = \sqrt[3]{2}$. Shu progressiya dastlabki to‘qqizta hadlari ko‘paytmasini toping

22. Agar geometrik progressiyada $b_1=3$, $q=2$ bo‘lsa, dastlabki S_6 ta hadi yig‘indisini toping.

23. Geometrik progressiyada $b_1=2$, $q=3$ bo‘lsa, S_8 ni toping.

24. Geometrik progressiyada $b_1=4$, $q=\frac{1}{2}$ bo‘lsa, dastlabki 10 ta hadi yig‘indisini toping.

25. Geometrik progressiyada $S_4=10+\frac{5}{8}$, $S_5=42+\frac{5}{8}$, $b_1=\frac{1}{8}$ bo‘lsa, q ni toping.

26. Geometrik progressiyada $b_1=1$ va $b_3+b_5=90$ bo‘lsa, q ni toping.

27. Uchta x, y va 12 sonlari kamayuvchi geometrik progressiya tashkil qiladi. Agar 12 ning o‘rniga 9 qo‘yilsa, uchta son arifmetik progressiya tashkil qiladi. $x + y$ ni toping.

28. Geometrik progressiyada $b_2 \cdot b_4 \cdot b_6 = 216$ va $b_3 = 12$ shu progressiya dastlabki oltita hadlari yig‘indisini toping.

29. Cheksiz kamayuvchi geometrik progressiyada $b_1=8$, $q=\frac{1}{2}$ bo‘lsa, uning yig‘indisini toping.

30. $12, 4, \frac{4}{3}, \ldots$ cheksiz kamayuvchi geometrik progressiya yig‘indisini toping.

31. Cheksiz kamayuvchi geometrik progressiya yig‘indisi 150 ga teng. Agar $q=\frac{1}{3}$ bo‘lsa, b_1 ni toping.

32. Cheksiz kamayuvchi geometrik progressiyada $b_1=\frac{1}{4}$, $S=16$ bo‘lsa, q ni toping.

33. Cheksiz kamayuvchi geometrik progressiyaning toq o‘rinlarda turgan hamma hadlarining yig‘indisi 36 ga teng. Juft o‘rinlarda turgan hamma hadlarining yig‘indisi 12 ga teng. Shu progressiya maxrajini va ikkinchi hadini toping.

34. Cheksiz kamayuvchi geometrik progressiyaning hamma hadlari musbat, birinchi hadi 4 ga teng, uchinchi hadi bilan beshinchi hadining ayirmasi $\frac{32}{81}$ bo‘lsa, progressiya yig‘indisini toping.

35. Cheksiz kamayuvchi geometrik progressiyaning birinchi va to‘rtinchi hadlarining yig‘indisi 54, ikkinchi va uchinchi hadlarining yig‘indisi 36 ga teng. Shu progressiyaning yig‘indisi topilsin.

36. 5, 9, 13, 17, ... arifmetik progressiyaning nechta hadi olinsa, yig‘indi 10877 chiqadi?



1-BOB. ELEMENTAR FUNKSIYALAR

- FUNKSIYA. FUNKSIYANING BERILISH USULLARI
- FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO‘PLAMI
- FUNKSIYALAR USTIDA ARIFMETIK AMALLAR
- MURAKKAB, TESKARI, DAVRIY FUNKSIYA
- FUNKSIYA XOSSALARI
- FUNKSIYA GRAFIKLARI USTIDA SODDA ALMASHTIRISHLAR
- CHIZIQLI VA KVADRATIK MODELLASHTIRISH
- LOYIHA ISHI

FUNKSIYA. FUNKSIYANING BERILISH USULLARI

Funksiya

Tabiat, ishlab chiqarish, iqtisodiyot va boshqa sohalarda qaraladigan miqdorlar orasidagi bog‘lanishlarni o‘rganishda ***funksiya*** deb ataluvchi tushunchaning ahamiyati nihoyatda kattadir. Funksiya va unga bog‘liq tushunchalarni bayon etamiz.

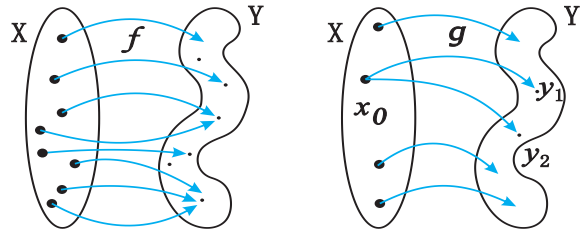
X va Y – sonli to‘plamlar bo‘lsin. Har bir $x \in X$ nuqtaga yagona $y \in Y$ nuqtani mos qo‘yuvchi qonuniyat ***funksiya*** deyiladi.

Funksiyani aniqlovchi qonuniyatlar f, g, ... harflar orqali belgilanadi. $y = f(x)$ yozuv f qonuniyat $x \in X$ nuqtaga $y \in Y$ nuqtani mos qo‘yganini anglatadi va bu holda X to‘plamning nuqtalarini Y to‘plamning nuqtalariga mos qo‘yuvchi f funksiya berilgan deyiladi.

Bunda x – *erkli o‘zgaruvchi* yoki *argument*, y esa – *erksiz o‘zgaruvchi* yoki *funksiya* deb yuritiladi. f funksiya odatda $y = f(x)$, yoki $f(x)$ ko‘rinishlarda ifodalanadi.

Quyida ayrim funksiyalar keltirilgan:

1-rasm



f qonuniyat funksiya bo‘ladi: X ning har bir x elementiga Y dan yagona y element mos qo‘yilgan.

g qonuniyat funksiya emas: $x_0 \in X$ elementga ikkita $y_1, y_2 \in Y$ elementlar mos qo‘yilgan.

Funksiya bo‘ladigan (f) va bo‘lmaydigan (g) qonuniyatlar.

1) *To‘g‘ri proporsionallik*: $y = kx$;

2) *Chiziqli funksiya*: $y = kx + b$;

3) *Kvadratik funksiya*: $y = ax^2 + bx + c$;

4) *Darajali funksiya*: $y = x^n$;

5) *Irratsional funksiya*: $y = \sqrt[n]{x}$;

6) *Teskari proporsionallik*: $y = \dfrac{k}{x}$ (bu yerda $k \neq 0$);

7) *Sonning moduli*: $|x| = \begin{cases} x, & agar\ x > 0, \\ 0, & agar\ x = 0, \\ -x, & agar\ x < 0. \end{cases}$

Funksiyaning berilish usullari

Funksiyalar quyidagi usullarda berilishi mumkin:

1. Funksiya berilishining ***analitik usuli.*** Agar funksiya bitta yoki bir nechta formula yoki tenglamalar bilan berilgan bo‘lsa, u holda bu funksiya *analitik usulda berilgan* deyiladi.

   Masalan, moddiy nuqtaning harakat tenglamasi $s = 20 - 5t + \dfrac{1}{4}t^2$ analitik usulda berilgan funksiya bo‘ladi.

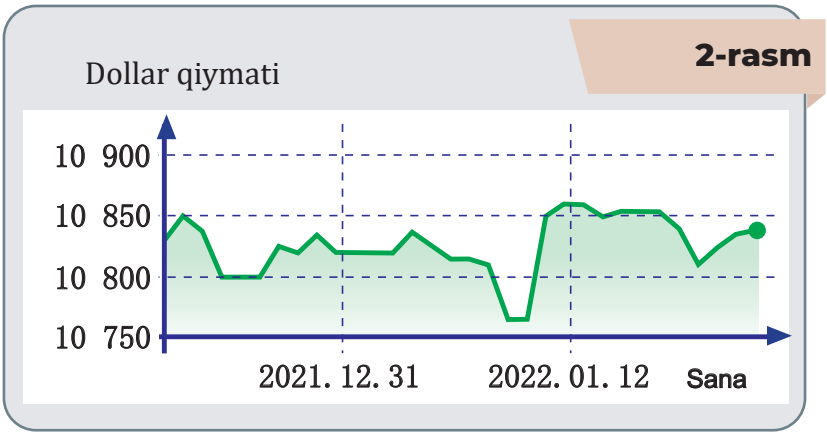
2. Funksiya berilishining ***jadval usuli*** odatda amaliy tajribalarda o‘zgaruvchilar orasidagi o‘zaro bog‘liqlikni o‘rnatadi. Masalan, haroratning kunlik o‘zgarishi jadval usulda berilishi mumkin.

Bu yerda kun soatlari – erkli o‘zgaruvchi (ya’ni argument), harorat esa – erksiz o‘zgaruvchi (ya’ni funksiya) bo‘ladi. Toshkent shahrida 2022-yil 20–26-yanvar kunlari havo haroratining haftalik o‘zgarishi quyidagi jadvalda keltirilgan.

Sana		2022. 01.20	2022. 01.21	2022. 01.22	2022. 01.23	2022. 01.24	2022. 01.25	2022. 01.26
Harorat, t °C	Kunduzi	13	9	3	4	6	7	8
	Kechasi	–2	–3	–1	–2	–3	–4	–3

3. Ayrim amaliy ishlarda o‘zgaruvchilarning bog‘liqligi ***grafik usulda*** beriladi. Masalan, so‘mning dollarga nisbatan qiymatining oylik, yillik o‘zgarishi grafik usulda ifodalanishi mumkin. Bu yerda sanalar – argument, so‘mning dollarga nisbatan qiymati esa – funksiya bo‘ladi.

2-rasm



4. Funksiya ***matn usulida*** berilishi ham mumkin. Masalan: 4 nafar a’zosi bor oila osh damlash uchun 1 kg guruch sarflaydi. Uyga 2 nafar mehmon kelganda qozonga osh uchun necha kg guruch solish maqsadga muvofiq? – degan masalada pishiriladigan oshdagi guruch miqdori uydagi kishilar sonining funksiyasi bo‘ladi. Ravshanki, kishilar soni – argument, guruch miqdori – funksiya bo‘ladi.

MISOLLAR

1. Matn bilan berilgan funksiyaning analitik ko‘rinishini yozing. (Masalan, “argumentni kvadratidan 5 ni ayiring” qoidasi quyidagi funksiyani beradi $f(x) = x^2 - 5$):
a) argumentni 3 ga ko‘paytirib, undan 5 ni ayiring
b) argumentning kvadratiga 2 ni qo‘shing
c) argumentdan 1 ni ayirib, keyin kvadratga ko‘taring
d) argumentga 1 ni qo‘shing, keyin kvadrat ildizini topib, 6 ga bo‘ling

2. Funksiyaning matnli qoidasi berilgan. Bu funksiyaning (a) ***analitik***, (b) ***jadval*** va (d) ***grafik*** ko‘rinishini toping:
a) $f(x)$ ni topish uchun argumentni 3 ga bo‘ling, keyin $\frac{2}{3}$ ni qo‘shing.
b) $g(x)$ ni topish uchun argumentdan 4 ni ayiring, keyin $\frac{3}{4}$ ga ko‘paytiring.
c) $T(x)$ funksiya x so‘mga sotib olingan mahsulotning soliq miqdori funksiyasi bo‘lsin. Soliq miqdorini topish uchun mahsulot narxini 8% ga ko‘paytiring.
d) $V(d)$ funksiya d diametrli sharning hajmini topish funksiyasi bo‘lsin. Hajmni topish uchun diametrning 3-darajasini π ga ko‘paytirib 6 ga bo‘ling.

3. Berilgan funksiyalar uchun qiymatlar jadvalini to'ldiring:

a) $f(x)=2(x-1)^2$;

x	$f(x)$
–1	
0	
1	
2	
3	

b) $g(x)=|2x+3|$.

x	$g(x)$
-3	
-2	
0	
1	
3	

4. Funksiyaning berilgan argumentdagi qiymatini toping.

a) $f(x)=x^2-6$; $f(-3), f(3), f(0), f(\frac{1}{2})$;

b) $f(x)=x^3+2x$; $f(-2), f(-1), f(0), f(\frac{1}{2})$;

c) $f(x)=\frac{|x|}{x}$; $f(-2), f(-1), f(0), f(5), f(x^2), f\left(\frac{1}{x}\right)$;

d) $f(x)=\frac{1-2x}{3}$; $f(2), f(-2), f\left(\frac{1}{2}\right), f(a), f(-a), f(a-1)$;

e) $h(x)=\frac{x^2+4}{5}$; $h(2), h(-2), h(a), h(-x), h(a-2), h\left(\sqrt{x}\right)$;

f) $f(x)=x^2+2x$; $f(0), f(3), f(-3), f(a), f(-x), f\left(\frac{1}{a}\right)$;

g) $h(t)=t+\frac{1}{t}$; $h(-1),\ h(2),\ h\left(\frac{1}{2}\right),\ h(x-1),\ h\left(\frac{1}{x}\right)$;

h) $g(x)=\frac{1-x}{1+x}$; $g(2), g(-1), g\left(\frac{1}{2}\right), g(a), g(a-1), g\left(x^2-1\right)$;

i) $g(t)=\frac{t+2}{t-2}$; $g(-2), g(2), g(0), g(a), g\left(a^2-2\right), g(a+1)$;

j) $k(x)=-x^2-2x+3$; $k(0), k(2), k(-2), k\left(\sqrt{2}\right), k(a+2), k(-x), k\left(x^2\right)$;

k) $k(x)=2x^3-3x^2$; $k(0), k(3), k(-3), k\left(\frac{1}{2}\right), k\left(\frac{a}{2}\right), k(-x), k\left(x^3\right)$;

l) $f(x)=2|x-1|$; $f(-2), f(0), f\left(\frac{1}{2}\right), f(2), f(x+1), f\left(x^2+2\right)$.

5. Berilgan tenglamalar funksiya bo'lishi yoki bo'lmasligini aniqlang.

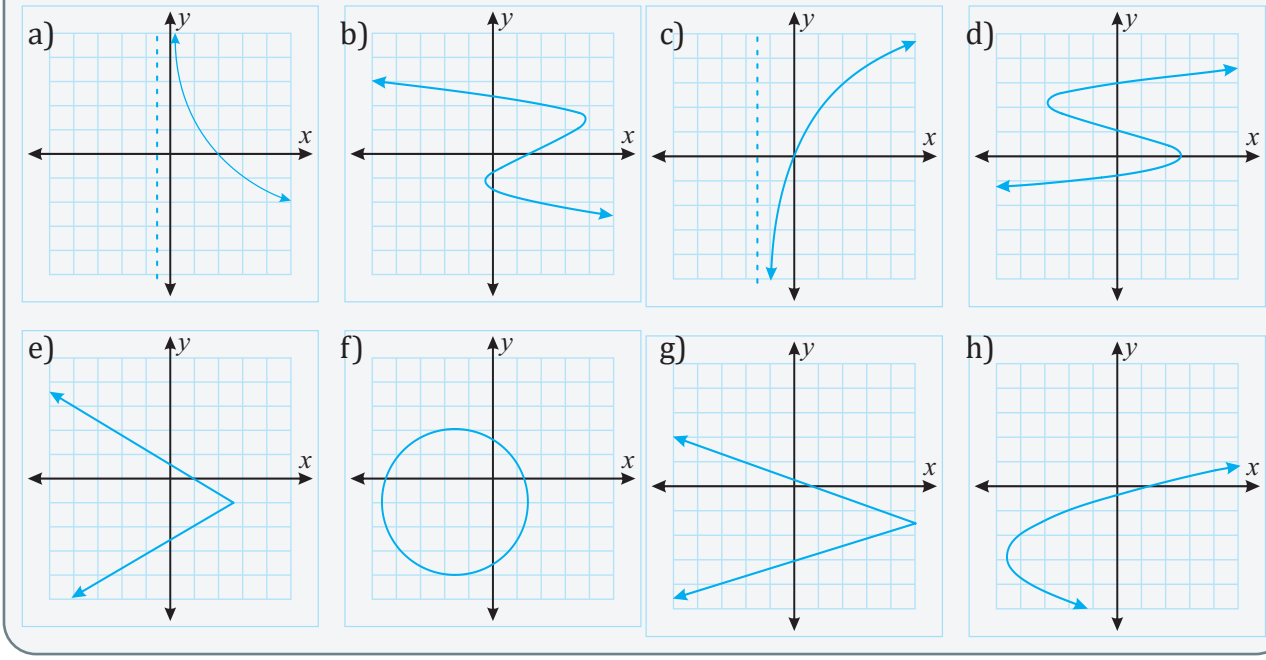
a) $3x-5y=7$ b) $3x^2-y=5$ c) $x=y^2$ d) $x^2+(y-1)^2=4$

e) $2x-4y^2=3$ f) $2x^2-4y^2=3$ g) $2xy-5y^2=4$ h) $\sqrt{y}-x=5$

i) $2|x|+y=0$ j) $2x+|y|=0$ k) $x=y^3$ l) $x=y^4$

6. 3-rasmdagi chizmalardan foydalanib, berilgan chiziqlar biror funksiyaning grafigi bo‘lishi yoki bo‘lmasligini aniqlang.

3-rasm



7. Berilgan jadvallardan qaysi birida funksiya aniqlangan.

a)

x	y
–5	–12
9	2
11	2

b)

x	y
15	9
16	12
17	15

c)

x	y
0	20
18	2
19	1

d)

x	y
–10	–9
$3\frac{1}{2}$	–6
8	–1

e)

x	y
2	0
–17	–3
–5	7
11	17
6	7

f)

x	y
–4	$\frac{1}{2}$
$3\frac{1}{2}$	$-3\frac{1}{2}$
$9\frac{3}{5}$	–10

FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO‘PLAMI

Funksiyaning aniqlanish sohasi va qiymatlar to‘plami

$y=f(x)$ funksiya qaralayotgan bo‘lsin. x argument qabul qilishi mumkin bo‘lgan elementlar to‘plami $y=f(x)$ funksiyaning ***aniqlanish sohasi***, y funksiya qabul qilishi mumkin bo‘lgan sonlar to‘plami $y=f(x)$ funksiyaning ***qiymatlar to‘plami*** deb ataladi va ular mos ravishda $D(f)$ va $E(f)$ kabi belgilanadi.

Ba’zi elementar funksiyalar uchun aniqlanish sohasi va qiymatlar to‘plami jadvali:

Funksiya	Aniqlanish sohasi	Qiymatlar to‘plami
1) $y=kx+b$	$D(f)=(-\infty;+\infty)$,	$E(f)=[0;+\infty)$.
2) $y=x^2$	$D(f)=(-\infty;+\infty)$,	$E(f)=[0;\ +\infty)$.
3) $y=\lvert x\rvert$	$D(f)=(-\infty;+\infty)$,	$E(f)=[0;\ +\infty)$.
4) $y=\frac{k}{x}$	$D(f)=(-\infty;0)\cup(0;+\infty)$,	$E(f)=(-\infty;0)\cup(0;+\infty)$,
5) $y=\sqrt{x}$	$D(f)=[0;+\infty)$,	$E(f)=[0;+\infty)$.
6) $y=\sqrt[2n]{x}$	$D(f)=[0;+\infty)$,	$E(f)=[0;+\infty)$.
7) $y=\sqrt[3]{x}$	$D(f)=(-\infty;+\infty)$,	$E(f)=[0;+\infty)$.
8) $y=\sqrt[2n+1]{x}$	$D(f)=(-\infty;+\infty)$,	$E(f)=[0;+\infty)$.

x argumentning $y=f(x)$ funksiyaning aniqlanish sohasiga tegishli bo‘lmagan har qanday qiymatida $y=f(x)$ funksiya aniqlanmagan bo‘ladi, yoki, boshqacha aytganda, $f(x)$ ifoda ma’noga ega bo‘lmaydi. Masalan, $y=\sqrt{x}$ funksiya uchun $x=-1$ bo‘lganda, $y=\frac{k}{x}$ funksiya uchun $x=0$ bo‘lganda ma’noga ega bo‘lmagan mos ravishda $\sqrt{-1}$ yoki $\frac{k}{0}$ ifoda hosil bo‘ladi.

1-misol. $y=\frac{1}{x^2-x}$ funksiyaning aniqlanish sohasini toping.

Yechish: Ratsional ifodaning maxraji nolga teng bo‘lishi mumkin emas, ya'ni:

$$x^2-x\neq 0$$
$$x(x-1)\neq 0$$
$$x\neq 0 \text{ va } x\neq 1$$

Demak, x argument 0 va 1 qiymatlarni qabul qilmaydi. Shuning uchun, berilgan funksiyaning aniqlanish sohasi $D(y)=(-\infty;0)\cup(0;1)\cup(1;+\infty)$.

Uni quyidagicha ham yozish mumkin: $\{x|x\neq 0, x\neq 1\}$.

Javob: $D(y)=(-\infty;0)\cup(0;1)\cup(1;+\infty)$.

2-misol. $y=\sqrt{9-x^2}$ funksiyaning aniqlanish sohasini toping.

Yechish: Kvadrat ildiz ostidagi ifoda manfiy bo‘lmaydi. Ya‘ni,

$$9-x^2\geq 0$$
$$(3-x)(3+x)\geq 0$$
$$-3\leq x\leq 3.$$

Demak, x argument faqat $[-3;3]$ kesmadan qiymat qabul qila oladi. Shuning uchun funksiyaning aniqlanish sohasi: $D(y)=[-3;3]$, yoki boshqacha ifodalansa: $\{x\,|\,-3\leq x\leq 3\}$.

Javob: $D(y)=[-3;3]$.

3-misol. $y=\dfrac{1}{\sqrt{x+1}}$ funksiyaning aniqlanish sohasini toping.

Yechish: Berilgan funksiya maxrajida kvadrat ildiz ostidagi ifoda berilgan, bu ifoda nolga teng bo‘lishi mumkin emas, hamda noldan kichik bo‘lmasligi kerak. Shuning uchun,

$$x+1>0$$
$$x>-1.$$

Demak, funksiyaning aniqlanish sohasi $D(y)=(-1;\infty)$ yoki $\{x|x>-1\}$.

Javob: $D(y)=(-1;\infty)$.

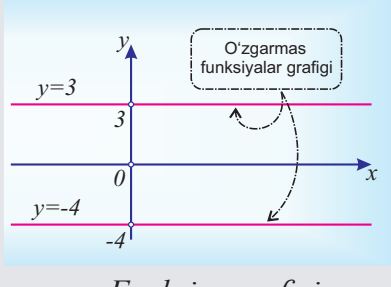
Funksiya grafigi

$y=f(x)$ funksiya o‘zining $D(f)$ aniqlanish sohasidan olingan har bir x elementga $E(f)$ qiymatlar to‘plamidan yagona $f(x)$ qiymatni mos qo‘yadi. Natijada har bir $x\in D(f)$ element Oxy koordinatalar tekisligida yagona $(x,f(x))$ nuqtani aniqlaydi.

Oxy koordinatalar tekisligida hosil qilingan barcha $(x,f(x))$ nuqtalar to‘plami $y=f(x)$ ***funksiyaning grafigi*** deyiladi.

1-, 2-rasmda funksiya grafiklari tasvirlangan. Undagi grafiklar yoki egri chiziq, yoki to‘g‘ri chiziqlardan iborat.

1-rasm



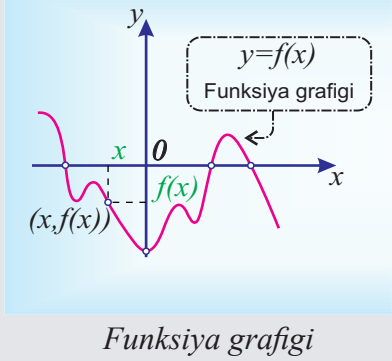
Funksiya grafigi

4-misol. Quyidagi funksiyalarning grafiklarini chizing.

$a)\ y=x^2$ $\quad b)\ y=x^3$ $\quad d)\ y=\sqrt{x}$

Yechish: Bu funksiyalarning grafiklarini chizish uchun avvalo, qiymatlar jadvalini tuzib olamiz. Keyin bu nuqtalarni koordinata tekisligida belgilaymiz va ularni silliq egri chiziq bilan tutashtiramiz.

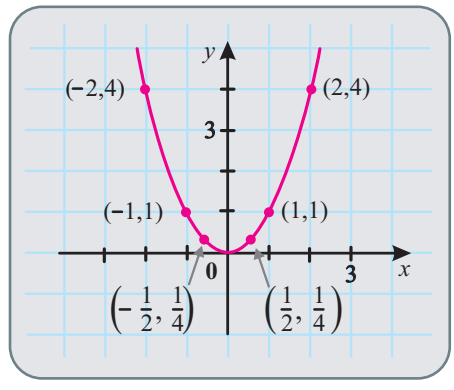
2-rasm



Funksiya grafigi

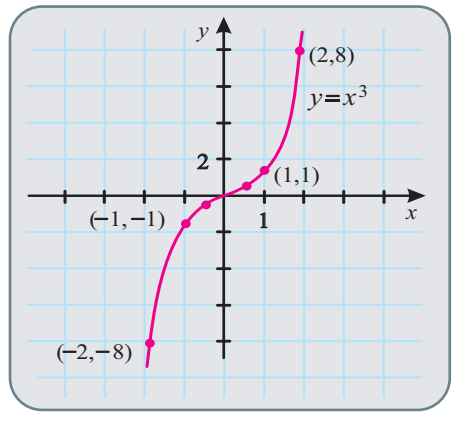
a)

x	–3	–2	–1	0	1	2	3
$y=x^2$	9	4	1	0	1	4	9



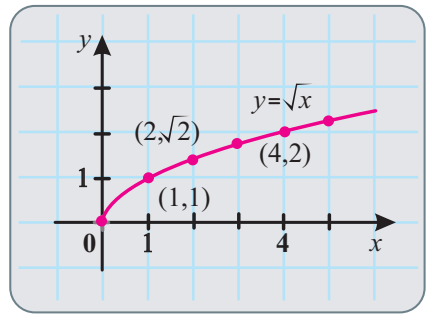
b)

x	–2	–1	–1/2	0	1/2	1	2
$y=x^3$	–8	–1	–1/8	0	1/8	1	8



d)

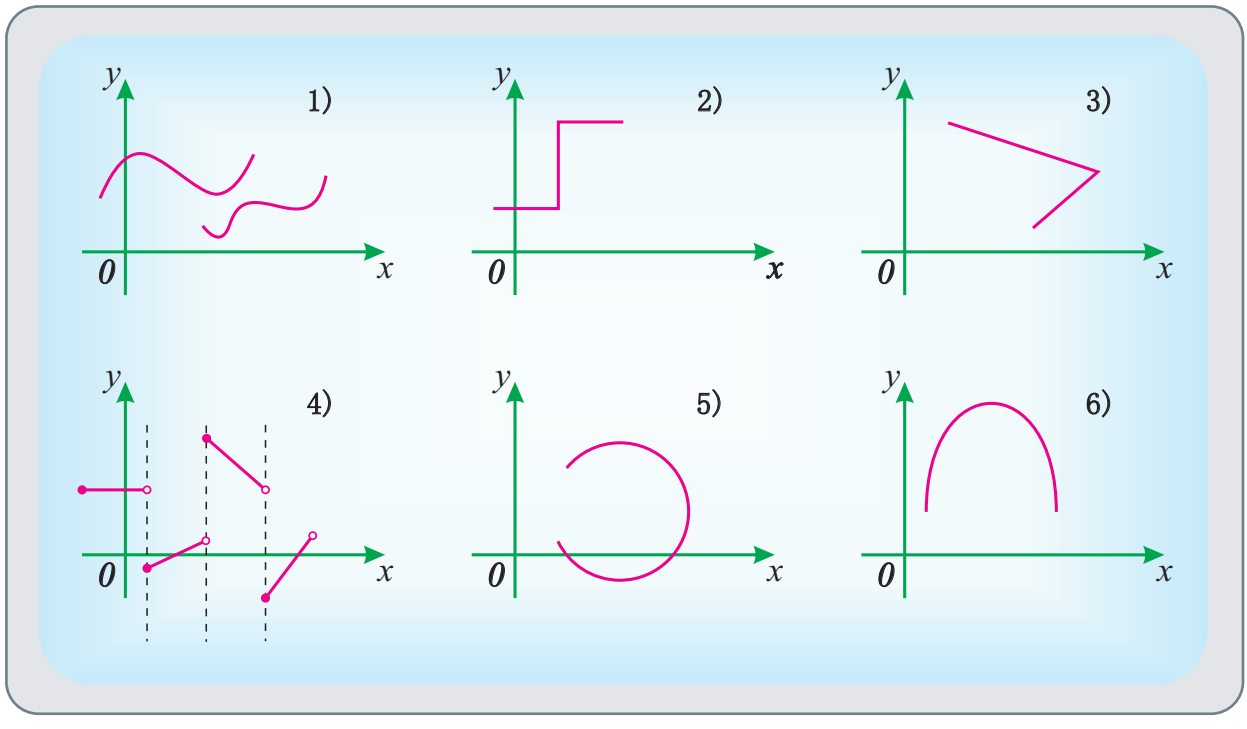
x	0	1/4	1	2	4	9
$y=\sqrt{x}$	0	1/2	1	$\sqrt{2}$	2	3



Ta’kidlash joizki, Oxy tekisligida tasvirlangan chiziq biror $y=f(x)$ funksiyaning grafigi bo‘lishi uchun Oy o‘qiga parallel har bir to‘g‘ri chiziq berilgan chiziqni bittadan ortiq bo‘lmagan

nuqtada kesib o‘tishi zarur. Aks holda, ya’ni Oy o‘qiga parallel qandaydir to‘g‘ri chiziq berilgan chiziqni bittadan ortiq nuqtada kesib o‘tsa, bu chiziq funksiya grafigi bo‘lmaydi.

Quyidagi rasmda keltirilgan chiziqlardan 4) va 6) chiziqlar biror funksiyaning grafigi bo‘ladi, 1), 2), 3) va 5) chiziqlar esa funksiya grafigi bo‘lmaydi.



MISOLLAR

1. Funksiyaning aniqlanish sohasini va qiymatlar to‘plamini toping.

a) $f(x)=3x$ b) $f(x)=3x,\ 2\le x\le 6$

c) $f(x)=5x^2+2$ d) $f(x)=5x^2+2,\ 0\le x\le 2$

2. Funksiyaning aniqlanish sohasini toping.

a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=\dfrac{1}{3x-6}$ c) $f(x)=\dfrac{x+2}{x^2-1}$

d) $f(x)=\dfrac{x^4}{x^2+x-6}$ e) $f(t)=\sqrt{t+1}$ f) $g(t)=\sqrt{t^2+9}$

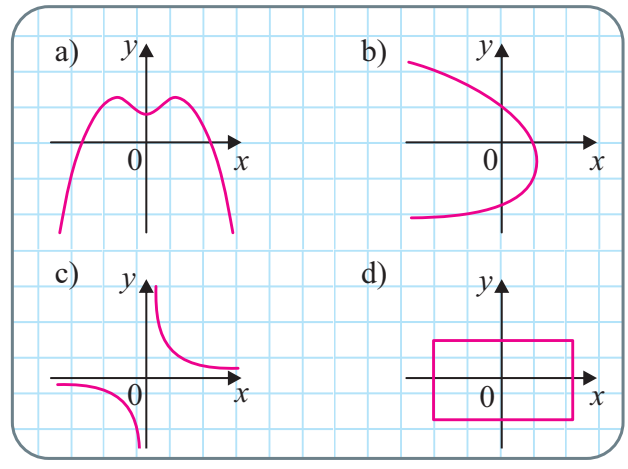
g) $f(t)=\sqrt[3]{t-1}$ h) $g(x)=\sqrt{7-3x}$ i) $f(x)=\sqrt{1-2x}$

j) $g(x)=\sqrt{x^2-4}$ k) $g(x)=\dfrac{\sqrt{2+x}}{3-x}$ l) $g(x)=\dfrac{\sqrt{x}}{2x^2+x-1}$

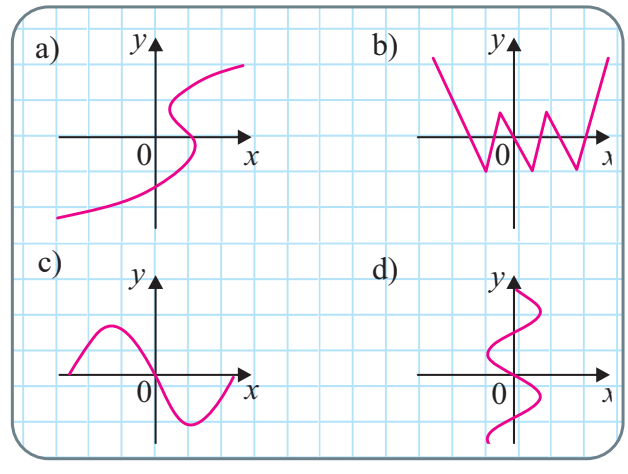
m) $g(x)=\sqrt[4]{x^2-6x}$ n) $g(x)=\sqrt{x^2-2x-8}$ o) $f(x)=\dfrac{3}{\sqrt{x-4}}$

p) $f(x)=\dfrac{x^2}{\sqrt{6-x}}$ q) $f(x)=\dfrac{(x+1)^2}{\sqrt{2x-1}}$ r) $f(x)=\dfrac{x}{\sqrt[4]{9-x^2}}$

3. Berilgan chiziqlardan qaysi biri funksiyaning grafigi ekanini aniqlang.

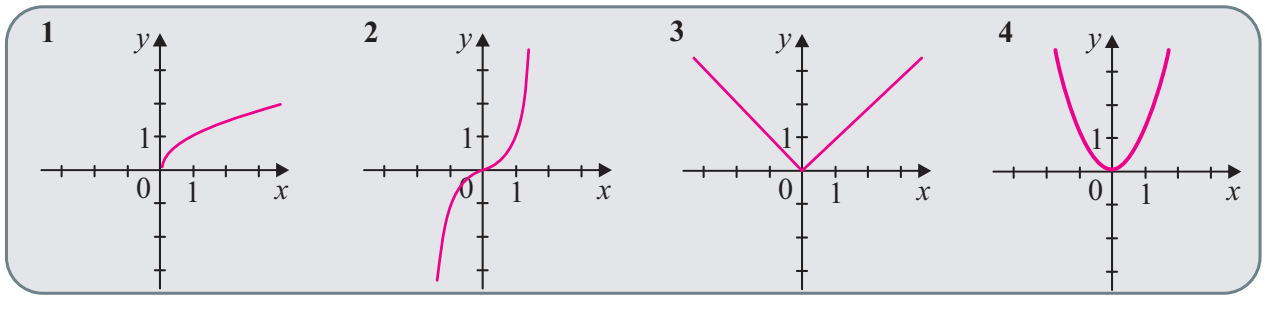


4. Berilgan chiziqlardan qaysi biri funksiyaning grafigi ekanini aniqlang.



5. Funksiyaga mos grafikni aniqlang.

a) $f(x)=x^2$ b) $f(x)=x^3$ c) $f(x)=\sqrt{x}$ d) $f(x)=|x|$



6. Berilgan funksiyalarning qiymatlar jadvalini tuzing va grafigini chizing.

a) $f(x)=-x^2$ b) $f(x)=x^2-4$ c) $g(x)=-(x+1)^2$

d) $r(x)=3x^4$ e) $r(x)=1-x^4$ f) $g(x)=x^3-8$

g) $k(x)=\sqrt[3]{-x}$ h) $k(x)=-\sqrt[3]{x}$ i) $f(x)=1+\sqrt{x}$

j) $C(t)=\dfrac{1}{t^2}$ k) $C(t)=-\dfrac{1}{t+1}$ l) $H(x)=|2x|$

m) $G(x)=|x|+x$ n) $G(x)=|x|-x$ o) $f(x)=|2x-2|$

7. Berilgan funksiyalarning grafiklarini chizing.

a) $f(x)=8x-x^2$ b) $g(x)=x^2-x-20$ c) $h(x)=x^3-5x-4$

FUNKSIYALAR USTIDA ARIFMETIK AMALLAR

Funksiyalar ustida arifmetik amallar

Funksiyalar ustida qo‘shish (+), ayirish (–), ko‘paytirish (×), bo‘lish (÷) arifmetik amallarini bajarish mumkin.

$f(x)$ va $g(x)$ funksiyalarning aniqlanish sohalari mos ravishda A va B to‘plamlar bo‘lsin. Bu funksiyalarning $A \cap B$ to‘plamdagi ***yig‘indisi*** deb, har bir $x \in A \cap B$ elementda $f(x) + g(x)$ qiymatni qabul qiladigan funksiyaga aytiladi. $f(x)$ va $g(x)$ funksiyalarning yig‘indisi $(f + g)(x)$ kabi belgilanadi. Demak,

$$(f + g)(x) = f(x) + g(x).$$

Xuddi shuningdek, $f(x)$ va $g(x)$ funksiyalarning ***ayirmasi, ko‘paytmasi, bo‘linmasini*** aniqlash mumkin:

$$(f - g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

Diqqat qiling!

1) $A \cap B = \varnothing$ bo‘lsa, bu amallar aniqlanmaydi.

2) Ikkita $f(x)$ va $g(x)$ funksiyalarning bo‘linmasini aniqlashda X dan olingan har bir x element uchun $g(x) \neq 0$ bo‘lishi talab etiladi.

MISOLLAR

1-misol. $f(x) = \frac{1}{x-2}$ va $g(x) = \sqrt{x}$ funksiyalar berilgan.

a) $f+g, f-g, fg$ va $\frac{f}{g}$ funksiyalarni va ularning aniqlanish sohasini toping.

b) $(f+g)(4), (f-g)(4), (fg)(4)$ va $\left(\frac{f}{g}\right)(4)$ ni toping

Yechish:

a) f ning aniqlanish sohasi $\{x \mid x \neq 2\}$, g niki esa $\{x \mid x \geq 0\}$. f va g ning aniqlanish sohalari kesishmasi $\{x \mid x \geq 0$ va $x \neq 2\} = [0; 2) \cup (2; \infty)$ bo‘ladi.

Shunda ular ustida amallar quyidagicha bajariladi:

$$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \sqrt{x}$$

$$(f-g)(x) = f(x) - g(x) = \frac{1}{x-2} - \sqrt{x}$$

$$(fg)(x)=f(x)g(x)=\frac{\sqrt{x}}{x-2}$$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{1}{(x-2)\sqrt{x}}.$$

$x = 4$ qiymat har bir yangi funksiyaning aniqlanish sohasiga tegishli ekanligidan quyidagi qiymatlar aniqlangan:

$$(f+g)(4)=f(4)+g(4)=\frac{1}{4-2}+\sqrt{4}=\frac{5}{2}$$

$$(f-g)(4)=f(4)-g(4)=\frac{1}{4-2}-\sqrt{4}=-\frac{3}{2}$$

$$(fg)(4)=f(4)g(4)=\left(\frac{1}{4-2}\right)\sqrt{4}=1$$

$$\left(\frac{f}{g}\right)(4)=\frac{f(4)}{g(4)}=\frac{1}{(4-2)\sqrt{4}}=\frac{1}{4}$$

2-misol. Funksiyalarni grafik usulda qo‘shish.

f va g funksiyalarning grafigi 1-rasmda berilgan bo‘lsin. Grafik usuldagi qo‘shish yordamida $f + g$ funksiyaning grafigini chizing.

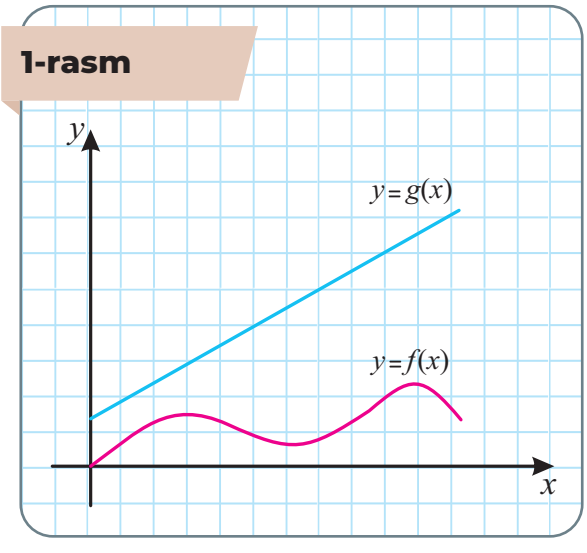
Yechish.

Ma’lumki, f funksiya grafigi Oxy to‘plamdagi $\{(x, f(x)) : x \in D(f)\}$ to‘plamdan iborat. Xuddi shuningdek, $\{(x, g(x)) : x \in D(g)\}$ to‘plam g funksiyaning grafigi bo'ladi. f va g funksiyalarni qo‘shishning grafik usuli deganda ushbu to‘plam tushuniladi:

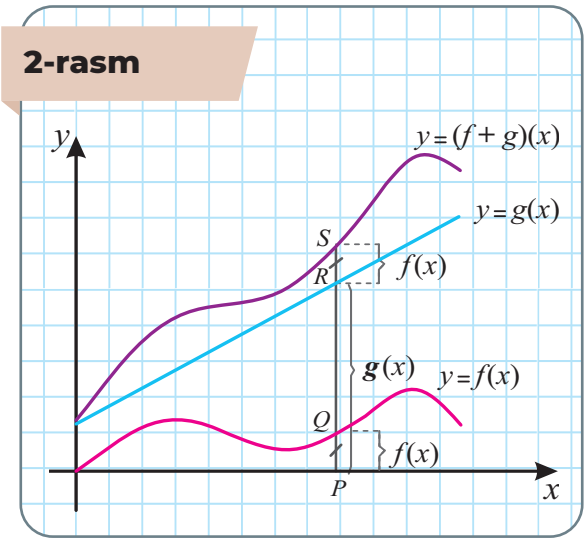
$\{(x, f(x) + g(x)) : x \in D(f) + D(g)\}$.

PQ kesmani PR kesmaning yuqorisiga $f + g$ funksiyaning S nuqtasini hosil qilish uchun nusxalab ko‘chirilgan ($PQ = RS$).

1-rasm



2-rasm



MISOLLAR

1. Funksiyalarni qo‘shing va ayiring.

a) $f(x)=5x+1$, $g(x)=-2x$ bo‘lsa, $(f+g)(x)=?$

b) $f(x)=-3x+3$, $g(x)=-5x+4$ bo‘lsa, $(f-g)(x)=?$

c) $f(x)=2x+1$, $g(x)=-5x+3$ bo‘lsa, $(f+g)(x)=?$

d) $f(x)=-3x^2+7x$, $g(x)=2x+4$ bo‘lsa, $(f\cdot g)(x)=?$

2. Funksiyalarni ko‘paytiring.

a) $f(x)=-x^2$, $g(x)=-3x+1$ bo‘lsa, $(f\cdot g)(x)=?$

b) $f(x)=-3x^2+3$, $g(x)=-x$ bo‘lsa, $(f\cdot g)(x)=?$

c) $f(x)=-x+3$, $g(x)=5x+6$ bo‘lsa, $(f\cdot g)(x)=?$

d) $f(x)=-4x+5$, $g(x)=-3x+1$ bo‘lsa, $(f\cdot g)(x)=?$

3. $f+g, f-g, fg, \frac{f}{g}$ larni va ularning aniqlanish sohasini toping.

a) $f(x)=x,\ g(x)=2x$

b) $f(x)=x,\ g(x)=\sqrt{x}$

c) $f(x)=x^2+x,\ g(x)=x^2$

d) $f(x)=3-x^2,\ g(x)=x^2-4$

e) $f(x)=5-x,\ g(x)=x^2-3x$

f) $f(x)=x^2+2x,\ g(x)=3x^2-1$

g) $f(x)=\sqrt{25-x^2},\ g(x)=\sqrt{x+3}$

h) $f(x)=\sqrt{16-x^2},\ g(x)=\sqrt{x^2-1}$

i) $f(x)=\frac{2}{x},\ g(x)=\frac{4}{x+4}$

j) $f(x)=\frac{2}{x+1},\ g(x)=\frac{x}{x+1}$

4. Funksiyaning aniqlanish sohasini toping.

a) $f(x)=\sqrt{x}+\sqrt{3-x}$

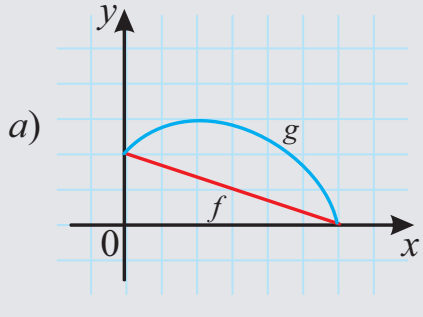
b) $f(x)=\sqrt{x+4}-\frac{\sqrt{1-x}}{x}$

c) $h(x)=(x-3)^{-\frac{1}{4}}$

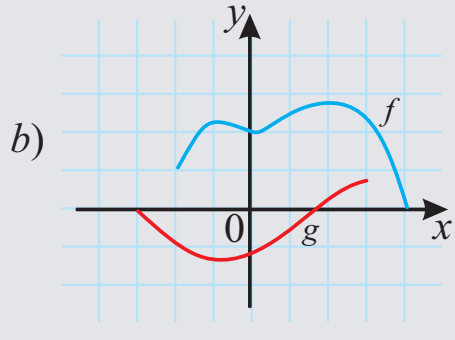
d) $k(x)=\frac{\sqrt{x+3}}{x-1}$

5. Grafik usuldagi qo‘shish yordamida $f+g$ funksiyaning grafigini chizing.

3-rasm



4-rasm



MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR

Murakkab funksiya

Funksiyalarni ketma-ket qo‘llash natijasida o‘zgaruvchilarning yangi bog‘lanishlari hosil bo‘ladi. Agar X to‘plamda $y = f(x)$ funksiya berilgan bo‘lib, x argument T to‘plamda aniqlangan biror $x = g(t)$ funksiya bo‘lsa, u holda T to‘plamda $y = f(g(t))$ ***murakkab funksiya*** aniqlangan deyiladi.

Masalan, $y=2x^2-3x$ funksiya $X=(-\infty,+\infty)$ to‘plamda, $x=\sqrt{t}$ funksiya esa $T=[0;+\infty)$ to‘plamda berilgan bo‘lsin. U holda $y=2t-3\sqrt{t}$ funksiya $T=[0,+\infty)$ to‘plamda $y=2x^2-3x$ va $x=\sqrt{t}$ funksiyalarning murakkab funksiyasi bo‘ladi.

1-misol.

$f(x)=x^2$ va $g(x)=x-3$ funksiyalar berilgan:

a) $f(g(x))$ va $g(f(x))$ murakkab funksiyalarni va ularning aniqlanish sohasini toping;

b) $f(g(5))$ va $g(f(7))$ ni toping.

Yechish.

a) Quyidagi tenglik o‘rinli:

$f(g(x)) = f(x-3)$ g ning berilishiga ko‘ra,

$f(g(x)) = (x-3)^2$ f ning berilishiga ko‘ra,

$g(f(x)) = g(x^2)$ f ning berilishiga ko‘ra,

$g(f(x)) = x^2-3$ g ning berilishiga ko‘ra.

Ikkala $f(g(x))$ va $g(f(x))$ funksiyaning aniqlanish sohasi R.

b) Topilgan murakkab funksiyalarda x ning o‘rniga berilgan qiymatni qo‘yamiz

$f(g(5)) = (5-3)^2 = 2^2 = 4,$ $g(f(7)) = 7^2 - 3 = 49 - 3 = 46.$

2-misol.

Agar $f(x)=\sqrt{x}$ va $g(x)=\sqrt{2-x}$ berilgan bo‘lsa, quyidagi funksiyalarni va ularning aniqlanish sohasini toping.

a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$ d) $g(g(x))$

Yechish.

a) Ta’rifiga ko‘ra,

$f(g(x)) = f\left(\sqrt{2-x}\right)$ f ning berilishiga ko‘ra,

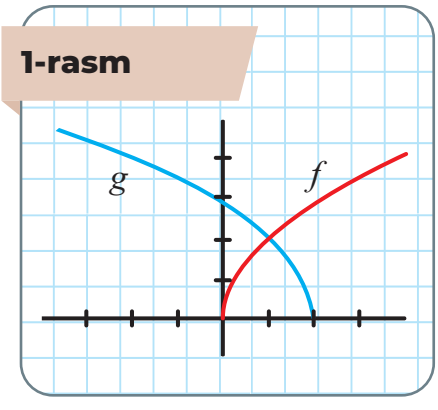
$g(f(x)) = \sqrt{\sqrt{2-\sqrt{x}}}$ g ning berilishiga ko‘ra.

$\sqrt{x}$ ning aniqlanish sohasi: $x \geq 0$.

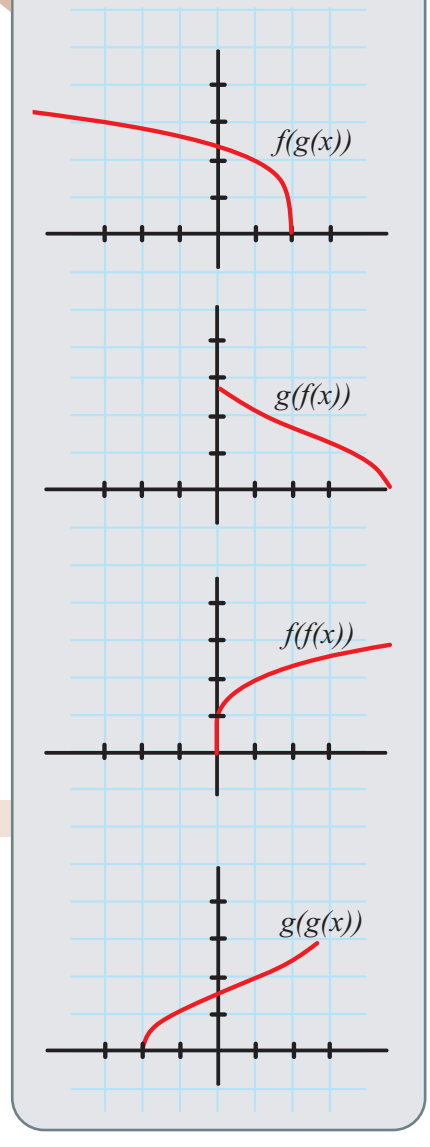
$\sqrt{2-\sqrt{x}}$ ning aniqlanish sohasi $2-\sqrt{x} \geq 0$, bundan, $\sqrt{x} \leq 2$, yoki $x \leq 4$. Demak, $0 \leq x \leq 4$.

$f(g(x))$ ning aniqlanish sohasi [0; 4] kesmadan iborat.

1-rasm



2-rasm



b) $g(f(x))=g\left(\sqrt{x}\right)$ f ning berilishiga ko‘ra,

$g(f(x))=\sqrt{2-\sqrt{x}}$ g ning berilishiga ko‘ra.

$\sqrt{x}$ ning aniqlanish sohasi: $x\geq 0$.

$\sqrt{2-\sqrt{x}}$ ning aniqlanish sohasi: $2-\sqrt{x}\geq 0$, bundan, $\sqrt{x}\leq 2$, yoki $x\leq 4$. Demak, $0\leq x\leq 4$.

c) $f(f(x))=f\left(\sqrt{x}\right)$ f ning berilishiga ko‘ra,

$f(f(x))=\sqrt{\sqrt{x}}=\sqrt[4]{x}$ f ning berilishiga ko‘ra.

$\sqrt[4]{x}$ ning aniqlanish sohasi: $[0;\infty)$.

d) $g(g(x))=g\left(\sqrt{2-x}\right)$ g ning berilishiga ko‘ra,

$g(g(x))=\sqrt{2-\sqrt{2-x}}$ g ning berilishiga ko‘ra.

$\sqrt{2-\sqrt{2-x}}$ ning aniqlanish sohasi: $2-x\geq 0$ va $\sqrt{2-x}\leq 2$ Bundan $x\leq 2$ va $x\geq -2\Rightarrow -2\leq x\leq 2$, demak, $g(g(x))$ ning aniqlanish sohasi: $[-2;\ 2]$.

3-misol. $f\left(x\right)=\dfrac{x}{x+1}, g\left(x\right)=x^{10}$ va $h\left(x\right)=x+3$ bo‘lsa, $f(g(h(x)))$ ni toping.

Yechish.

$f(g(h(x)))=f\left(g\left(x+3\right)\right)$ h ning berilishiga ko‘ra

$f(g(h(x)))=f\left(\left(x+3\right)^{10}\right)$ g ning berilishiga ko‘ra

$f(g(h(x)))=\dfrac{\left(x+3\right)^{10}}{\left(x+3\right)^{10}+1}$ f ning berilishiga ko‘ra

Shu vaqtgacha biz sodda funksiyalardan murakkab funksiyalarni hosil qilish hollarini qarab chiqdik. Lekin matematik analizda “teskari amal” – murakkab funksiyadan sodda funksiyalarni aniqlay olish juda foydalidir. Quyidagi misolda buni qarab chiqamiz:

4-misol.

$F\left(x\right)=\sqrt[4]{x+9}$ funksiya berilgan. Agar $F\left(x\right)=f(g(x))$ bo‘lsa, f va g funksiyalarni aniqlang.

Yechish.

F funksiyaning qoidasi: “x ga 9 ni qo‘shib keyin 4-darajali ildizni toping”.

Biz f va g funksiyalarni quyidagi ko‘rinishda olishimiz mumkin. $g\left(x\right)=x+9$ va $f\left(x\right)=\sqrt[4]{x}$

Keyin,

$f(g(x)) = f(x+9)$ g ning berilishiga ko‘ra

$f(g(x)) = \sqrt[4]{x+9}$ f ning berilishiga ko‘ra.

5-misol. Murakkab funksiyaning qo‘llanilishi.

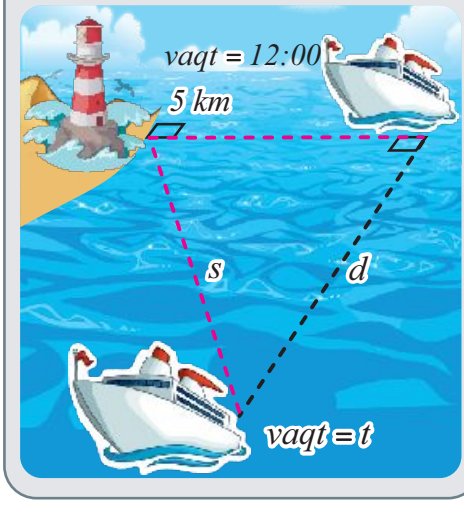
Kema 20 km/h o‘zgarmas tezlikda qirg‘oqqa parallel ravishda harakat qilmoqda. Kema mayoqning ro‘parasidan soat 12:00 da, qirg‘oqdan 5 km uzoqlikda o‘ tadi.

a) Mayoq va kema orasidagi s masofani d ga, kemaning soat 12:00 dan keyin yurgan masofasiga, nisbatan olingan funksiya, ya’ni quyidagi funksiya ko‘rinishida yozing: $s = f(d)$.

b) d ni t ga, soat 12:00 dan keyin o‘tgan vaqtga, nisbatan olingan funksiya, ya’ni quyidagi funksiya ko‘rinishida yozing: $d = g(t)$.

c) $f(g(t))$ murakkab funksiyani toping. Bu funksiya nimani anglatadi?

3-rasm



Yechish.

3-rasmdagi kabi chizmani chizib olamiz.

a) s va d masofalarni bog‘liqligini Pifagor teoremasi yordamida ko‘rsatamiz. Ya’ni, s ning d ga nisbatan funksiya ekanligini quyidagicha ifodalaymiz:

$$s = f(d) = \sqrt{25 + d^2}$$

b) kema 20 km/h o‘zgarmas tezlikda harakatlanayotgani uchun, d masofa t ga nisbatan funksiyasini quyidagicha ifodalashimiz mumkin:

$$d = g(t) = 20t$$

c) shunday qilib,

$f(g(t)) = f(20t)$ g ning berilishiga ko‘ra

$f(g(t)) = \sqrt{25 + (20t)^2}$ f ning berilishiga ko‘ra

$f(g(t))$ funksiya kema va mayoq orasidagi masofaning vaqtga nisbatan funksiyasini anglatadi.

Teskari funksiya

Agarda $f(x) = y$ tenglama har bir y uchun x ga nisbatan yagona ildizga ega bo‘lsa, u holda $x = g(y)$ funksiya $y = f(x)$ funksiyaga ***teskari funksiya*** deyiladi. $x = g(y)$ funksiya o‘rniga odatdagi belgilashlarga ko‘ra $y = g(x)$ yozuvi ishlatiladi. $y = f(x)$ funksiyaga teskari funksiya $y = f^{-1}(x)$ kabi yoziladi.

1-misol. $y = 3x - 5$ funksiyani qaraylik. Bu yerdan x ni y orqali ifodalaylik:

$$3x - 5 = y \Rightarrow 3x = y + 5 \Rightarrow x = \frac{y+5}{3}.$$

Oxirgi tenglikda x va y larning o ‘rinlarini almashtirib,

$$f^{-1}(x)=y=\frac{x+5}{3}$$

funksiyaga ega bo‘lamiz. Bu funksiya $y=3x-5$ funksiyaga teskari funksiya deb aytiladi.

Eslatma. Berilgan $y=f(x)$ funksiya va unga teskari $y=f^{-1}(x)$ funksiya uchun $D\left(f^{-1}\right)=E(f)$ hamda $E\left(f^{-1}\right)=D(f)$ bo‘ladi.

Diqqat qiling! $\left(f(x)\right)^{-1}=\frac{1}{f(x)}$ bo‘lib, bu tenglikdagi (-1) daraja ko‘rsatkichini anglatadi. $f^{-1}(x)$ yozuvdagi (-1) esa teskari funksiyani bildiradi. Umuman olganda, $\left(f(x)\right)^{-1}\neq f^{-1}(x)$. Masalan,

$f(x)=3x-5$ funksiya uchun $f^{-1}(x)=\frac{x+5}{3}$ hamda $\left(f(x)\right)^{-1}=\frac{1}{3x-5}$ bo‘ladi.

2-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{x^5-3}{2}$.

Yechish. Funksiyani $y=\frac{x^5-3}{2}$ kabi yozib olamiz va uni x ga nisbatan yechamiz.

$$y=\frac{x^5-3}{2}$$

$$2y=x^5-3$$

$$x^5=2y+3$$

$$x=(2y+3)^{\frac{1}{5}}.$$

Endi x va y larning o‘rnini almashtiramiz: $y=(2x+3)^{\frac{1}{5}}$. Demak, teskari funksiya quyidagicha: $f^{-1}(x)=(2x+3)^{\frac{1}{5}}$

3-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{2x+3}{x-1}$.

Yechish. Funksiyani quyidagicha yozib olamiz $y=\frac{2x+3}{x-1}$ va uni x ga nisbatan yechamiz.

$$y=\frac{2x+3}{x-1}$$

$$y(x-1)=2x+3$$

$$yx-y=2x+3$$

$$yx-2x=y+3$$

$$x(y-2)=y+3$$

$$x=\frac{y+3}{y-2}$$

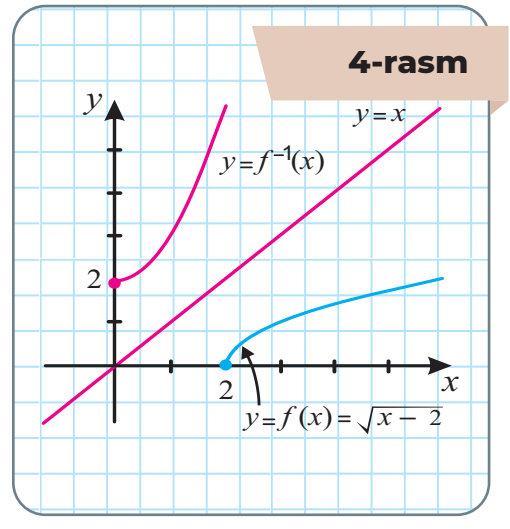
Demak, $f^{-1}(x)=\frac{x+3}{x-2}$ teskari funksiya bo‘ladi.

4-misol. Teskari funksiyaning grafigini chizish.

a) $f(x)=\sqrt{x-2}$ funksiyaning grafigidan foydalanib, f^{-1} funksiyaning grafigini chizing.

c) f^{-1} funksiyaning ko‘rinishini toping.

Yechish. 1) $y=\sqrt{x-2}$ funksiyani grafigi 4-rasmda keltirilgan.



4-rasm

2) f^{-1} funksiyaning grafigi f funksiyani grafigini $y=x$ to‘g‘ri chiziqqa nisbatan akslantirish yordamida chiziladi (4-rasm).

3) $y=\sqrt{x-2}$ funksiya x ga nisbatan yechiladi, bunda $y\ge 0$ ekanligi inobatga olinadi.

$$\sqrt{x-2}=y$$

$$x-2=y^2$$

$$x=y^2+2,\quad y\ge 0.$$

Endi x va y larning o‘rnini almashtiramiz: $y=x^2+2,\quad x\ge 0$.

Demak, teskari funksiya $f^{-1}(x)=x^2+2$ bo‘lar ekan, $x\ge 0$.

Bu topilgan f^{-1} teskari funksiya $y=x^2+2$ parabolaning o‘ng tarmog‘idan iborat. Buni grafikdan ham ko‘rsa bo‘ladi.

Davriy funksiyalar

$y=f(x)$ funksiya berilgan bo‘lib, $D(f)$ uning aniqlanish sohasi bo‘lsin. Shunday eng kichik musbat T_0 son topilib, har bir $x\in D(f)$ uchun

1) $(x\pm T_0)\in D(f)$ va

2) $f(x\pm T_0)=f(x)$ munosabatlar bajarilsa, u holda $y=f(x)$ ***davriy funksiya*** deyiladi. T_0 son esa $y=f(x)$ funksiyaning ***asosiy davri*** deb yuritiladi.

Agar T_0 soni $y=f(x)$ funksiyaning asosiy davri bo‘lsa, u holda har bir n butun son uchun nT_0 soni ham $y=f(x)$ funksiyaning davri bo‘ladi:

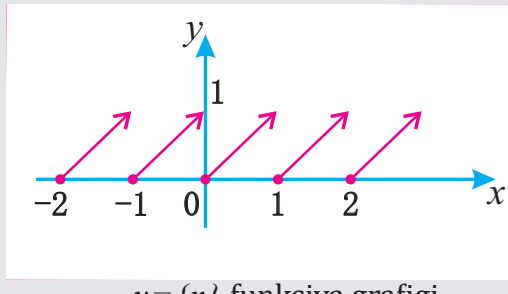
$$f(x\pm nT_0)=f(x),\quad n\in\mathbb{Z}.$$

Funksiyaning davriy ekanligini bilish uning grafigini chizishda amaliy ahamiyatga ega. Davriy funksiyaning grafigi bitta davr oralig‘ida chizish kifoya qiladi, boshqa davr oraliqlarida shu grafik takrorlanadi.

Masalan, sonning kasr qismi $\{x\}$ – berilgan x songa uning kasr qismini mos qo‘yuvchi funksiya (5-rasm) – davriy funksiya bo‘ladi. Uning asosiy davri $T_0=1$, ya’ni ixtiyoriy $x\in(-\infty,+\infty)$ son uchun $(x+1)\in(-\infty,+\infty)$ hamda $\{x+1\}=\{x\}$ munosabatlar o‘rinli bo‘ladi.

Agar $y=f(x)$ funksiyaning asosiy davri T_0 bo‘lsa, u holda $y=kf(ax+b)+c$ funksiyaning asosiy davri $T_1=\dfrac{T_0}{|a|}$ bo‘ladi $(a\neq 0)$.

2-rasm



$y=\{x\}$ funksiya grafigi

MISOLLAR

1. $f(g(x))$, $g(f(x))$, $f(f(x))$ va $g(g(x))$ funksiyalarni va ularning aniqlanish sohasini toping

a) $f(x)=2x+3$ $g(x)=4x-1$ b) $f(x)=6x-5$, $g(x)=\dfrac{x}{2}$

c) $f(x)=x^2$, $g(x)=x+1$ d) $f(x)=x^3+2$, $g(x)=\sqrt[3]{x}$

e) $f(x)=\dfrac{1}{x}$, $g(x)=2x+4$ f) $f(x)=x^2$, $g(x)=\sqrt{x-3}$

g) $f(x)=|x|$, $g(x)=2x+3$ h) $f(x)=4-x$, $g(x)=|x+4|$

i) $f(x)=\dfrac{x}{x+1}$, $g(x)=2x-1$ j) $f(x)=\dfrac{1}{\sqrt{x}}$, $g(x)=x^2-4x$

k) $f(x)=\dfrac{x}{x+1}$, $g(x)=\dfrac{1}{x}$ l) $f(x)=\dfrac{2}{x}$, $g(x)=\dfrac{x}{x+2}$

2. $f(x)=2x-3$ va $g(x)=4-x^2$ dan foydalanib, ifodalarning qiymatini toping.

a) $f(g(0))$ b) $g(f(0))$ c) $f(f(2))$ d) $g(g(3))$

e) $f(g(-2))$ f) $g(f(-2))$ g) $f(f(-1))$ h) $g(g(-1))$

3. $f(x)=3-x$ va $g(x)=x^2+1$ dan foydalanib, funksiyalarni toping.

a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$ d) $g(g(x))$

4. $f(g(h(x)))$ murakkab funksiyani toping.

a) $f(x)=x-1$, $g(x)=\sqrt{x}$, $h(x)=x-1$

b) $f(x)=\dfrac{1}{x}$, $g(x)=x^3$, $h(x)=x^2+2$

c) $f(x)=x^4+1$, $g(x)=x-5$, $h(x)=\sqrt{x}$

d) $f(x)=\sqrt{x}$, $g(x)=\dfrac{x}{x-1}$, $h(x)=\sqrt[3]{x}$

5. $F(x)=f(g(x))$ ko'rinishidagi murakkab funksiyadan f va g sodda funksiyalarni aniqlang.

a) $F(x)=(x-9)^5$ b) $F(x)=\sqrt{x}+1$ c) $F(x)=\dfrac{x^2}{x^2+4}$

d) $F(x)=\dfrac{1}{x+3}$ e) $F(x)=|1-x^3|$ f) $F(x)=\sqrt{1+\sqrt{x}}$

6. Berilgan f funksiyaga teskari funksiyani toping.

a) $f(x)=3x+5$ b) $f(x)=7-5x$ c) $f(x)=5-4x^3$

d) $f(x)=3x^3+8$ e) $f(x)=\dfrac{1}{x+2}$ f) $f(x)=\dfrac{x-2}{x+2}$

g) $f(x)=\dfrac{x}{x+4}$ h) $f(x)=\dfrac{3x}{x-2}$ i) $f(x)=\dfrac{2x+5}{x-7}$

j) $f(x)=\dfrac{4x-2}{3x+1}$ k) $f(x)=\dfrac{2x+3}{1-5x}$ l) $f(x)=\dfrac{3-4x}{8x-1}$

m) $f(x)=4-x^2,\ x\ge 0$ n) $f(x)=x^2+x,\ x\ge -\dfrac{1}{2}$ o) $f(x)=x^6,\ x\ge 0$

p) $f(x)=\dfrac{1}{x^2},\ x>0$ q) $f(x)=\dfrac{2-x^3}{5}$ r) $f(x)=(x^5-6)^7$

s) $f(x)=\sqrt{5+8x}$ t) $f(x)=2+\sqrt{3+x}$ u) $f(x)=\dfrac{1}{x^2-4},\ x<-2$

v) $f(x)=2+\sqrt[3]{x}$ w) $f(x)=\sqrt{4-x^2},\ 0\le x\le 2$

7. f funksiya berilgan: a) f funksiyaning grafigini chizing; b) f funksiyaning grafigidan foydalanib, f^{-1} funksiyaning grafigini chizing; d) f^{-1} funksiyani toping.

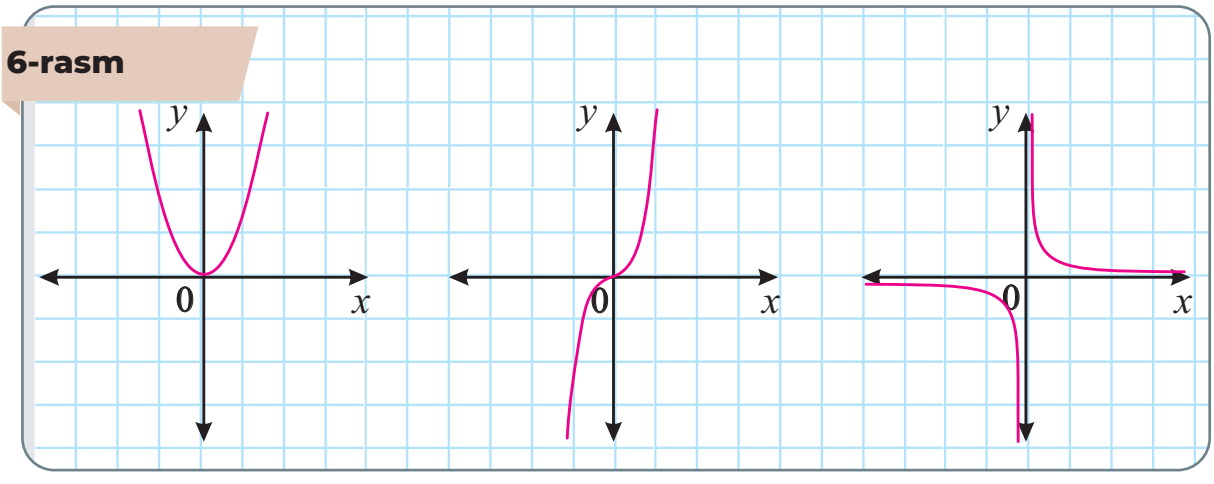
a) $f(x)=3x-6$ b) $f(x)=16-x^2,\ x\ge 0$

c) $f(x)=\sqrt{x+1}$ d) $f(x)=x^3$

8. 6-rasmda berilgan grafiklarga mos funksiyalarni tanlang va ularga teskari funksiya grafigini chizing:

$f(x)=x^2;\quad f(x)=x^3;\quad f(x)=\sqrt{x}$

6-rasm



FUNKSIYA XOSSALARI

Juft va toq funksiyalar

X sonli to‘plamni qaraylik. Agar har bir $x\in X$ nuqtani olganda ham $x\in X$ bo‘laversa, u holda X to‘plam nolga nisbatan ***simmetrik to‘plam*** deyiladi.

Masalan, $X=[-2;2]$ to‘plam, yoki $Y=[-7;-2)\cup(2,7]$ to‘plam nolga nisbatan simmetrik to‘plamlar bo‘ladi.

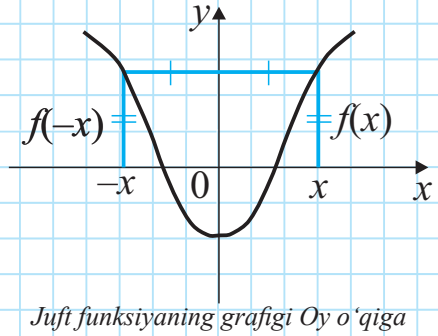
$D(f)$ aniqlanish sohasi nolga nisbatan simmetrik to‘plam bo‘lgan $y=f(x)$ funksiyani qaraylik.

Agarda ixtiyoriy $x\in D(f)$ nuqta uchun $f(-x)=f(x)$ tenglik bajarilsa, u holda $y=f(x)$ ***juft funksiya*** deyiladi.

Agarda ixtiyoriy $x\in D(f)$ element uchun $f(-x)=-f(x)$ tenglik bajarilsa, u holda $y=f(x)$ ***toq funksiya*** deyiladi.

1-rasm

f funksiya juft, agar $f(-x)=f(x)$.



Juft funksiyaning grafigi Oy o‘qiga nisbatan simmetrik

1-misol. $f(x)=2x^2+5$ funksiya uchun

$$f(-x)=2(-x)^2+5=2x^2+5=f(x),$$

ya’ni $f(-x)=f(x)$ ekanligidan $f(x)=2x^2+5$ funksiya juft funksiya bo‘ladi.

2-misol. $f(x)=2x^2+5$ funksiya uchun

$$f(-x)=2(-x)^3+5(-x)=-(2x^3+5x)=-f(x),$$

ya’ni $f(-x)=-f(x)$ ekanligidan $f(x)=2x^3+5x$ funksiya toq funksiya bo‘ladi.

3-misol. $f(x)=2x^3+5x^2-3x+1$ funksiyani juft - toqligini tekshiring.

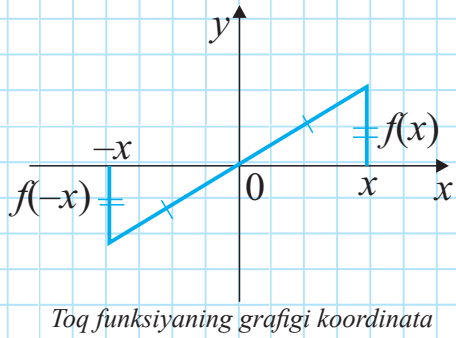
Yechish.

$$f(-x)=2(-x)^3+5(-x)^2-3(-x)+1=-2x^3+5x^2+3x+1=$$
$$=-(2x^3-5x^2-3x-1)\neq -f(x),$$
$$f(-x)\neq f(x).$$

Demak, bu funksiya juft ham emas, toq ham emas.

2-rasm

f funksiya toq, agar $f(-x)=-f(x)$.



Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik.

Funksiyalarning o‘sishi va kamayishi

$y=f(x)$ funksiya X to‘plamda berilgan bo‘lsin. Agar $x_1<x_2$ tengsizlikni qanoatlantiruvchi barcha x_1, $x_2\in X$ lar uchun:

$f(x_1)<f(x_2)$ bo‘lsa, $y=f(x)$ o‘suvchi funksiya;

$f(x_1)>f(x_2)$ bo‘lsa, $y=f(x)$ kamayuvchi funksiya;

$f(x_1)\geq f(x_2)$ bo‘lsa, $y=f(x)$ o‘smaydigan funksiya;

$f(x_1)\leq f(x_2)$ bo‘lsa, $y=f(x)$ kamaymaydigan funksiya deyiladi.

O‘suvchi, kamayuvchi, o‘smaydigan va kamaymaydigan funksiyalar umumiy nom bilan ***monoton funksiyalar*** deyiladi.

$f(x)$ funksiya uchun shunday M soni topilib, ixtiyoriy $x \in X$ uchun $f(x) \le M$ (yoki $f(x) < M$) bo'lsa, $f(x)$ ***yuqoridan chegaralangan funksiya*** deyiladi.

$f(x)$ funksiya uchun shunday m soni topilib, ixtiyoriy $x \in X$ uchun $f(x) \ge m$ (yoki $f(x) > m$) bo'lsa, $f(x)$ ***quyidan chegaralangan funksiya*** deyiladi.

Ham quyidan, ham yuqoridan chegaralangan funksiya ***chegaralangan funksiya*** deyiladi. Boshqacha aytganda, $f(x)$ funksiya uchun shunday M soni topilib, ixtiyoriy $x \in X$ uchun $|f(x)| \le M$ (yoki $|f(x)| < M$) bo'lsa, u holda $f(x)$ chegaralangan funksiya deyiladi.

Quyida chegaralanmagan, chegaralangan, quyidan chegaralangan, yuqoridan chegaralangan funksiyalar keltirilgan.

1) $y = \frac{1}{2}x - 4$ funksiya quyidan ham, yuqoridan ham chegaralanmagan.

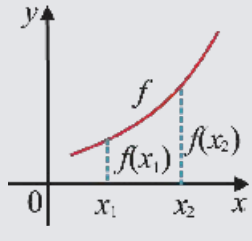
2) $y = x^2$ funksiya quyidan chegaralangan, chunki, x argumentning har bir qiymatida $x^2 \ge 0$ bo'ladi. Lekin bu funksiya yuqoridan chegaralanmagan.

3) $y = 4 - |x|$ funksiya yuqoridan chegaralangan, chunki, x argumentning har bir qiymatida $4 - |x| \le 4$ bo'ladi. Lekin bu funksiya quyidan chegaralanmagan.

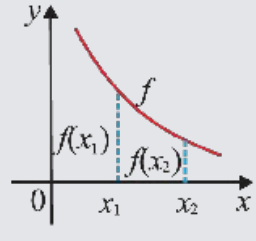
4) $y = \frac{1}{1+x^2}$ funksiya quyidan ham, yuqoridan ham chegaralangan. x argumentning har bir qiymatida $0 < \frac{1}{1+x^2} \le 1$ bo 'ladi.

3-rasm O'suvchi va kamayuvchi funksiyalar

f funksiya o'suvchi, ya'ni $x_1 < x_2$ uchun $f(x_1) < f(x_2)$ bajariladi.



f funksiya kamayuvchi, ya'ni $x_1 < x_2$ uchun $f(x_1) > f(x_2)$ bajariladi.



4-misol.

a) $f(x) = 12x^2 + 4x^3 - 3x^4$ funksiyaning grafigini chizing.

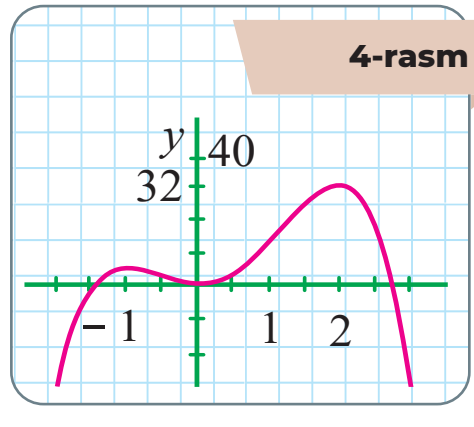
b) $f(x)$ funksiyaning aniqlanish sohasini va qiymatlar to'plamini toping.

c) $f(x)$ funksiyaning o'sish va kamayish oraliqlarini toping.

Yechish.

a) 4-rasmda berilgan grafikni chizib olamiz. Buning uchun maxsus online dasturlardan foydalanish mumkin.

4-rasm

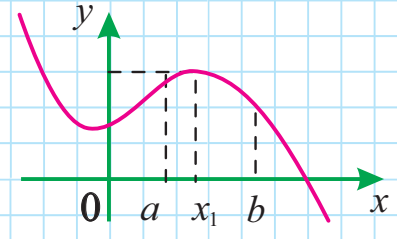


b) f funksiyaning aniqlanish sohasi $\mathbb{R}$. Funksiyaning eng katta qiymatini aniqlab olamiz: $f(2)=32$. Demak, funksiyaning qiymatlar to'plami $(-\infty,32]$.

c) $f(x)$ ning grafigidan, funksiya $(-\infty;-1)$ va (0; 2) oraliqlarda o'sishini va (–1; 0) hamda $(2;\infty)$ oraliqlarda kamayishini aniqlaymiz.

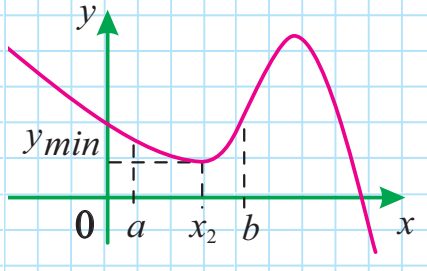
Funksiya ekstremum nuqtalari va ekstremumlari

5-rasm



$y=f(x)$ funksiya grafigi.
$y=f(x)$ uchun $(a;b)$ intervalda
x_1 – funksiyaning maksimum nuqtasi;
$y_{max}=f(x_1)$ – funksiyaning maksimumi.

6-rasm



$y=f(x)$ funksiya grafigi.
$y=f(x)$ uchun $(a;b)$ intervalda
x_2 – funksiyaning minimum nuqtasi;
$y_{min}=f(x_2)$ – funksiyaning minimumi.

Agar:

1) $f(x)$ funksiya x_1 nuqta tegishli bo'lgan biror $(a;b)$ intervalda aniqlangan bo'lib;

2) $(a;b)$ intervalning x_1 dan farqli barcha x nuqtalarida $f(x)<f(x_1)$ shart bajarilsa, u holda x_1 nuqta $f(x)$ ***funksiyaning maksimum nuqtasi*** deyiladi. (5-rasm)

Agar $x_1\in D(f)$ nuqta $f(x)$ funksiya uchun maksimum nuqta bo'lsa, u holda $f(x)$ funksiyaning x_1 nuqtadagi $f(x_1)$ qiymati $f(x)$ ***funksiyaning maksimumi*** deyiladi va y_{max} kabi begilanadi. Demak,

$$y_{max}=f(x_1) \text{ ekan.}$$

Agar:

1) $f(x)$ funksiya x_2 tegishli bo'lgan biror $(a;b)$ intervalda aniqlangan bo'lib;

2) $(a;b)$ intervalning x_2 dan farqli barcha x nuqtalarida $f(x)>f(x_2)$ shart bajarilsa, u holda x_2 nuqta $f(x)$ ***funksiyaning minimum nuqtasi*** deyiladi. (6-rasm)

Agar $x_2\in X$ nuqta $f(x)$ funksiya uchun minimum nuqta bo'lsa, u holda $f(x)$ funksiyaning x_2 nuqtadagi $f(x_2)$ qiymati $f(x)$ ***funksiyaning minimumi deyiladi*** va y_{min} kabi begilanadi. Demak, $y_{min}=f(x_2)$ bo'ladi.

Maksimum nuqta va minimum nuqta iboralari uchun umumiy nom – ekstremum nuqta iborasi ishlatiladi.

MISOLLAR

1. Berilgan funksiyalarni juft yoki toqlikka tekshiring. Agar funksiya juft yoki toq bo'lsa, grafigini chizish uchun simmetriyadan foydalaning.

a) $f(x)=x^4$ b) $f(x)=x^3$

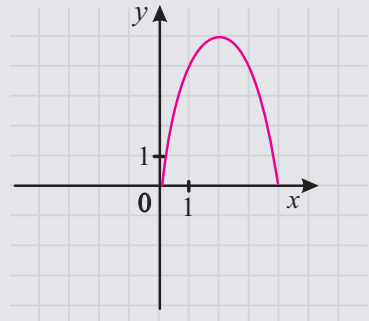
c) $f(x)=x^2+x$ d) $f(x)=x^4-4x^2$

e) $f(x)=x^3-x$ f) $f(x)=3x^3+2x^2+1$

g) $f(x)=1-\sqrt[3]{x}$ h) $f(x)=x+\frac{1}{x}$

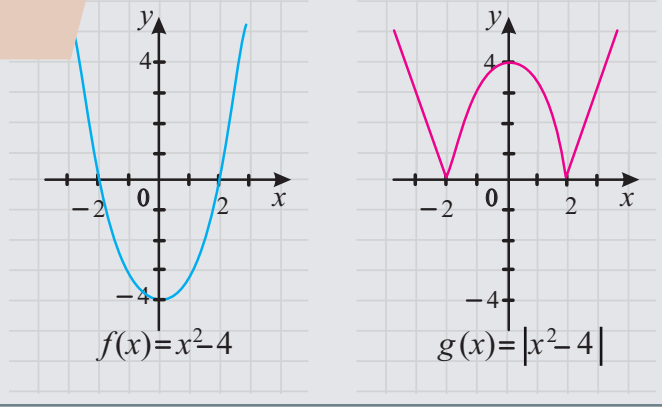
2. 7-rasmda $x \geq 0$ soha uchun funksiyaning grafigi berilgan. $x < 0$ sohada grafikni shunday quringki,
a) juft funksiya;
b) toq funksiya grafigi hosil bo'lsin.

7-rasm



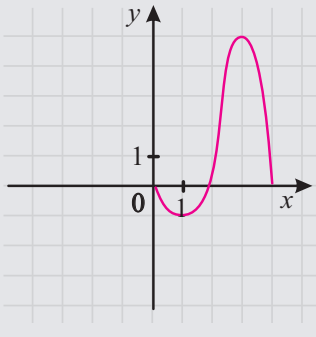
3. 8-rasmda $f(x) = x^2 - 4$ va $g(x) = |x^2 - 4|$ funksiya grafiklari berilgan. g funksiyaning grafigi f funksiyaning grafigidan qanday hosil qilinganini tushuntirib bering.

8-rasm

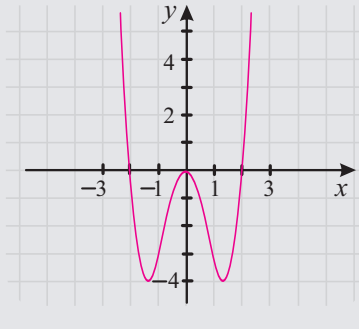


. 9-rasmda $x \geq 0$ soha uchun funksiyaning grafigi berilgan. $x < 0$ sohada grafikni shunday quringki,
a) juft funksiya;
b) toq funksiya grafigi hosil bo'lsin.

9-rasm



10-rasm

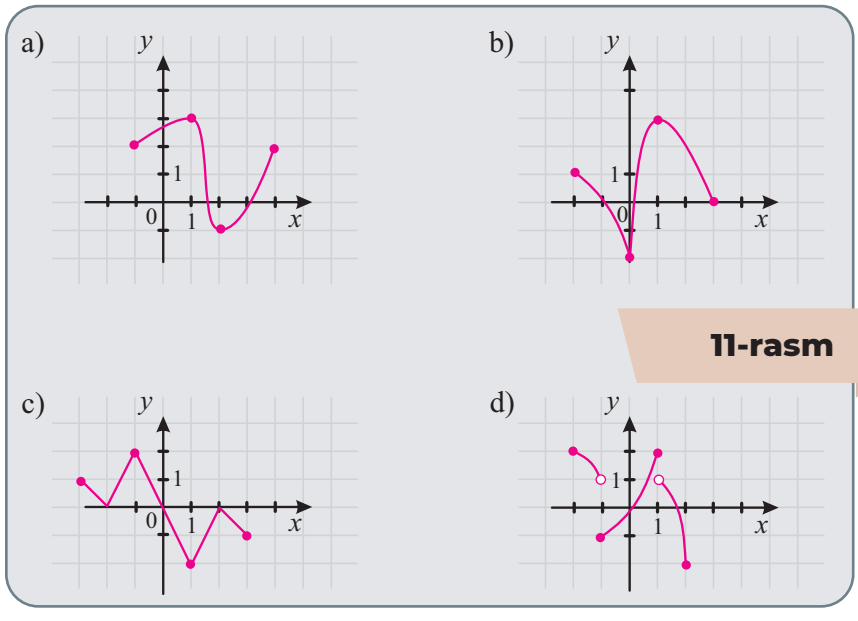


5. $f(x) = x^4 - 4x^2$ funksiyaning grafigi berilgan (10- rasm). Undan foydalanib, $g(x) = |x^4 - 4x^2|$ funksiyaning grafigini chizing.

6. 11-rasmda f funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagilarni taxminiy aniqlang:

1) f funksiyaning aniqlanish sohasini va qiymatlar to‘plamini.

2) f ning o‘sish va kamayish intervallarini.



11-rasm

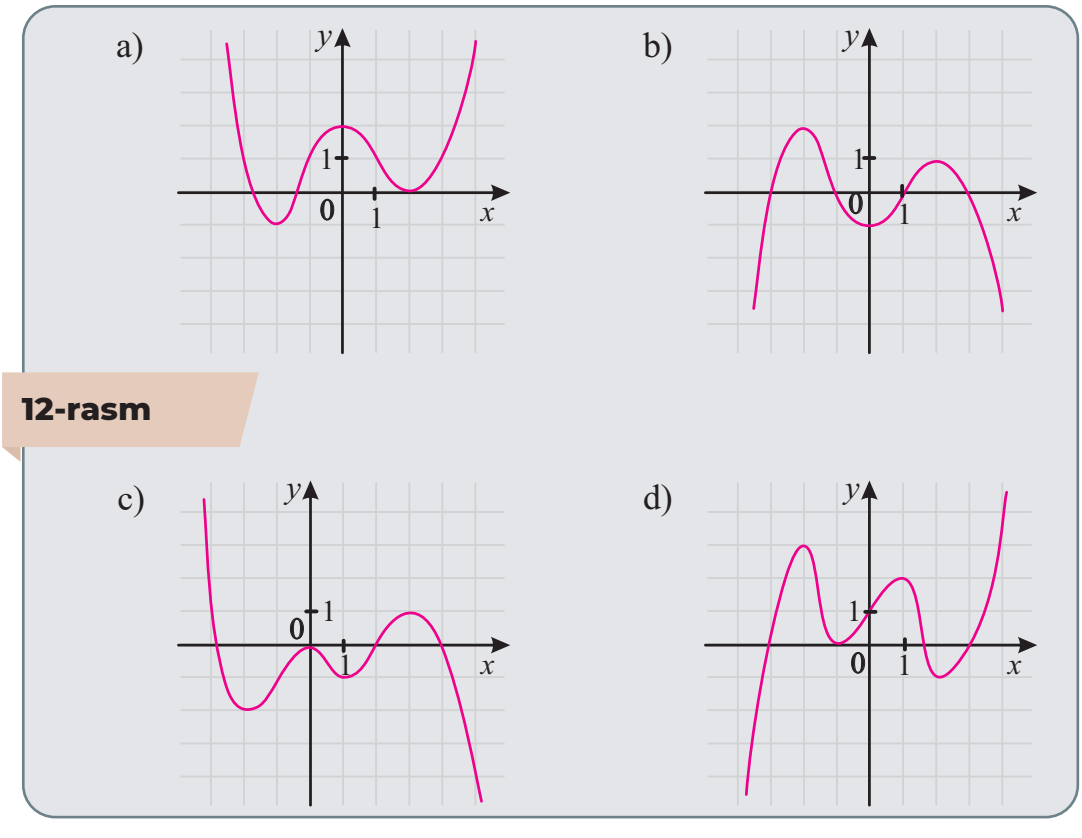
7. Berilgan funksiyalarning grafigini chizing, aniqlanish sohasini va qiymatlar to‘plamini aniqlang, o‘sish va kamayish intervallarini taxminiy toping.

a) $f(x)=x^2-5x$ b) $f(x)=x^3-4x$ c) $f(x)=x^4-16x^2$

8. f funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagilarni taxminiy aniqlang:

1) funksiyaning barcha ekstremum nuqtalarini va ekstremumlarini;

2) f ning o‘sish va kamayish intervallarini.



12-rasm

FUNKSIYA GRAFIKLARINI SODDA ALMASHTIRISHLAR

Funksiya grafigi ta'rifidan Oxy koordinatalar sistemasi kiritilgan tekislikdagi

$$\{(x, f(x)):x\in D(f)\}$$

to'plam $y=f(x)$ funksiyaning grafigini aniqlashi kelib chiqadi. Bu yerda $D(f)$ orqali $y=f(x)$ funksiyaning aniqlanish sohasi belgilangan.

Funksiya grafigini siljitish

Berilgan $y=f(x)$ funksiyaning grafigini Oxy tekisligida siljitish mumkin. Funksiya grafigining quyida keltiriladigan siljitishlarini ko'rib o'tamiz.

1) Funksiya grafigini Ox o'qi bo'ylab sijitish.
2) Funksiya grafigini Oy o'qi bo'ylab sijitish.
3) Funksiya grafigini biror vektor yo'nalishida sijitish.

1) Funksiya grafigini Ox o'qi bo'ylab x_0 birlik sijitish. Bunda berilgan $y=f(x)$ funksiya $y=f(x-x_0)$ ko'rinishga keladi. $y=f(x-x_0)$ funksiyaning aniqlanish sohasi ham Ox o'qi bo'ylab siljiydi va $D(f)+x_0=\{x+x_0 : x\in D(f)\}$ ko'rinishda bo'ladi.

$y=f(x-x_0)$ funksiyaning grafigi Oxy tekisligidagi

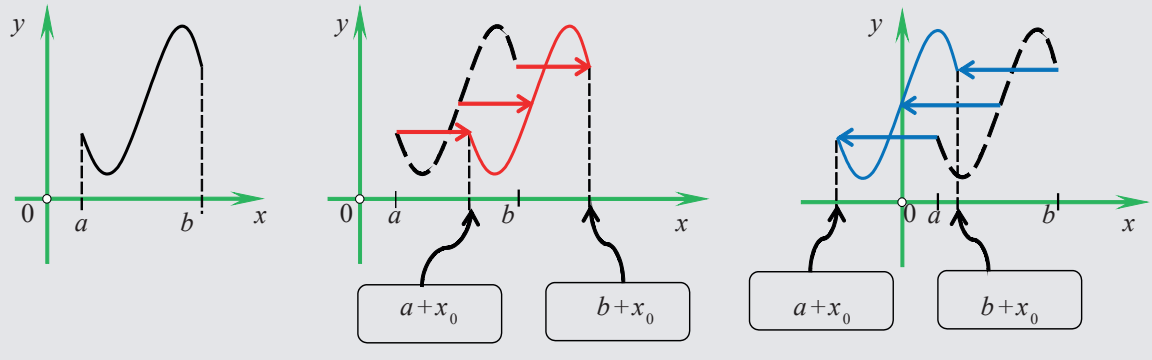
$$\{(x,f(x-x_0)):x\in D(f)+x_0\}$$

to'plamdan iborat bo'ladi. Bunda:

a) agar $x_0>0$ bo'lsa, grafik Ox o'qi yo'nalishida x_0 birlik siljiydi;

b) agar $x_0<0$ bo'lsa, grafik Ox o'qi yo'nalishiga qarshi $|x_0|$ birlik siljiydi.

1-rasm **$y=f(x)$ funksiya grafigini Ox o'qi bo'yliab siljitish.**



a) Berilgan $y=f(x)$ funksiya grafigi.

b) $x_0>0$ bo'lganda $y=f(x-x_0)$ funksiya grafigi. $y=f(x)$ funksiya grafigi o'ngga x_0 birlik siljigan.

c) $x_0<0$ bo'lganda $y=f(x-x_0)$ funksiya grafigi. $y=f(x)$ funksiya grafigi chapga $|x_0|$ birlik siljigan.

1-misol. Gorizontal siljitish.

$f(x)=x^2$ funksiya grafigidan foydalanib, quyidagi funksiyalar grafigini chizing.

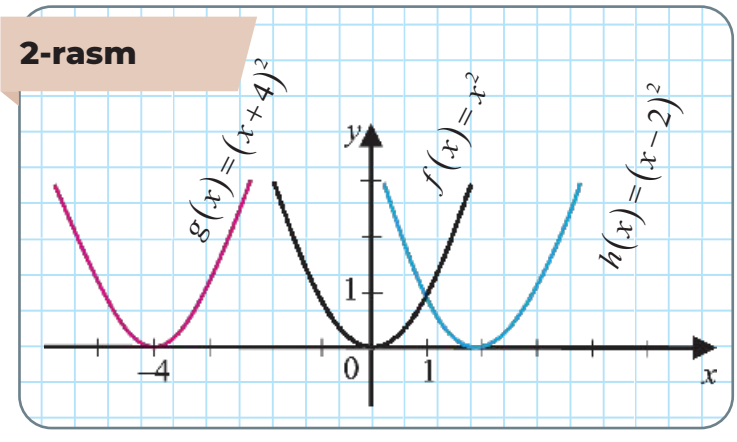
a) $g(x)=(x+4)^2$ b) $h(x)=(x-2)^2$

Yechish. 2-rasmda ko‘rsatilgandek

a) g funksiyaning grafigini yasash uchun, f funksiyaning grafigini chapga 4 birlikka siljitamiz.

b) h funksiyaning grafigini yasash uchun, f funksiyaning grafigini o‘ngga 2 birlikka siljitamiz.

2-rasm



2) Funksiya grafigini Oy o‘qi bo‘ylab y_0 birlik sijitish. Bunda berilgan $y=f(x)$ funksiya

$$y=f(x)+y_0$$

ko‘rinishga keladi. $y=f(x)+y_0$ funksiyaning $D(f+y_0)$ aniqlanish sohasi $y=f(x)$ funksiyaning $D(f)$ aniqlanish sohasi bilan ustma-ust tushadi:

$$D(f+y_0)=D(f).$$

$y=f(x)+y_0$ funksiyaning grafigi Oxy tekisligidagi

$$\{(x,f(x)+y_0): x\in D(f)\},$$

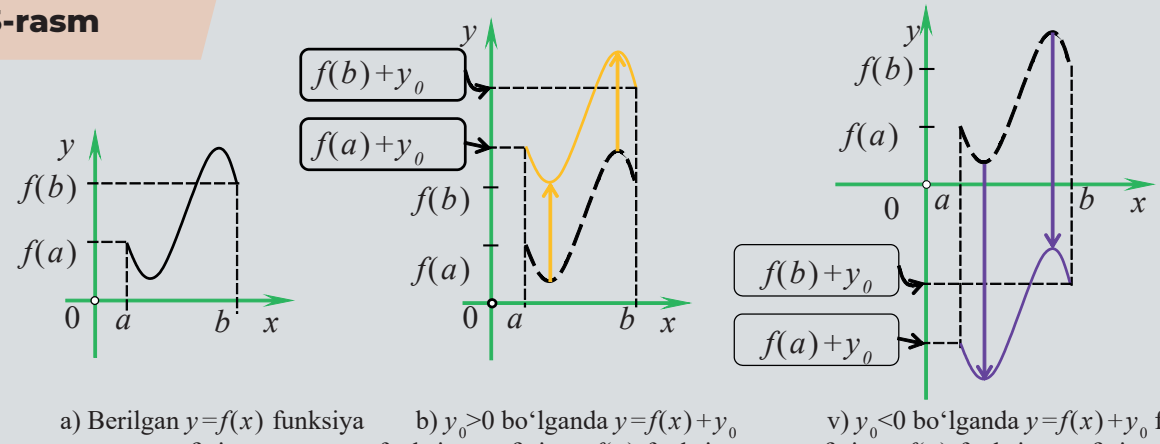
to‘plamdan iborat bo‘ladi. Bunda:

a) agar $y_0>0$ bo‘lsa, grafik Oy o‘qi yo‘nalishida y_0 birlik siljiydi;

b) agar $y_0<0$ bo‘lsa, grafik Oy o‘qi yo‘nalishiga qarshi $|y_0|$ birlik siljiydi. (3-rasm)

3-rasm

$y=f(x)$ funksiya grafigini Oy o‘qi bo‘ylab siljitish.



a) Berilgan $y=f(x)$ funksiya grafigi.

b) $y_0>0$ bo‘lganda $y=f(x)+y_0$ funksiya grafigi. $y=f(x)$ funksiya grafigi yuqoriga y_0 birlik siljigan.

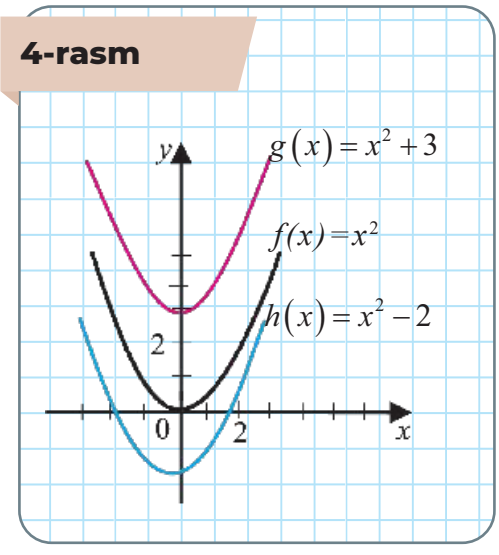
v) $y_0<0$ bo‘lganda $y=f(x)+y_0$ funksiya grafigi. $y=f(x)$ funksiya grafigi pastga $|y_0|$ birlik siljigan.

2-misol. Vertikal siljitish.

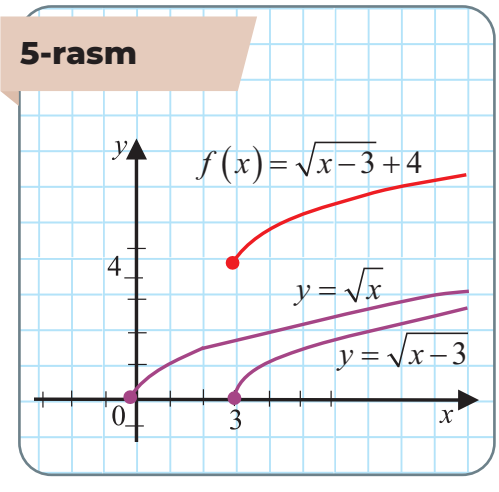
$f(x)=x^2$ funksiyadan foydalanib, quyidagi funksiyalarning grafigini chizing.

a) $g(x)=x^2+3$ b) $h(x)=x^2-2$

4-rasm



5-rasm



Yechish.

a) Quyidagiga e'tibor bering:

$$g(x) = x^2 + 3 = f(x) + 3$$

Demak, 4-rasmda ko'rsatilgandek g funksiyaning grafigini chizish uchun, f funksiyaning grafigini yuqoriga 3 birlikka siljitamiz (ko'taramiz).

b) Xuddi shunday, h funksiyaning grafigini chizish uchun, f funksiyaning grafigini pastga 3 birlikka siljitamiz (tushiramiz).

3-misol. Gorizontal va vertikal siljitish kombinatsiyasi.

$f(x) = \sqrt{x-3} + 4$ funksiyaning grafigini chizing.

Yechish.

Grafikni $y = \sqrt{x}$ funksiyaning grafigini chizishdan boshlaymiz. Uni 3 birlikka o'ngga siljitamiz va $y = \sqrt{x-3}$ funksiyaning grafigini hosil qilamiz. Keyin bu grafikni 4 birlikka yuqoriga siljitamiz va $f(x) = \sqrt{x-3} + 4$ funksiyaning grafigini hosil qilamiz (5-rasm).

3) Funksiya grafigini $\vec{p} = (x_0, y_0)$ vektor yo'nalishida sijitish. Bunda berilgan $y = f(x)$ funksiya

$$y = f(x - x_0) + y_0$$

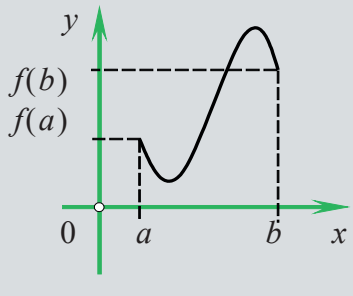
ko'rinishga keladi. $y = f(x - x_0) + y_0$ funksiyaning $D(f) + x_0$ aniqlanish sohasi Ox o'qi bo'ylab siljiydi va

$D(f) + x_0 = \{x + x_0 : x \in D(f)\}$ ko'rinishda bo'ladi.

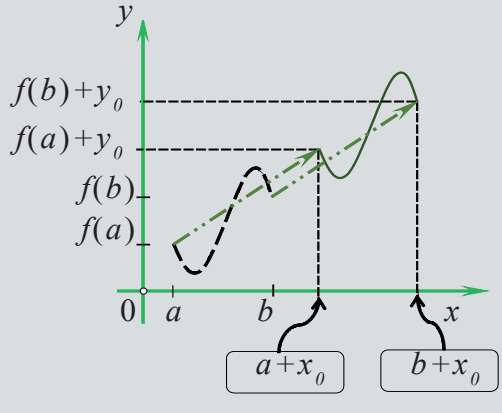
$y = f(x - x_0) + y_0$ funksiyaning grafigi Oxy tekisligidagi $\{(x, f(x - x_0) + y_0) : x \in D(f) + x_0\}$ to'plamdan iborat bo'ladi. Grafik $\vec{p} = (x_0, y_0)$ vektor yo'nalishida $\sqrt{x_0^2 + y_0^2}$ birlik siljiydi (6-rasm).

6-rasm

$y=f(x)$ funksiya grafigini $\vec{p} = (x_0, y_0)$ vektor yo'nalishida siljitish.



a) Berilgan $y=f(x)$ funksiya grafigi.



b) $y=f(x-x_0)+y_0$ funksiya grafigi. $y=f(x)$ funksiya grafigi $\vec{p} = (x_0, y_0)$ vektor yo'nalishida $\sqrt{x_0^2 + y_0^2}$ birlik siljigan.

Funksiya grafiklarini siqish, cho‘zish

Berilgan $y=f(x)$ funksiyaning grafigini Oxy tekisligida deformatsiyalash (siqish yoki cho‘zish) mumkin. Ikkita muhim holni ko ‘rib o‘tamiz.

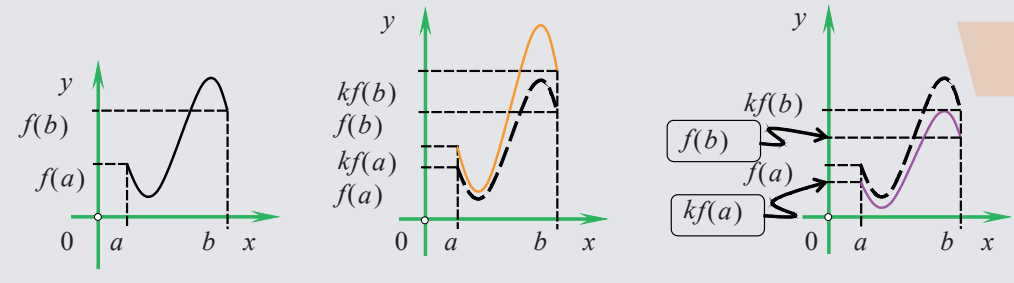
1-hol. Bunda berilgan $y=f(x)$ funksiya $y=kf(x)$ ko‘rinishga keladi va ushbu funksiyaning $D(kf)$ aniqlanish sohasi $y=f(x)$ funksiyaning $D(f)$ aniqlanish sohasi bilan bir xil bo‘ladi: $D(kf)=D(f)$. Bunda:

a) agar $k>1$ bo‘lsa, grafik Ox o‘qidan Oy o‘qi bo‘ylab k marta cho‘ziladi;

b) agar $0<k<1$ bo‘lsa, grafik Ox o‘qiga Oy o‘qi bo‘ylab k marta siqiladi.

y=f(x) funksiya grafigining deformatsiyasi

7-rasm



a) Berilgan $y=f(x)$ funksiya grafigi.

b) $k>1$ bo‘lganda $y=kf(x)$ funksiya grafigi. $y=f(x)$ funksiya grafigi cho‘zilgan.

c) $0<k<1$ bo‘lganda $y=kf(x)$ funksiya grafigi. $y=f(x)$ funksiya grafigi siqilgan.

Funksiya grafigini manfiy k songa ham ko‘paytirish mumkin. Bunda $y=kf(x)$ funksiya grafigi $y=|k|f(x)$ funksiya grafigiga Ox o‘qqa nisbatan simmetrik bo‘ladi.

2-hol. Endi $y=f(x)$ funksiyaning $D(f)$ aniqlanish sohasi 0 nuqtaga nisbatan simmetrik, aytaylik $D(f(x))=[-a,a]$ bo‘lsin. U holda, $y=f(mx)$ funksiyaning aniqlanish sohasi $D(f(mx))=\left[-\frac{a}{m},\frac{a}{m}\right]$ bo‘ladi. Bunda:

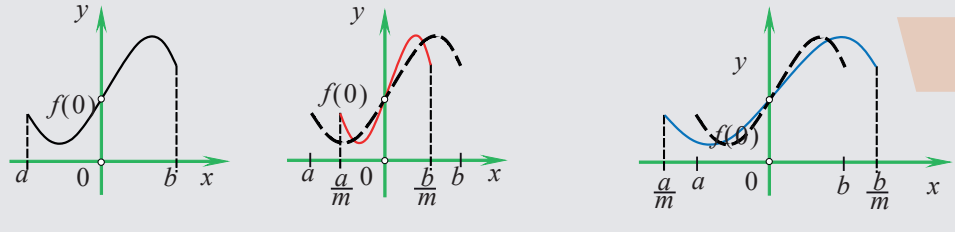
a) agar $m>1$ bo‘lsa, grafik Oy o‘qiga Ox o‘qi bo‘ylab m marta siqiladi;

b) agar $0<m<1$ bo‘lsa, grafik Oy o‘qidan Ox o‘qi bo‘ylab m marta cho‘ziladi.

Funksiya argumentini manfiy m songa ham ko‘paytirish mumkin. Bunda $y=f(mx)$ funksiyaning grafigi $y=f(|m|x)$ funksiya grafigiga Oy o‘qqa nisbatan simmetrik bo‘ladi.

y=f(x) funksiya grafigining deformatsiyasi

8-rasm



a) Berilgan $y=f(x)$ funksiya grafigi.

b) $m>1$ bo‘lganda $y=f(mx)$ funksiya grafigi. $y=f(x)$ funksiya grafigi siqilgan.

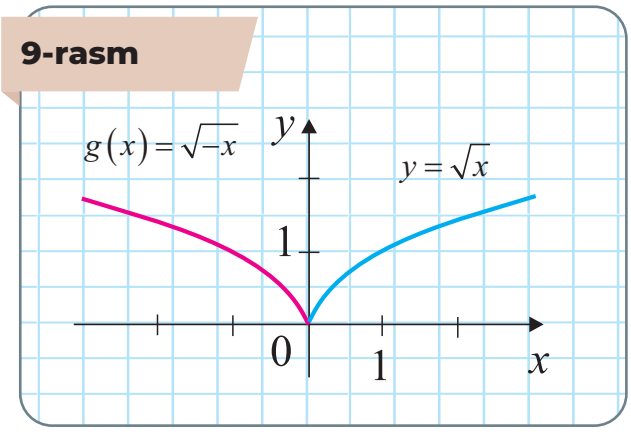
c) $0<m<1$ bo‘lganda $y=f(mx)$ funksiya grafigi. $y=f(x)$ funksiya grafigi cho‘zilgan.

4-misol. Grafiklarni akslantirish

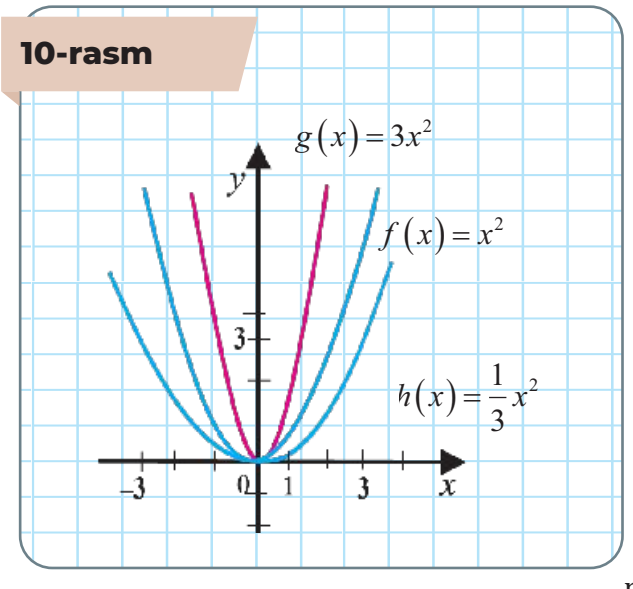
Quyidagi funksiyalarning grafigini chizing.

a) $f(x)=-x^2$ b) $g(x)=\sqrt{-x}$

9-rasm



10-rasm



Yechish.

9-rasmda $y=\sqrt{x}$ funksiyaning grafigini chizamiz. Bu grafikni y o‘qiga nisbatan simmetrik akslantirish orqali $g(x)=\sqrt{-x}$ funk- siyaning grafigi hosil qilinadi.

E‘tibor qiling: $g(x)=\sqrt{-x}$ funksiyaning aniqlanish sohasi: $\{x|x\le 0\}$ dan iborat.

5-misol. Vertikal cho‘zish va siqish.

10-rasmda $f(x)=x^2$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini chizing.

a) $g(x)=3x^2$

b) $h(x)=\frac{1}{3}x^2$

Yechish.

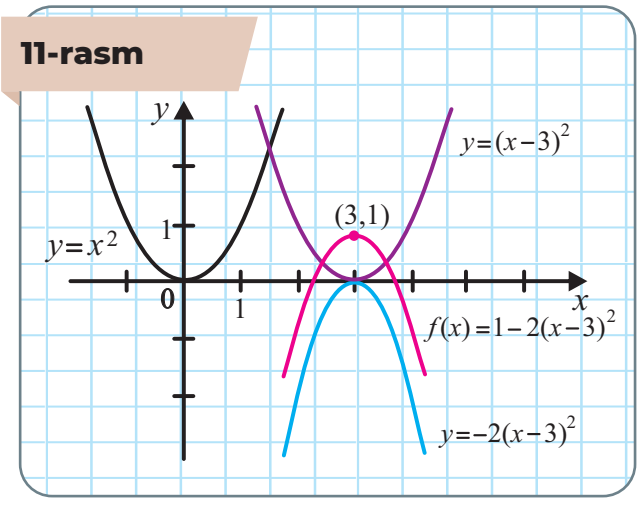
a) g funksiyaning grafigi f funksiyaning har bir nuqtasining y koorinatasini 3 ga ko‘paytirishdan hosil bo‘ladi. Ya‘ni, g funksiyaning grafigini hosil qilish uchun, f funksiyaning grafigini vertikal 3 baravar cho‘zish kerak. Hosil bo‘lgan parabola, dastlabki parabolaga nisbatan "ingichka" bo‘ladi.

b) h funksiyaning grafigi f funksiyaning har bir nuqtasining y koorinatasini $\frac{1}{3}$ ga ko‘paytirishdan hosil bo‘ladi. Ya‘ni, h funksiyaning grafigini hosil qilish uchun, f funksiyaning grafigini vertikal yo‘nalishda x o‘qqa 3 baravar siqish kerak. Hosil bo‘lgan parabola, dastlabki parabolaga nisbatan "yo‘g‘on" bo‘ladi. (10-rasm)

6-misol. Siljitish, vertikal cho‘zish va akslantirishning kombinatsiyasi.

$f(x)=1-2(x-3)^2$ funksiyaning grafigini chizing.

11-rasm



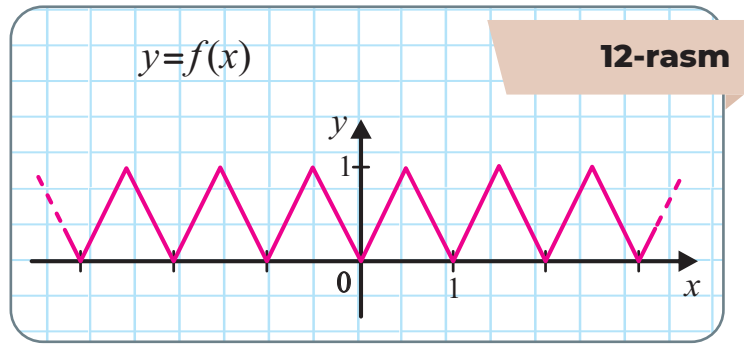
Yechish.

Avval $y=x^2$ funksiyaning grafigini o‘ngga 3 birlikka gorizontal siljitamiz va $y=(x-3)^2$ funksiyaning grafigini hosil qilamiz. Keyin bu grafikni Ox o‘qiga nisbatan akslantiramiz va 2 baravar cho‘zishni bajaramiz va $y=-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. Nihoyat, bu grafikni yuqoriga 1 birlikka siljitamiz va $f(x)=1-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. (11-rasm)

7-misol. Gorizontal cho‘zish va siqish.

12- rasmda $y=f(x)$ funksiyaning grafigi berilgan. Bu grafikdan foydalanib, quyidagi funksiyalarning grafigini chizing.

a) $y=f(2x)$; b) $y=f\left(\frac{1}{2}x\right)$.

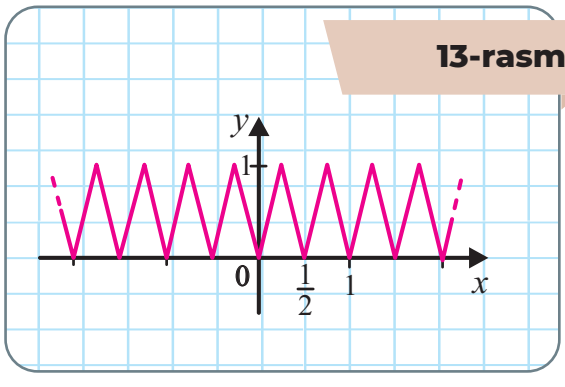


12-rasm

Yechish.

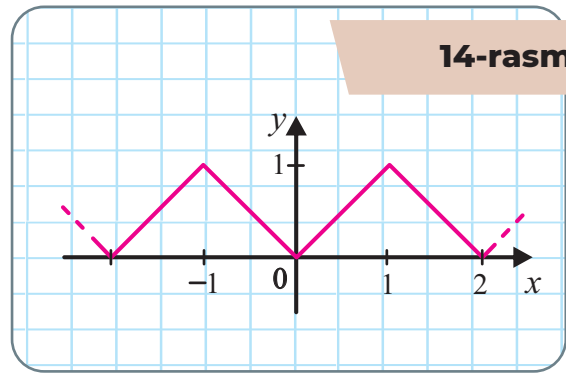
a) Har bir nuqtaning x koordinatasini 2 ga ko‘paytirib f funksiyaning grafigini gorizontal yo‘nalishda siqamiz (13-rasm).

b) Har bir nuqtaning x koordinatasini $\frac{1}{2}$ va ko‘paytirib f funksiyaning grafigini gorizontal yo‘nalishda cho‘zamiz (14- rasm).



13-rasm

$y=f(2x)$



14-rasm

$y=f\left(\frac{1}{2}x\right)$

MISOLLAR

1. $f(x)$ funksiya grafigi berilgan bo‘lsa, quyidagi funksiyalarning grafigi qanday chizilishini tushuntiring.

a) $y=f(x)-1$ b) $y=f(x-2)$ c) $y=f(x+5)$

d) $y=f(x)+4$ e) $y=f(-x)$ f) $y=3f(x)$

g) $y=-f(x)$ h) $y=\frac{1}{3}f(x)$ i) $y=f(x-5)+2$

j) $y=f(x+1)-1$ k) $y=f(x+3)+2$ l) $y=f(x-7)-3$

m) $y=-f(x)+5$ n) $y=3f(x)-5$ o) $y=1-f(-x)$

p) $2-\frac{1}{5}f(x)$ q) $y=2f(x+5)-1$ r) $y=\frac{1}{4}f(x-3)+5$

s) $y=\frac{1}{3}f(x-2)+5$ t) $y=4f(x+1)+3$ u) $y=f(4x)$

v) $y=f\left(\frac{1}{4}x\right)$ x) $y=f(2x)-1$ y) $y=2f\left(\frac{1}{2}x\right)$

2. g funksiyaning grafigi f funksiyaning grafigidan qanday almashtirishlar yordamida hosil qilinganini tushuntiring.

a) $f(x)=x^2,\ g(x)=(x+2)^2$
b) $f(x)=x^2,\ g(x)=x^2+2$
c) $f(x)=x^3,\ g(x)=(x-4)^3$
d) $f(x)=x^3,\ g(x)=x^3-4$
e) $f(x)=|x|,\ g(x)=|x+2|-2$
f) $f(x)=|x|,\ g(x)=|x-2|+2$
g) $f(x)=\sqrt{x},\ g(x)=-\sqrt{x}+1$
h) $f(x)=\sqrt{x},\ g(x)=\sqrt{-x}+1$

3. $y=x^2$ funksiyaning grafigidan foydalanib, quyidagi funksiyalarning grafigini chizing.

a) $g(x)=x^2+1$
b) $g(x)=(x-1)^2$
c) $g(x)=-x^2$
d) $g(x)=(x-1)^2+3$

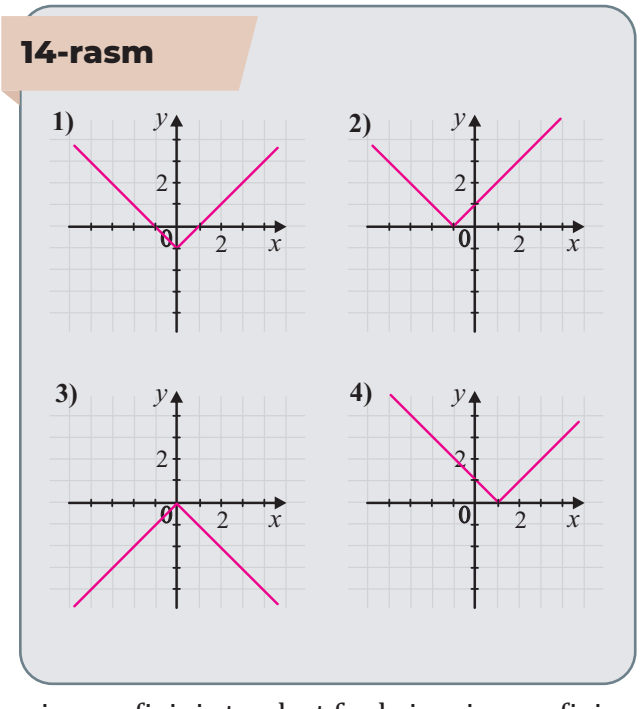
4. $y=\sqrt{x}$ funksiyaning grafigidan foydalanib, quyidagi funksiyalarning grafigini chizing.

a) $g(x)=\sqrt{x-2}$
b) $g(x)=\sqrt{x}+1$
c) $g(x)=\sqrt{x+2}+2$
d) $g(x)=-\sqrt{x}+1$

5. Berilgan funksiyalarga 14-rasmda berilgan grafiklardan mosini toping.

a) $y=|x|-1$
b) $y=|x|-1$
c) $y=|x-1|$
d) $y=-|x|$

14-rasm



6. Quyidagi funksiyalarning grafigini standart funksiyaning grafigi ustida mos almashtirishlarni bajarib chizing.

a) $f(x)=x^2+3$
b) $f(x)=\sqrt{x}+1$
c) $f(x)=|x|-1$
d) $f(x)=\sqrt{x}+1$
e) $f(x)=(x-5)^2$
f) $f(x)=(x+1)^2$

g) $f(x)=|x+2|$
h) $f(x)=\sqrt{x-4}$
i) $f(x)=-x^3$
j) $f(x)=-|x|$
k) $y=\sqrt[4]{-x}$
l) $y=\sqrt[3]{-x}$
n) $y=\frac{1}{4}x^2$
o) $y=-5\sqrt{x}$
p) $y=3|x|$
q) $y=\frac{1}{2}|x|$
r) $y=(x-3)^2+5$
s) $y=\sqrt{x+4}-3$
t) $y=3-\frac{1}{2}(x-1)^2$
u) $y=2-\sqrt{x+1}$
v) $y=|x+2|+2$
x) $y=2-|x|$
y) $y=\frac{1}{2}\sqrt{x+4}-3$
z) $y=3-2(x-1)^2$.

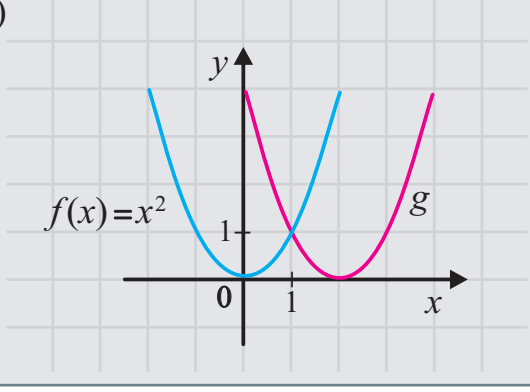
7. Berilgan f funksiyaning grafigiga berilgan ketma-ketlikda ko‘rsatilgan almashtirishlar qo‘llanilgan. Yakuniy funksiyaning formulasini yozing.

a) $f(x)=x^2$, 3 birlik pastga siljiting.
b) $f(x)=x^3$, 5 birlik yuqoriga siljiting.
d) $f(x)=\sqrt{x}$, 2 birlik chapga siljiting.
e) $f(x)=\sqrt[3]{x}$, 1 birlik o‘ngga siljiting.
f) $f(x)=|x|$, 2 birlik chapga va 5 birlik pastga siljiting.
g) $f(x)=|x|$, x o‘qiga nisbatan akslantirib, 4 birlik o‘ngga va 3 birlik yuqoriga siljiting.
h) $f(x)=\sqrt[4]{x}$, y o‘qiga nisbatan simmetrik akslantiring va 1 birlik yuqoriga siljiting.
i) $f(x)=x^2$, 2 birlik chapga siljiting va x o‘qiga nisbatan simmetrik akslantiring.
j) $f(x)=x^2$, 2 baravar vertikal cho‘zib, 2 birlik pastga va 3 birlik o‘ngga siljiting.
k) $f(x)=|x|$, $\frac{1}{2}$ baravar vertikal yo‘nalishda siqishni bajarib, 1 birlik chapga va 3 birlik yuqoriga siljiting.

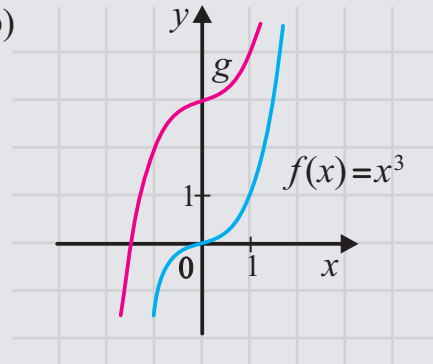
8. f va g funksiyalarning grafigi berilgan (15-rasm). f funksiyadan foydalanib g funksiyaning formulasini toping.

15-rasm

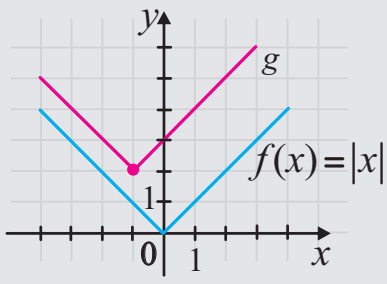
a)



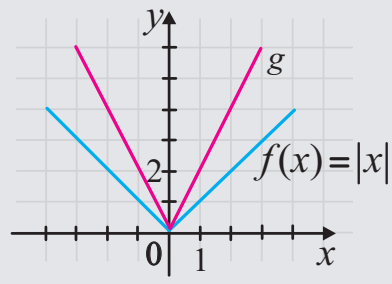
b)



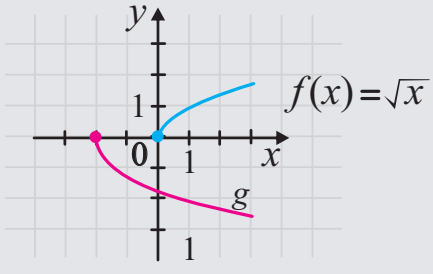
c)



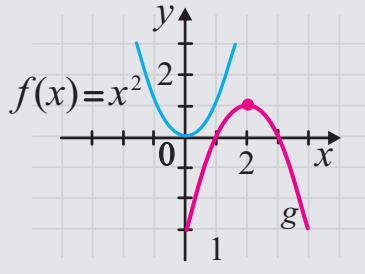
d)



e)



f)



9. $y=f(x)$ funksiya berilgan, 16-rasmda quyidagilarga mos grafikni toping.

a) $y=f(x-4)$

b) $y=f(x)+3$

c) $y=2f(x+6)$

d) $y=-f(2x)$

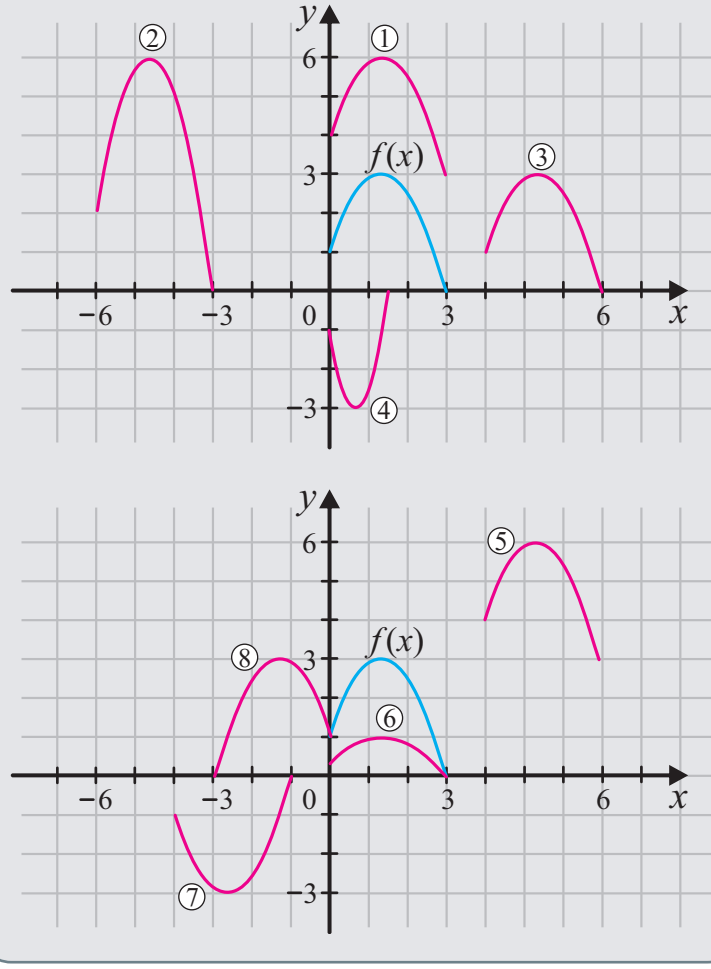
e) $y=\frac{1}{3}f(x)$

f) $y=-f(x+4)$

g) $y=f(x-4)+3$

h) $f(-x)$

16-rasm



CHIZIQLI VA KVADRATIK MODELLASHTIRISH

Matematik modellashtirish iqtisodiy jarayonlarni o‘rganishning asosiy analitik vositasi hisoblanadi.

Ushbu masalani qaraylik.

1-misol. Korxona ikki xil kiyim: yubka va shim tikishga ixtisoslashgan. Bunda 1 ta shim ishlab chiqarishdan 25 000 so‘m, 1 ta yubka ishlab chiqarishdan esa 20 000 so‘m foyda olinishi ma’lum. 1 ta shimni tikish uchun 1,5 metr mato, 3 ta tugma va 1 ta zigzag talab etiladi. 1 ta yubka tikish uchun esa 1,8 metr mato, 4 ta tugma va 3 ta zigzag lozim bo‘ladi.

Korxona omborida 900 m mato, 2100 ta tugma va 1200 ta zigzag zahirasi borligi ma’lum. Ishlashining shunday rejasini tuzingki, bu reja asosida ishlab chiqarilgan mahsulotlar eng ko‘p foyda keltirsin.

Yechish. Masalada keltirilgan shartlarni quyidagi jadval shaklida yozib olaylik.

Xomashyo turi	Xomashyo zahirasi	Mahsulot turi va xomashyo sarfi	
		Shim	Yubka
Mato	900 m	1,5 m	1,8 m
Tugma	2100 ta	3 ta	4 ta
Zigzag	1200 ta	1 ta	3 ta
Bitta tayyor mahsulot ishlab chiqarishdagi foyda		25 000 so‘m	20 000 so‘m

Masalaning **matematik modelini quramiz.**

x_1 orqali ishlab chiqarilishi rejalashtirilgan shim miqdorini,

x_2 orqali esa ishlab chiqarilishi rejalashtirilgan yubka miqdorini belgilaymiz.

Ravshanki, x_1, x_2 miqdorlar manfiy bo‘la olmaydi, ya’ni

$$x_1 \geq 0,\ x_2 \geq 0 \text{ bo‘ladi.}$$

x_1 ta shim ishlab chiqarishda $25\,000x_1$ so‘m foyda ko‘riladi;

x_2 ta yubka ishlab chiqarishda $20\,000x_2$ so‘m foyda ko‘riladi.

Bunda umumiy foyda

$$F = 25\,000x_1 + 20\,000x_2$$

so‘m bo‘ladi.

x_1 ta shim ishlab chiqarishda $1{,}5x_1$ metr mato sarflanadi;

x_2 ta yubka ishlab chiqarishda $1{,}8x_2$ metr mato sarflanadi.

Lekin mato miqdori cheklangan, 900 metr mato bor xolos. Demak,

$$1{,}5x_1 + 1{,}8x_2 \leq 900$$

tengsizlik bajarilishi kerak.

Xuddi shuningdek,

x_1 ta shim ishlab chiqarishda $3x_1$ ta tugma tikiladi;

x_2 ta yubka ishlab chiqarishda $4x_2$ ta tugma tikiladi.

Tugma zahirasi 2100 ta bo‘lgani uchun tikilgan barcha tugma 2100 dan oshmasligi kerak:

$$3x_1 + 4x_2 \leq 2100.$$

Va nihoyat,

x_1 ta shim ishlab chiqarishda x_1 ta zigzag ishlatiladi;

x_2 ta yubka ishlab chiqarishda $3x_2$ ta zigzag ishlatiladi.

Ishlatilgan zigzaglar miqdori ombordagi mavjud 1200 ta zigzagdan oshib keta olmaydi:

Shunday qilib, qo‘yilgan masalada ushbu

$$\begin{cases} x_1+3x_2\le 1200. \\ 1,5x_1+1,8x_2\le 900, \\ 3x_1+4x_2\le 2100, \\ x_1+3x_2\le 1200, \end{cases}$$

$$x_1\ge 0,\ x_2\ge 0$$

shartlar bajarilganda

$$F=25\ 000x_1+20\ 000x_2$$

kattalikning eng katta qiymatini topish talab etilmoqda ekan. Natijada qo‘yilgan masalaning ushbu **matematik modeli**ga ega bo‘ldik:

$$F=25\ 000x_1+20\ 000x_2\to\max,$$

$$\begin{cases} 1,5x_1+1,8x_2\le 900, \\ 3x_1+4x_2\le 2100, \\ x_1+3x_2\le 1200, \end{cases}$$

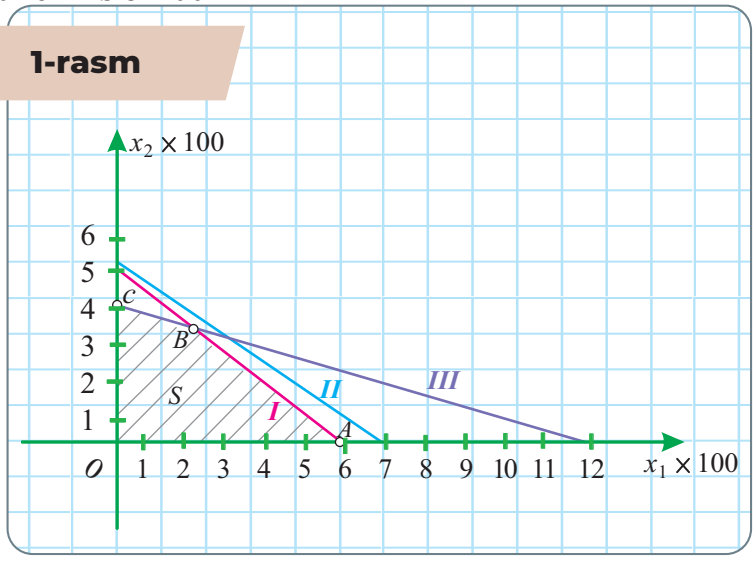
$$x_1\ge 0,\ x_2\ge 0.$$

Bu modeldagi o‘zgaruvchilar birinchi darajali bo‘lib, o‘zaro chiziqli amallar (qo‘shish va songa ko‘paytirish) orqali bog‘langan. Shuning uchun bu tipdagi matematik modellar ***chiziqli modellar*** deyiladi. Qo‘yilgan masalani chiziqli model shakliga olib kelish jarayoni ***chizqli modellashtirish*** deyiladi.

Hosil qilingan chiziqli model yechimini ko‘rsatish uchun Ox_1x_2 Dekart koordinatalar sistemasi kiritiladi. Unda ushbu to‘g‘ri chiziqlar chizib olinadi:

$I: 1,5x_1+1,8x_2=9,$
$II: 3x_1+4x_2=21,$
$III: x_1+3x_2=12.$

1-rasm



Tekislikda Ox_1 va Ox_2 o‘qlari hamda va to‘g‘ri chiziqlar bilan chegaralangan soha hosil bo‘ldi (bu soha to‘g‘ri chiziq bilan chegaralanmagan). Shuning uchun va to‘g‘ri chiziqlar kesishish nuqtasi topiladi:

$$\begin{cases} 1,5x_1+1,8x_2=9, \\ x_1+3x_2=12, \end{cases}\Rightarrow\begin{cases} x_1=2, \\ x_2=3\frac{1}{3}. \end{cases}$$

s sohaning uchlari O(0, 0), A(6, 0), B(2, $3\frac{1}{3}$), C(0, 4) topiladi. Aslida bu nuqtalarning koordinatalari O(0, 0), A(600, 0), B(200,33 $3\frac{1}{3}$), C(0, 400)

bo‘ladi (**savol:** nima uchun?). Ishlab chiqariladigan yubka butun son bo‘lishi kerak, shuning uchun B nuqtada x_2 o‘zgaruvchining qiymati 333 bo‘lishi kerak: $B(200,333)$

Diqqat! $F=25\ 000x_1+20\ 000x_2$ miqdor o‘zining eng katta qiymatiga s sohaning uchlarida erishadi. Bu qiymatlarni hisoblaymiz: $F(0, 0)=0$,

$F(600,0)=25\ 000\cdot600=15\ 000\ 000$,
$F=25\ 000\cdot200+20\ 000\cdot333=11\ 660\ 000$,
$F=20\ 000\cdot400=8\ 000\ 000$.

Javob: Ombordagi zahiraning hammasini shim tikishga sarflansa, sex eng ko ‘p foyda olar ekan.

2-misol. O‘quvchi Oxy koordinatalar tekisligini shunday tanladiki, bunda o‘z uyini koordinata boshi $O(0;0)$ deb oldi. Keyin o‘zi o‘qiydigan maktab $C(4,3)$ nuqtada joylashganini aniqladi. Yo‘lning uyi va maktab orasidan o‘tadigan to‘g‘ri chiziqli qismi Ox o‘qini $(6,0)$ nuqtada, Oy o‘qini $(0,4)$ nuqtada kesib o‘tishini hisoblab chiqdi.

Maktabga uyali aloqa kompaniyasining antennasi o‘rnatilganligi ma’lum. O‘quvchi yo‘lda harakatlanayotgan avtomobildagi yo‘lovchining uyali aloqa vositasi antennadan tarqalayotgan to‘lqinni eng yaxshi tutadigan nuqtani topishga qiziqib qoldi.

Topshiriq. Siz bu masalani qanday yechgan bo‘lar edingiz?

Yechish. Ravshanki, yo‘lning maktabga eng yaqin nuqtasida uyali aloqa vositasi to‘lqinni eng yaxshi tutadi. Bu masalani yechishda yo‘lni tavsiflovchi (AB) to‘g‘ri chiziq tenglamasini tuzish va uning maktabga eng yaqin nuqtasining koordinatalarini topish kerak. Buning uchun avvalo bayon etilganlar asosida vaziyatning chizmasi chiziladi (rasmga qarang).

Keyin $A(6,0)$ va $B(0,4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi tuziladi. Buning uchun to‘g‘ri chiziqning

$$y=kx+b$$

tenglamasiga $A(6,0)$ va $B(0,4)$ nuqtalarning koordinatalarini qo‘yib, ushbu

$$0=k\cdot6+b,$$
$$4=k\cdot0+b$$

tengliklar hosil qilinadi. Ulardan

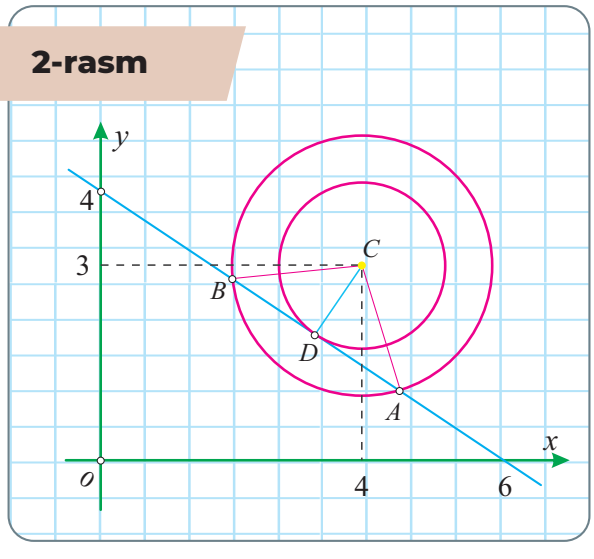
$$b=4,$$
$$k=-\frac{2}{3}$$

koeffitsiyentlar topiladi. Demak, (AB) to‘g‘ri chiziq tenglamasi

$$y=-\frac{2}{3}x+4$$

bo‘ladi.

2-rasm



Masalaning yechimi (AB) to‘g‘ri chiziqning $C(4,3)$ nuqtaga eng yaqin $D(x, y)$ nuqtasini topishdan iborat. Bu vaziyatning matematik modeli quyidagicha yoziladi:

$$F=\sqrt{(x-4)^2+(y-3)^2}\to min,$$
$$y=-\frac{2}{3}x+4$$

Bu modeldagi o‘zgaruvchilar birinchi va ikkinchi darajali bo‘lgani uchun bu tipdagi matematik modellar ***kvadratik modellar*** deyiladi. Qo‘yilgan masalani kvadrat model shakliga olib kelish jarayoni ***kvadratik modellashtirish*** deyiladi.

MISOLLAR

1. Har bir berilgan topshiriqning chiziqli modelini burchak koeffitsiyenti bilan yozing:

a) Siz velosipedni 10 000 so‘m boshlang‘ich to‘lov va soatiga 5000 so‘m tarif bo‘yicha ijaraga oldingiz.

b) Avtomobillarni ta’mirlash ustaxonasi 50 000 so‘m bazaviy to‘lov hamda soatiga 15 000 so‘mdan to‘lov belgiladi.

c) Shamning uzunligi 30 cm va u soatiga 1,4 cm tezlikda yonadi.

d) Dasturlash bo‘yicha mutaxassis maslahat uchun alohida $75 va undan so‘ng soatiga $35 oladi.

e) Hozirgi harorat 25 °C va uning kechasi har soatda 2 °C ga tushishi kutilmoqda.

f) Langar aholisi 6791 kishini tashkil etadi va yiliga 7 taga kamayib bormoqda.

2. Berilgan jadvaldagi funksiya chiziqli yoki kvadratik ekanini aniqlang.

x	0	1	3	4	6
y	5	10	20	25	35

3. To‘p tepaga va pastga sakraganda, uning erishadigan balandligi doimiy ravishda kamayadi. Quyidagi jadvalda vaqt bo‘yicha sakrash balandligi ko‘rsatilgan.

a) Eng mos keladigan kvadrat funksiyani toping.

b) To'pning maksimal balandligini toping.

c) 2,5 sekundda to'pning qancha balandlikda bo‘lganini taxmin qiling.

t (s)	2	2,2	2,4	2,6	3
h (dyum)	2	16	26	33	42

4. Agar tosh 70 metrli binoning tepasidan otilgan bo‘lsa, toshning vaqtga bog‘liq balandligi $h(t) = -9{,}8t^2 - 10t + 270$ kvadrat funksiya bilan berilgan, bu erda t sekundda, balandligi esa metrda. Necha soniyadan keyin tosh yerga tegadi?

5. Malikaga xonasini tozalash uchun Umidadan ikki baravar ko‘p vaqt kerak bo‘ladi. Azizaga xonasini tozalash uchun Umidadan 10 minut ko‘proq vaqt ketadi. Hammasi bo‘lib ular xonalarini tozalash uchun 90 minut sarflaydi. Malika xonasini tozalashi uchun qancha vaqt sarflaydi?

6. Dilshod dengizga durni olish uchun sho‘ng‘idi. Uning t sekunddan keyingi sho‘ng‘ish chuqurligi $h(t) = -4t^2 + 4t + 3$ metr bo‘ldi, $t \geq 0$.

a) durlar qanday chuqurlikda joylashgan?

b) Dilshod durni olish uchun qancha vaqt sarflaydi?

c) Dilshod qanday balandlikdan suvga sho‘ng‘idi?

7. Jasmina ko‘ylak tikish uchun buyurtma oldi. U bir kunda x ta ko‘ylak tiksa, $P(x)=-x^2 + 20x$ so‘m miqdorida daromad oladi.

a) Eng katta daromad olish uchun u qancha ko‘ylak tikish kerak?

b) Eng katta daromad necha so‘mga teng?

8. 2005 yilda Zarafshon shahri aholisi 55 000 ga yaqin edi. O‘sha paytda aholi soni yiliga 2000 ga yaqin sur’atlarda o‘sib borardi. Har qanday yil uchun Zarafshon aholisini topish uchun uning chiziqli modelini tuzing. 2010 yilda Zarafshon aholisi qancha bo‘lgan? 2025 yil uchun Zarafshon aholisi soni qancha bo‘lishini hisoblang.

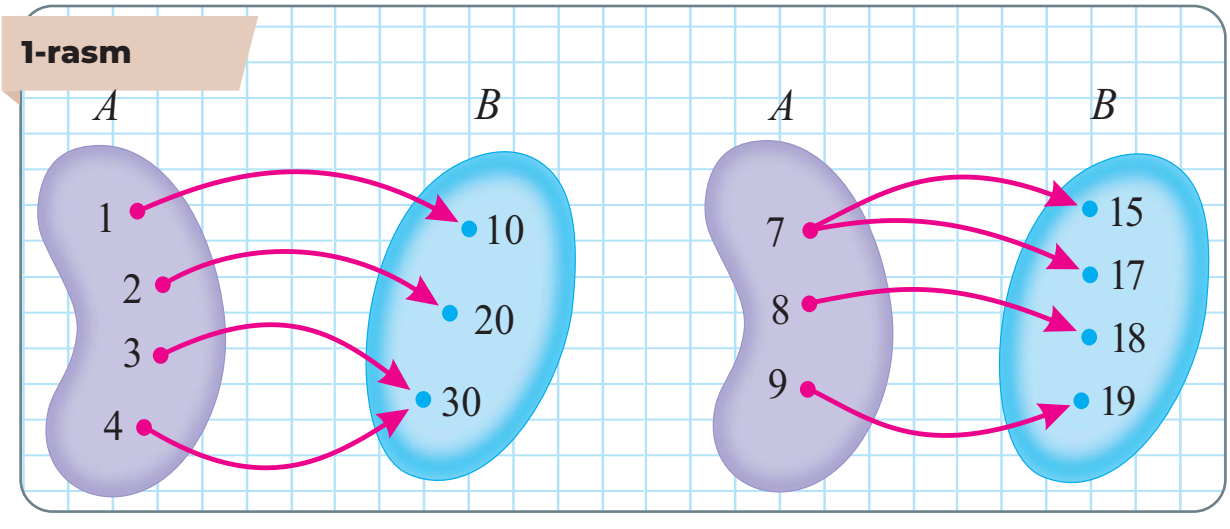
LOYIHA ISHI

MUNOSABATLAR VA FUNKSIYALAR

f funksiyani tartiblangan juftliklar to‘plami (x, y) sifatida ko‘rsatish mumkin, bunda x kirish va $y=f(x)$ chiqish. Masalan, har bir natural sonni kvadratga aylantiruvchi funksiya {(1,1), (2,4), (3,9), ...} tartiblangan juftliklar bilan ifodalanishi mumkin.

Munosabat - bu tartiblangan juftliklarning har qanday to‘plami. Agar munosabatlardagi tartiblangan juftliklarni (x, y) bilan belgilasak, u holda x-qiymatlar (yoki kirishlar) to‘plami aniqlanish soha, y-qiymatlar (yoki chiqishlar) to‘plami esa qiymatlar sohasidir. Ushbu terminologiyada funksiya har bir x-qiymati uchun bitta y-qiymatni (yoki har bir kirish uchun aynan bitta chiqish) bo‘lgan munosabatdir. Quyidagi rasmdagi munosabatlardan – birinchisi funksiyadir, lekin ikkinchisi funksiya emas, chunki A dagi 7 kirish B dagi ikki xil chiqishga, 15 va 17ga mos kelishi ko‘rsatilayapti.

1-rasm

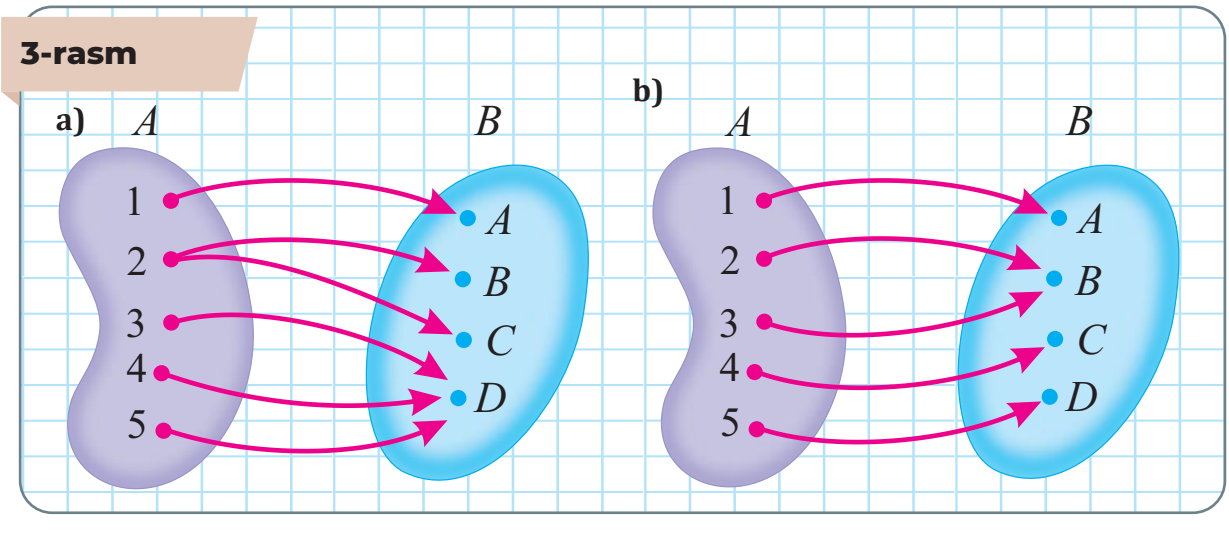


1. A={1,2,3,4} va B={−1,0,1} bo‘lsin. Quyida berilgan munosabat A dan B ga bo‘lgan funksiyami?

a) {(1,0), (2,−1), (3,0), (4,1)} b) {(1,0), (2,−1), (3,0), (3,−1), (4,0)}

2. Quyidagi muvofiqlik funksiya ekanligini aniqlang.

3-rasm



3. Quyidagi ma'lumotlar maktabning 10-sinf o'quvchilaridan to'plangan. Tartiblangan juftliklar to'plami (x, y) funksiyami?

a)

x **Balandligi**	y **Og'irligi**
182 cm	82 kg
152 cm	72 kg
152 cm	54 kg
160 cm	65 kg
178 cm	80 kg

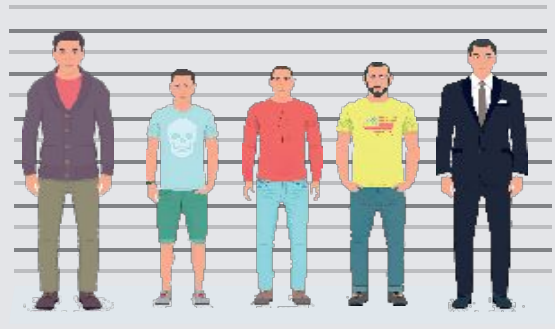
b)

x **Yosh**	y **ID raqami**
16	82–4090
17	80–4133
15	66–8295
16	64–9110
16	20–6666

c)

x **Bitiruv yili**	y **Bitiruvchilar soni**
2018	2
2019	12
2020	18
2021	7
2022	1

4-rasm

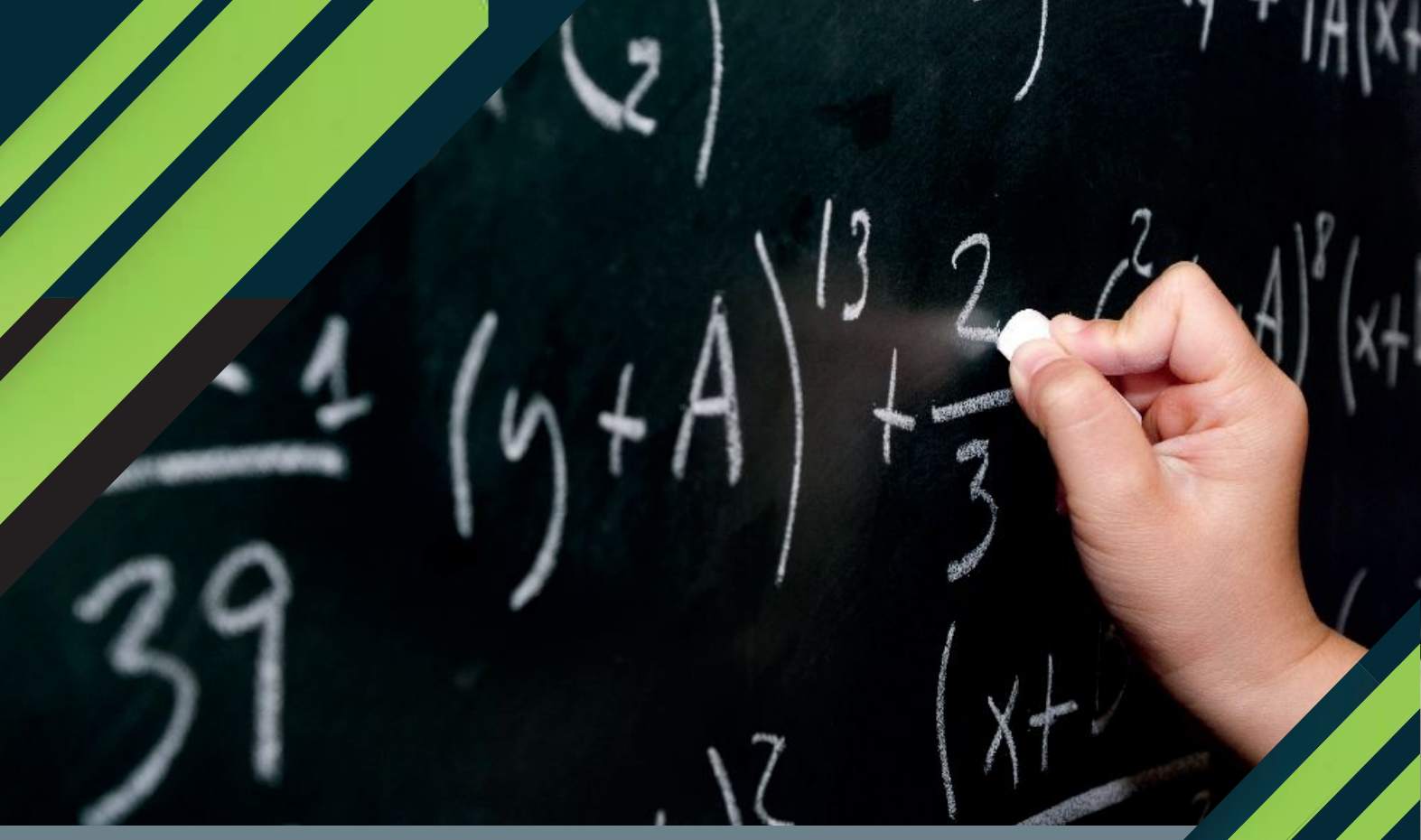


4. x va y bilan berilgan tenglama funksiya bo'lishi yoki bo'lmasligi mumkin bo'lgan munosabatni belgilaydi. Quyidagi shartni qanoatlantiruvchi (x, y) haqiqiy sonlarning barcha tartiblangan juftliklardan tashkil topgan munosabat funksiya ekanligini aniqlang.

a) $y = x^2$ b) $x = y^2$ c) $x \le y$ d) $2x + 7y = 11$

5. Kundalik hayotda biz funksiyalarni belgilaydigan yoki aniqlamaydigan ko'plab munosabatlarga duch kelamiz. Misol uchun, biz odamlarni telefon raqamlari bilan, futbolchilarni o'rtacha zarbalar soni bilan taqqoslaymiz. Bularning qay biri funksiyani belgilaydi? Quyidagi kundalik munosabatlardan qaysi biri funksiya hisoblanadi?

1. x – y ning qizi (x va y – O'zbekistondagi ayollar)
2. x y dan baland (x va y – Buxorodagi odamlardir)
3. x y dan stomatologik davolanishni oldi (x va y – O'zbekistondagi stomatologlar)
4. x telefon raqamidagi raqam (0 dan 9 gacha) va y telefon tugmasidagi mos keladigan harf.
5. Funksiya bo'ladigan munosabatlarga 5 ta misol keltiring.



2-BOB. RATSIONAL TENGLAMALAR VA TENGSIZLIKLAR. IRRATSIONAL TENGLAMALAR

- RATSIONAL TENGLAMALAR
- RATSIONAL TENGLAMALAR SISTEMASI
- RATSIONAL TENGSIZLIKLAR
- RATSIONAL TENGSIZLIKLAR SISTEMASI
- IRRATSIONAL TENGLAMALAR
- IRRATSIONAL TENGLAMALAR SISTEMASI

RATSIONAL TENGLAMALAR

Asosiy ta‘rif va tushunchalar

Ta‘rif:

$f(x)=g(x)$ ko‘rinishidagi tenglik ***bir noma‘lumli tenglama*** deyiladi, (bu yerda $f(x)$ va $g(x)$ lar x noma‘lumli funksiyalar).

Agar tenglamada x ning o‘rniga shunday $x=a$ qiymat qo‘yilganda $f(a)=g(a)$ to‘g‘ri tenglik hosil bo‘lsa, $x=a$ qiymat $f(x)=g(x)$ ***tenglamaning ildizi*** deyiladi.

Tenglamani yechish deganda – uning barcha ildizlarini topish yoki uning ildizi mavjud emasligini isbotlash tushuniladi.

Agar tenglamaning ildizlari $a_1, a_2, \ldots, a_n$ sonlar bo‘lsa, ular $\{a_1, a_2, \ldots, a_n\}$ to‘plam ko‘rinishida, yoki $x_1=a_1, x_2=a_2, \ldots, x_n=a_n$ kabi yoziladi.

Tenglamaning barcha ildizlari to‘plami ***tenglamaning yechimi*** deyiladi.

Tenglamaning ildizi mavjud bo‘lmagan holda **“Tenglamaning ildizi yo‘q”** yoki **“Tenglamaning yechimi – bo‘sh to‘plam”** iborasi ishlatiladi, bu holat $x \in \varnothing$ kabi ham yozish mumkin.

1-misol. $(x+3)(2x-1)(x-2)=0$ tenglamani yeching.

Bu tenglamaning o‘ng tarafi nolga teng, chap tarafi esa 3 ta ifodaning ko‘paytmasidan iborat. Ko‘paytuvchilaridan hech bo‘lmaganda bittasi nolga teng bo‘lgandagina ko‘paytma nolga teng bo‘lganligi uchun, har bir ko‘paytuvchi ifodani nolga tenglashtirib olamiz: $x+3=0$, $2x-1=0$, $x-2=0$. Hosil bo‘lgan ushbu tenglamalardan tenglamaning ildizlari

$$x_1=-3,\ x_2=\frac{1}{2},\ x_3=2$$ ekanligini aniqlab olamiz.

2-misol. Ildizlari 0, -1 va $\sqrt{2}$ ga teng bo‘lgan tenglama tuzing.

Turli ko‘rinishdagi tenglamalar javob tariqasida berilishi mumkin. Eng sodda tenglama $x(x+1)\left(x-\sqrt{2}\right)=0$ ko‘rinishida bo‘lishini eslatib o‘tamiz.

Bu sonlar yana quyidagi tenglamaning ham ildizi bo‘la oladi:

$$(x^2+x^3)\left(x-\sqrt{2}\right)\left(x^2+3\right)=0$$

Ta‘rif:

Agar $f(x)=g(x)$ tenglamaning barcha ildizlari $f_1(x)=g_1(x)$ tenglamaning ildizlari bo‘lsa, va aksincha, $f_1(x)=g_1(x)$ tenglamaning barcha ildizlari $f(x)=g(x)$ tenglamaning ildizlari bo‘lsa, ya‘ni ularning yechimlari ustma-ust tushsa, bunday tenglamalar ***teng kuchli tenglamalar*** deyiladi.

3-misol. $3x-6=0$ va $2x-1=3$ tenglamalarni teng kuchliligini tekshiring.

$3x-6=0$ va $2x-1=3$ tenglamalar teng kuchli, chunki har birining ildizi $x=2$ ga teng.

Yechimi bo‘sh to‘plam bo‘lgan har qanday ikkita tenglama ham teng kuchli bo‘ladi.

Teng kuchli tenglamalar quyidagicha belgilanadi: $3x-6=0 \Leftrightarrow 2x-1=3$

Tenglama quyidagi holatlarda o'ziga teng kuchli bo'lgan teglamaga o'tadi:

a) Tenglamaning biror-bir hadi tenglikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o'tkazilganda.

Masalan, $f(x)=g(x)+t(x) \Leftrightarrow f(x)-g(x)=t(x)$

b)Tenglamaning ikkala tarafini noldan farqli songa ko'paytirilganda yoki bo'lganda.

Butun ratsional tenglamalar

Agar $f(x)$ va $g(x)$ funksiyalar butun ratsional ifodalar bilan berilgan bo'lsa,

$$f(x)=g(x)$$

tenglama, ***butun ratsional tenglama*** deyiladi.

Bunday tenglamaning aniqlanish sohasi barcha haqiqiy sonlar to'plami bo'ladi.

Ta'rif:

Quyidagi ko'rinishidagi tenglama $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0,\ a_0\neq 0,$ **standart ko'rinishdagi *n*-darajali butun ratsional tenglama** deb ataladi.

Agar $a_0=1$ bo'lsa, $x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0$ **tenglama keltirilgan *n*-darajali butun ratsional tenglama** deb ataladi. $a_0,\ a_1,\ \ldots,\ a_{n-1}$ – koeffitsiyentlar, a_n – ozod had deb ataladi.

Ma'lumki, *n*- darajali ko'phad n tadan ko'p bo'lmagan ildizlarga ega bo'lishi mumkin, demak, har bir standart ko'rinishidagi *n*- darajali butun ratsional tenglama ham n tadan ko'p bo'lmagan ildizlarga ega bo'ladi.

Teorema: Butun koeffitsiyentli keltirilgan butun ratsional tenglamaning ildizlari butun son bo'lsa, ular ozod hadining bo'luvchilari bo'ladi.

Ko'phadni ko'phadga burchakli bo'lish usuli yordamida yechish

4-misol: $x^4+2x^3=11x^2-4x-4$ tenglamani yeching.

Yechish: Avval uni standart ko'rinishga keltiramiz: $x^4+2x^3-11x^2+4x+4=0$

Bu tenglamaning butun ildizlari borligini tekshirish uchun ozod hadi 4 ning barcha bo'luvchilarini yozib olamiz: $\pm1, \pm2, \pm4$. Bu sonlarni ketma-ket tenglamaga qo'yib ko'rib, $x_1=1$ va $x_2=2$ sonlar tenglamaning ildizlari bo'lishini aniqlab olamiz. Demak, $x^4+2x^3-11x^2+4x+4$ ko'phad $(x-1)(x-2)=x^2-3x+2$ ko'phadga qoldiqsiz bo'linadi.

$$\begin{array}{r|l} x^4+2x^3-11x^2+4x+4 & x^2-3x+2 \\ \underline{-\,x^4-3x^3+2x^2} & \underline{x^2+5x+2} \\ 5x^3-13x^2+4x+4 & \\ \underline{-\,5x^3-15x^2+10x} & \\ 2x^2-6x+4 & \\ \underline{-\,2x^2-6x+4} & \\ 0 & \end{array}$$

Demak, tenglamani quyidagi ko‘paytuvchilarga ajratish mumkin: $(x-1)(x-2)(x^2+5x+2)=0$.

Demak, hosil bo‘lgan tenglama birinchi tenglamaga teng kuchli tenglamadir. Har bir ko‘paytuvchini nolga tenglashtirib, tenglamaning ildizlarini topamiz.

Javob: $x_1=1;\ x_2=2,\ x_{3,4}=\dfrac{-5\pm\sqrt{17}}{2}$.

Teorema: Agar $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0, a_0\neq 0,$ n-darajali butun koeffitsiyentli ratsional tenglama $x_0=\dfrac{p}{q}$ ratsional ildizga ega bo‘lsa, unda $p-$ozod had (a_n) ning bo‘luvchisi, $q-$ esa bosh had (a_0) ning bo‘luvchisi bo‘ladi.

5-misol: $6x^3-11x^2-2x+8=0$ tenglamani yeching.

Yechish. Ozod hadining bo‘luvchilarini: $\pm1,\pm2,\pm4,\pm8$ va bosh hadining natural bo‘luvchilarini yozib olamiz: 1, 2, 3, 6.

Tanlash yo‘li bilan $x_1=\dfrac{4}{3}$ tenglamaning ildizi bo‘lishini aniqlaymiz.

Tenglamaning chap tarafidagi ko‘phadni $x-\dfrac{4}{3}$ ikkihadga bo‘lib, quyidagi ko‘paytuvchilarga ajralgan teng kuchli tenglamaga keltiramiz: $\left(x-\dfrac{4}{3}\right)\left(6x^2-3x-6\right)=0$

Ikkinchi qavsdagi kvadrat tenglamani yechib, $x_2=\dfrac{1-\sqrt{17}}{4}$, $x_3=\dfrac{1+\sqrt{17}}{4}$ larni topamiz.

Javob: $x_1=\dfrac{4}{3}, x_2=\dfrac{1-\sqrt{17}}{4}$, $x_3=\dfrac{1+\sqrt{17}}{4}$.

Simmetrik tenglamalar va ularga keltiriladigan tenglamalar

Ushbu $ax^n+bx^{n-1}+cx^{n-2}+\ldots+cx^2+bx+a=0$ ko‘rinishdagi butun ratsional tenglama ***simmetrik tenglama*** deyiladi.

Bunda tenglamaning boshidan va oxiridan bir xil uzoqlikda yotgan hadlarning koeffitsiyentlari bir-biriga teng bo‘ladi. Simmetrik tenglamaning ildizlaridan hech biri nolga teng emasligini ko‘rish oson.

Agar $x=0$ tenglamaning ildizi bo‘lsa, u holda biz $a=0$ ga ega bo‘lamiz va tenglamaning darajasi pastroq bo‘ladi.

1. Oldin juft ($n=2k$) darajali simmetrik tenglamani ko‘rib chiqamiz.

Tenglamaning har ikkala qismini x^k ga bo‘lib, hadlarni guruhlash natijasida uni quyidagi ko‘rinishga keltiramiz:

$$a\left(x^k+\frac{1}{x^k}\right)+b\left(x^{k-1}+\frac{1}{x^{k-1}}\right)+\ldots+b\left(x+\frac{1}{x}\right)+a=0$$

Agar bu tenglamada $x+\dfrac{1}{x}=t$ belgilash kiritsak, ketma-ket quyidagilarni topamiz:

$$x^2+\frac{1}{x^2}=t^2-2, x^3+\frac{1}{x^3}=t^3-3t;\ldots$$

Bu ifodalarni yuqoridagi tenglamaga qo‘yib, t ga nisbatan k darajali tenglamani hosil qilamiz. x ning qiymatlarini esa $x^2-tx+1=0$ tenglamadan topamiz.

6-misol. $x^4 - 5x^3 + 8x^2 - 5x + 1 = 0$ tenglamani yeching.

Yechish: Berilgan tenglama 4-darajali qaytma (simmetrik) tenglama. Uni yechish uchun tenglamaning ikkala tomonini $x^2 \neq 0$ ga bo‘lamiz va unga teng kuchli tenglamani hosil qilamiz:

$$x^2-5x+8-\frac{5}{x}+\frac{1}{x^2}=0,$$

Qo‘shiluvchilarni guruhlab, tenglamani quyidagi ko‘rinishga keltirib olamiz:

$$x^2+\frac{1}{x^2}-5\left(x+\frac{1}{x}\right)+8=0$$

$x+\frac{1}{x}=t, x^2+\frac{1}{x^2}=t^2-2$ belgilash kiritib, $t^2-5t+6=0$ tenglamani hosil qilamiz. Bu tenglamani yechimlari $t_1=2$ va $t_2=3$. Bu qiymatlarni belgilashga qayta qo‘yib, berilgan tenglamaning yechimi $x+\frac{1}{x}=2$ va $x+\frac{1}{x}=3$ tenglamalarning yechimi birlashmasiga teng bo‘lishini ko‘ramiz.

Bu tenglamalarni yechib, $x_1=1, x_2=\frac{3+\sqrt{5}}{2}$ va $x_3=\frac{3-\sqrt{5}}{2}$ ekanligini topamiz.

Javob: $x_1=1, x_2=\frac{3+\sqrt{5}}{2}, x_3=\frac{3-\sqrt{5}}{2}$

7-misol. $21x^6+82x^5+103x^4+164x^3+103x^2+82x+21=0$ tenglamani yeching.

Yechish: Tenglamaning har ikkala qismini $x^3 \neq 0$ ga bo‘lamiz:

$$21\left(x^3+\frac{1}{x^3}\right)+82\left(x^2+\frac{1}{x^2}\right)+103\left(x+\frac{1}{x}\right)+164=0$$

Agar $x+\frac{1}{x}=t$ desak, $x^2+\frac{1}{x^2}=t^2-2$, $x^3+\frac{1}{x^3}=t^3-3t$ bo‘ladi. Natijada t ga nisbatan tenglamaga ega bo‘lamiz: $21t^3+82t^2+40t=0 \Rightarrow t\left(21t^2+82t+40\right)=0$

Har bir ko‘paytuvchini nolga tenglashtiramiz: $t=0$ va $21t^2+82t+40=0$. Bu tenglamalarni yechib, ildizlarni topamiz: $t_1=0;\ t_2=-\frac{4}{7};\ t_3=-\frac{10}{3}$.

Agar: 1) $t_1=0$ bo‘lsa, $x+\frac{1}{x}=0 \Rightarrow x^2+1=0$ tenglamaga ega bo‘lamiz. Uning haqiqiy ildizlari mavjud emas.

2) $t_2=-\frac{4}{7}$ bo‘lsa, $7x^2+4x+7=0$ bo‘ladi. Uning ham haqiqiy ildizlari mavjud emas.

3) $t_3=-\frac{10}{3}$ bo‘lsa, $3x^2+10x+3=0$ tenglamaga ega bo‘lamiz.

Uning ildizlari: $x_1=-\frac{1}{3}; x_2=-3$.

Javob: $x_1=-\frac{1}{3};\ x_2=-3$.

2. Toq darajali ($n=2k+1$) simmetrik tenglamani yechish juft darajali simmetrik tenglamani yechishga keltiriladi.

Ushbu $ax^{2k+1}+bx^{2k}+cx^{2k-1}+\ldots+cx^2+bx+a=0$ tenglamaning $x=-1$ ildizga ega ekanligini ko‘rish qiyin emas. Demak, bu tenglamaning chap qismi $x+1$ ga bo‘linadi. Tenglamaning ikkala qismini har biri $x+1$ ga bo‘linadigan qo‘shiluvchilar yig‘indisi ko‘rinishida ifodalaymiz:

$$a(x^{2k+1}+1)+bx\left(x^{2k-1}+1\right)+cx^2(x^{2k-3}+1)+\ldots+lx^k\left(x+1\right)=0$$

$$\left(x+1\right)\left(ax^{2k}+b_1x^{2k-1}+\ldots+b_1x+a\right)=0$$

Shunday qilib, masala juft ko‘rsatkichli ushbu $ax^{2k}+b_1x^{2k-1}+\ldots+b_1x+a=0$ simmetrik tenglamani yechishga keltiriladi.

Simmetrik tenglamaning yana o‘ziga xos bir xususiyati bor. Agar $x=x_0$ soni simmetrik tenglamaning ildizi bo‘lsa, u holda $x=\frac{1}{x_0}$ soni ham shu tenglamaning ildizi bo‘ladi.

8-misol. $3x^3+4x^2+4x+3=0$ tenglamani yeching.

Yechish: Berilgan tenglama 3-darajali qaytma (simmetrik) tenglama. Uni yechish uchun avval ko‘paytuvchilarga ajratamiz va unga teng kuchli tenglamani hosil qilamiz:

$$3(x^3+1)+4x(x+1)=0;$$

$$(x+1)(3x^2-3x+3+4x)=0;$$

$$(x+1)(3x^2+x+3)=0.$$

Bu tenglamaning yechimi quyidagi 2 ta tenglamaning yechimi birlashmasiga teng:

$$x+1=0 \quad \text{va} \quad 3x^2+x+3=0.$$

1-tenglamaning yechimi $x=-1$, 2-tenglama esa haqiqiy yechimga ega emas.

Javob: $x=-1$

9-misol. $x^7+2x^6-5x^5-13x^4-13x^3-5x^2+2x+1=0$ tenglamaning ildizlarini toping.

Yechish. Toq darajali simmetrik tenglamaning ildizi $x=-1$ bo‘ladi. Tenglamaning chap tarafidagi ifodani $x+1$ ikkihadga bo‘lib quyidagi ko‘paytuvchilarga ajratamiz:

$$\left(x+1\right)\left(x^6+x^5-6x^4-7x^3-6x^2+x+1\right)=0$$

Har birini nolga tenglashtiramiz $x+1=0\Rightarrow x_1=-1$ yoki $x^6+x^5-6x^4-7x^3-6x^2+x+1=0$

Bu tenglamada $x\neq 0$ bo‘lgani uchun, tenglamaning ikkala tarafini $x^3\neq 0$ ga bo‘lish mumkin:

$$x^3+x^2-6x-7+\frac{6}{x}+\frac{1}{x^2}+\frac{1}{x^3}=0;$$

$$\left(x^3+\frac{1}{x^3}\right)+\left(x^2+\frac{1}{x^2}\right)-6\left(x+\frac{1}{x}\right)-7=0.$$

Belgilash kiritamiz: $x+\frac{1}{x}=t$; $x^2+\frac{1}{x^2}=t^2-2$; $x^3+\frac{1}{x^3}=t^3-3t$ va quyidagini hosil qilamiz:

$$t^3+t^2-9t-9=0;$$

$$(t+1)(t-3)(t+3)=0.$$

Bundan, $t=-1$, yoki $t=3$, yoki $t=-3$.

1) $t=-1\Rightarrow x+\frac{1}{x}=-1\Rightarrow \frac{x^2+x+1}{x}=0\Rightarrow x^2+x+1=0\Rightarrow$ haqiqiy ildizi yo‘q.

2) $t=3\Rightarrow x+\frac{1}{x}=3\Rightarrow \frac{x^2-3x+1}{x}=0\Rightarrow x_{2,3}=\frac{3\pm\sqrt{5}}{2}$.

3) $t=-3\Rightarrow x+\frac{1}{x}=-3\Rightarrow \frac{x^2+3x+1}{x}=0\Rightarrow x_{4,5}=\frac{-3\pm\sqrt{5}}{2}$.

Javob: $x_1=-1; x_{2,3}=\frac{3\pm\sqrt{5}}{2}; x_{4,5}=\frac{-3\pm\sqrt{5}}{2}$.

3. Ushbu $ax^4+bx^3+cx^2+dx+l=0,\ (l\neq 0)$ tenglama simmetrik tenglama bo‘lishi uchun uning koeffitsiyentlari quyidagicha bog‘langan bo‘lishi kerak: $d=\lambda b,\ l=\lambda^2 a$

Bu holda berilgan tenglama $t=x+\frac{\lambda}{x}$ almashtirish bilan kvadrat tenglamaga keladi.

4. Quyidagi $(x+a)(x+b)(x+c)(x+d)=m$ tenglamani simmetrik tenglamaga keltirish uchun uning koeffitsiyentlari orasida $a+b=c+d$ (yoki $a+c=b+d$, yoki $a+d=b+c$) tenglik bajarilishi kerak. Bunda avval $(x-a)(x-b)$ va $(x-c)(x-d)$ lar ko‘paytirilib, keyin almashtirish bajariladi.

Kasr-ratsional tenglamalar.

$\frac{f(x)}{g(x)}=0$ ko‘rinishiga keltirish mumkin bo‘lgan tenglamalarga ***kasr-ratsional tenglama*** deyiladi.

$\frac{f(x)}{g(x)}=0$ ko‘rinishidagi ratsional tenglamaning **aniqlanish sohasi** $g(x)\neq 0$.

Ratsional tenglamalarni yechish qadamlari:

- Tenglamadagi barcha ifodalarni tenglikning chap tarafiga o‘tkaziladi;
- Barcha ifodalar umumiy maxrajga keltiriladi;
- Tenglama $\frac{f(x)}{g(x)}=0$ ko‘rinishiga keltiriladi;
- Suratining nollari topiladi;
- Aniqlanish sohasi topiladi;
- Aniqlanish sohasini qanoatlantiruvchi suratining nollari tenglamaning ildizlari bo‘ladi.

Yoki $\frac{f(x)}{g(x)}=0$ ratsional tenglamaning yechimini topish uchun uni quyidagi $\begin{cases} f(x)=0 \\ g(x)\neq 0 \end{cases}$ teng kuchli sistema ko‘rinishida yozib olinadi va yechiladi.

Ba‘zi hollarda bir tenglamadan unga teng kuchli tenglamaga o‘tishda **chet ildizlar** paydo bo‘lishi mumkin, masalan, quyidagi tenglamani qaraylik, $\frac{x^2+x-2}{x-1}=0$,

Kasrning suratini nolga tenglashtiramiz:

$$x^2+2-2=0 \Rightarrow x_1=-2,\ x_2=1.$$

Bu tenglamaning aniqlanish sohasi $x\neq 1$, ya'ni $x=1$ qiymat berilgan tenglamaning yechimi bo'la olmaydi, demak, $x=1$ – **chet ildiz bo'ladi.**

10-misol. Tenglamaning ildizini toping: $\dfrac{2x+3}{x-1}=0$.

Yechish. $\begin{cases}2x+3=0\\ x-1\neq 0\end{cases}\Rightarrow\begin{cases}2x=-3\\ x\neq 1\end{cases}\Rightarrow\begin{cases}x=-1,5\\ x\neq 1\end{cases}$

Javob: $x=-1,5.$

11-misol. Tenglamani yeching: $\dfrac{4x+4}{3(x+2)-3}=0$.

Yechish. $\begin{cases}4x+4=0\\ 3(x+2)-3\neq 0\end{cases}\Rightarrow\begin{cases}4x=-4\\ 3x+6-3\neq 0\end{cases}\Rightarrow\begin{cases}x=-1\\ x\neq -1\end{cases},$

ko'rinib turibdiki, x ning qiymati -1 ga teng bo'lishi mumkin emas. Shuning uchun,

Javob: $x\in\varnothing$

12-misol. Tenglamaning ildizini toping: $\dfrac{-2x-4}{x^2-4}=\dfrac{x+5}{x-2}$.

Yechish. Barcha ifodalarni tenglikdan chap tarafga o'kazamiz va umumiy maxrajga keltiramiz.

$$\frac{x+5}{x-2}+\frac{2x+4}{x^2-4}=0\Rightarrow\frac{(x+5)(x+2)+2x+4}{x^2-4}=0\Rightarrow$$
$$\Rightarrow\frac{x^2+7x+10+2x+4}{x^2-4}=\frac{x^2+9x+14}{x^2-4}=0.$$

Kasr-ratsional ifodaning suratini nolga tenglashtiramiz va nollarini topamiz. Viyet teoremasidan foydalanamiz:

$$x^2+9x+14=0\Rightarrow x=-2;x=-7.$$

Kasr-ratsional ifodaning maxrajini nolga tenglashtiramiz va nollarini topamiz. Ko'paytuvchilarga ajratish usulidan foydalansak bo'ladi:

$$x^2-4=(x-2)(x+2)=0\Rightarrow x=-2;x=2.$$

Ko'rinib turibdiki, $x=-2$ ham suratning, ham maxrajning nolidir. Maxraj hech qachon nolga teng emas, demak, $x=-2$ tenglamaning ildizi bo'la olmaydi.

Demak, tenglamaning bitta ildizi bor, $x=-7$.

Javob: $x=-7$.

Diqqat qiling: Kasr-ratsional tenglamani yechishda har doim suratining nollarini tenglama aniqlanish sohasiga tegishli ekanligini tekshiring!

13-misol. Tenglamani yeching: $\dfrac{(x^2-x-56)(x-3)}{x^2+5x+6}=0$

Yechish. Berilgan tenglama kasr-ratsional tenglamadir. Avval suratining nollarini topamiz.

$$(x^2-x-56)(x-3)=0\Rightarrow x=3;\ \ x^2-x-56=0$$

$$D=(-1)^2-4\cdot1\cdot(-56)=225=15^2$$
$$x_{1,2}=\frac{1\pm15}{2}\Rightarrow x_1=8;\ \ x_2=-7$$

Suratning 3 ta nolini topdik: $x_1=8;\ \ x_2=-7;\ \ x_3=3$

Endi maxrajining nollarini topamiz. Buning uchun Viyet teoremasidan foydalanamiz.

$$\begin{cases}x_1+x_2=-5\\ x_1\cdot x_2=6\end{cases}\Rightarrow\begin{cases}x_1=-2\\ x_2=-3\end{cases}$$

Surat va maxraj umumiy nollarga ega emas, shuning uchun suratning topilgan 3 ta ildizi $x_1=8;\ \ x_2=-7;\ \ x_3=3$ tenglamaning iltizlari bo‘ladi.

Javob: $x_1=8;\ \ x_2=-7;\ \ x_3=3$.

14-misol. Tenglamaning ildizlarini toping:

$$\frac{2}{(x-2)(x+2)}-\frac{1}{x(x-2)}=\frac{4-x}{x(x+2)}.$$

Yechish: Tenglikning o‘ng tarafidagi ifodani chap tarafga o‘tkazamiz:

$$\frac{2}{(x-2)(x+2)}-\frac{1}{x(x-2)}-\frac{4-x}{x(x+2)}=0.$$

va umumiy maxrajga keltiramiz:

$$\frac{2x-(x+2)-(4-x)(x-2)}{x(x-2)(x+2)}=0.$$

Suratidagi qavslarni ochib, kvadrat tenglamaga keltiramiz:

$$\frac{2x-x-2-4x+x^2+8-2}{x(x-2)(x+2)}=0\Rightarrow\frac{x^2-5x+6}{x(x-2)(x+2)}=0$$

Hosil bo‘lgan kasr-ratsional tenglama quyidagi tenglamalar sistemasiga teng kuchli.

$$\begin{cases}x^2-5x+6=0\\ x(x-2)(x+2)\neq0\end{cases}$$

Suratining nollarini topamiz:

$$x^2-5x+6=0\Rightarrow D=(-5)^2-4\cdot6=1,\ x_{1,2}=\frac{5\pm1}{2}\Longrightarrow x_1=2;\ \ x_2=3.$$

Maxrajining nollarini topamiz:

$$x(x-2)(x+2)\neq0\Rightarrow x\neq0;\ \ x\neq2;\ \ x\neq-2$$

$x=2$ qiymatni chiqarib tashlaymiz, demak, ildiz $x=3$ ekan.

Javob: $x=3$.

15-misol. Tenglamani yeching:

$$x^2+x+1=\frac{15}{x^2+x+3}.$$

Yechish. $x^2+x+1=t$ belgilash kiritamiz. Tenglama quyidagi ko‘rinishga keladi:

$$t=\frac{15}{t+2}$$

$t\neq-2$, bo‘lishini inobatga olib, quyidagi tenglamani yechamiz:

$$t(t+2)=15;$$
$$t^2+2t-15=0;$$
$$t_1=-5;\ t_2=3.$$

Qayta o‘rniga qo‘yib, $x^2+x+1=-5$ va $x^2+x+1=3$ tenglamalarga ega bo‘ldik. Ularning har birini alohida yechamiz:

$x^2+x+6=0 \Rightarrow$ haqiqiy ildizi yo‘q; $\quad x^2+x-2=0 \Rightarrow x_1=-2;\ x_2=1$

Javob: $\{-2;1\}$

Matnli masalalar yechishda ham ratsional tenglamalar ishlatiladi. Quyida harakatga va ishga doir masalalar ratsional tenglama ko‘rinishida modellashtirilib yechilgan.

Harakatga doir masala.

Vertolyot 6 soatda shamol yo‘nalishi bo‘yicha 120 km masofani uchib o‘tdi va orqaga qaytdi. Agar vertolyotning shamolsiz havodagi tezligi 45km/h ga teng bo‘lsa, shamolning tezligini toping.

Yechish. Shamolning tezligini formulada x km/h bilan belgilaylik. Unda shamol yo‘nalishi bo‘yicha vertolyotning tezligi formulada$(45+x)$ km/h va shamolga qarshi yo‘nalishda esa formulada $(45-x)$ km/h ga teng bo‘ladi. Masalaning sharti bo‘yicha, vertolyot jami 6 soat vaqt sarflagan. Masofani tezlikka bo‘lib qo‘shsak, jami vaqtga teng bo‘ladi:

$$\frac{120}{45+x}+\frac{120}{45-x}=6$$

Kasr ratsional tenglama hosil bo‘ldi: $\frac{120}{45+x}+\frac{120}{45-x}-6=0;$

$$\frac{120(45-x)+120(45+x)-6(45+x)(45-x)}{(45+x)(45-x)}=0;$$

$$\begin{cases}6x^2-12150+10800=0\\(45+x)(45-x)\neq 0\end{cases}.$$

Birinchi tenglamani yechamiz. $6x^2-12150+10800=0$

$$6x^2-1350=0$$
$$x^2=225$$
$$x_1=-15;\ x_2=15$$

Ikkinchi tenglamaning yechimi; $x_1\neq -45;\ x_2\neq 45$.

Tezlik manfiy qiymat qabul qilmaydi, shuning uchun $x=-15$ ildiz bo‘la olmaydi. Demak, shamolning tezligi 15 km/h.

Javob: Shamolning tezligi 15 km/h.

Ishga doir masala

Ikkita traktorchi birgalikda dalani 4 kunda shudgor qildi. Agar 1-traktorchiga shudgorni alohida bajarishi uchun 2-traktorchiga nisbatan 6 kun kam vaqt kerak bo‘lsa, har bir traktorchi ishni necha kunda bajaradi?

Yechish. 1-traktorchi x kunda dalani shudgor qilsin. Unda 2-traktorchi shu dalani $(x+6)$ kunda shudgor qiladi. Demak, 1-traktorchi 1 kunda dalaning $\frac{1}{x}$ qismini, 2-traktorchi esa $\frac{1}{x+6}$ qismini shudgor qiladi. Masalaning shartiga ko‘ra, shu dalani ular birgalikda 4 kunda shudgor qiladi. Ya‘ni, ikkalasi 1 kunda dalaning $\frac{1}{4}$ qismini shudgor qiladi.

Tenglamani tuzamiz va yechamiz: $\frac{1}{x}+\frac{1}{x+6}=\frac{1}{4}$,

$$\frac{4(x+6)+4x-x(x+6)}{4x(x+6)}=0,$$

$$\frac{-x^2+2x+24}{4x(x+6)}=0.$$

Hosil bo‘lgan ratsional tenglama quyidagi tenglamalar sistemasiga teng kuchli.

$$\begin{cases} x^2-2x-24=0; \\ 4x(x+6)\neq 0; \end{cases} D=(-2)^2-4\cdot(-24)=100;\ x_{1,2}=\frac{2\pm 10}{2}\Rightarrow x_1=6;\ \ x_2=-4$$

$x_2=-4$ yechim manfiy bo‘lgani uchun javob bo‘la olmaydi. Demak, 1-traktorchi shudgorni 6 kunda, 2-traktorchi esa $x+6=6+6=12$ kunda bajaradi.

Javob: 1-traktorchi 6 (kun), 2-traktorchi 12 (kun).

MISOLLAR

Kasr-ratsional tenglamalarni yeching (1–10).

1. $\frac{1}{x}-\frac{2x}{x+1}=0$

2. $\frac{2y-5}{y+5}=\frac{3y+21}{2y-1}$

3. $\frac{5x-7}{x-3}=\frac{4x-3}{x}$

4. $\frac{x+1}{2(x-1)}=\frac{9}{2(x+4)}+\frac{1}{x-1}$

5. $\frac{2x}{x-1}-\frac{1}{x+1}=\frac{4x}{x^2-1}$

6. $\frac{x^2-2x}{x-2}=x^2-2$

7. $\frac{7}{2x+9}-6=5x$

8. $\frac{15}{x-2}=\frac{14}{x}+1$

9. $\frac{4}{x-2}+\frac{4}{x+2}=\frac{3}{2}$

10. $\frac{3x}{x^2-1}=2\left(\frac{2x-1}{x+1}\right)$

11. Bir ishni birinchi ishchining yolg‘iz o‘zi a kunda bajaradi, ikkinchi ishchi shu ishni bajarish uchun b kun ortiq vaqt sarf qilsa, uchinchi ishchining yolg‘iz o‘zi b kun tezroq bajara oladi. Shu ishni uchala ishchi birga ishlasa, necha kunda bajaradi?

12. Bir guruh sayohatchilar motorli katerda pristandan oqimga qarshi ketishdi. Ular 4 soat-u 40 minutdan so‘ng qaytib kelishlari kerak. Qayiqning turg‘un suvdagi tezligi 12 km/h, oqim tezligi esa 3 km/h bo‘lsa, sayohatchilar pristandan necha kilometr masofaga borib, 2 soat dam olib, o‘z vaqtida qaytib kelishlari mumkin?

13. 58 °C haroratli 100 l suv hosil qilish uchun qancha qaynoq (100 °C) suv va qancha uy haroratidagi (16 °C) suv olish kerak?

Kasr-ratsional tenglamalarni yeching (14–33).

14. $\frac{1}{x^2-12x+36}+\frac{12}{36-x^2}=\frac{1}{x+6}$

15. $\frac{8c-3}{4c^2-2c+1}+\frac{6}{8c^3+1}=\frac{2}{2c+1}$

16. $\frac{3x-2}{x-1}+\frac{x-4}{x+3}=\frac{3x^2+1}{(x-1)(x+3)}$

17. $\frac{2-3x}{x+1}-\frac{4}{3}\cdot\frac{x+1}{2-3x}=\frac{4}{3}$

18. $\frac{x-49}{x+6}+\frac{2x+50}{x+5}=2$

19. $\frac{(x+2)^2-9}{x-1}\cdot(x-5)=-24$

20. $(x+4)(x^2-1)=4x^2+24x-\frac{4x^2+20x}{5x+x^2}$

21. $\frac{25x-21}{2x^2+5x-12}=\frac{x-4}{2x-3}-\frac{2x-3}{x+4}$

22. $\frac{3}{x^2-2x+1}+\frac{2}{1-x^2}=\frac{1}{1+x}$

23. $\frac{6}{x-1}+\frac{6}{(x-1)(x-3)}+\frac{3}{3-x}=7$

24. $\frac{x^5-4x^3}{x-2}=16+2x^3$

25. $\frac{1}{(x-2)(x-3)}-\frac{9}{(x+2)(x-7)}=1$

26. $x^2+x+1=\frac{15}{x^2+x+3}$

27. $\frac{x^2+2}{3x-2}-\frac{3x-2}{x^2+2}=2\frac{2}{3}$

28. $x^2-5x+\frac{24}{x^2-5x}+10=0$

29. $\frac{x^2+1}{x}+\frac{x}{x^2+1}=2\frac{1}{2}$

30. $\frac{2}{x^2+3}+\frac{4}{x^2+7}=1$

31. $\frac{1}{x(x+2)}-\frac{1}{(x+1)^2}=\frac{1}{12}$

32. $\frac{x^2-3x-1}{x^2-2x+4}=\frac{x^2-2x}{x^2-x+1}$

33. $\frac{x^2-4x-1}{x^2-3x+5}=\frac{x^2-3x+1}{x^2-2x+2}$

34. Ikki pristan orasidagi masofa daryo yo‘li bilan 80 km. Kema shu pristanlarning biridan ikkinchisiga borib-kelish uchun 8 soat 20 minut vaqt sarf qildi. Daryo oqimining tezligi 4 km/h bo‘lsa, kemaning turg‘un suvdagi tezligini toping.

35. Ikki ishchi ayni bir ishni birgalashib ishlasa, 12 kunda tamom qiladi. Agar oldin bittasi ishlab, ishning yarmini tamom qilgandan keyin uning o‘rniga ikkinchisi ishlasa, ish 25 kunda tamom bo‘ladi. Shu ishni har qaysi ishchi yolg‘iz o‘zi ishlasa, necha kunda tamom qiladi?

36. “Belorus” traktori 1 soatda 2 ha, “Case” traktori esa 1 soatda 7 ha yerni shudgorlay oladi. Fermer ho‘jaligida 2 ta “Belorus” va 1 ta “Case” traktori bor. Agar bu traktorlar birgalikda ishlatilsa, 210 ha yerni necha kunda shudgorlasa bo‘ladi?

37. Avtomobil yo‘lning “Radar” qurilmasi o‘rnatilmagan 10 kilometrlik qismida 120 km/h, radar o‘rnatilgan 25 km qismida 70 km/h, 35 km qismida 50 km/h tezlik bilan harakatlandi. Uning butun yo‘l davomidagi o‘rtacha tezligini toping.

RATSIONAL TENGLAMALAR SISTEMASI

Ta‘rif:

Ikki noma‘lumli tenglamalar sistemasi deb quyidagi ko‘rinishdagi sistemaga aytiladi:

$$\begin{cases} f_1(x,y)=g_1(x,y) \\ f_2(x,y)=g_2(x,y) \end{cases}.$$

Tenglamalar sistemasini yechimi deb – ikkala tenglamani qanoatlantiruvchi barcha (x,y) juftliklar to‘plamiga aytiladi.

Tenglamalar sistemasini yechish – uning yechimini topishdan yoki yechimga ega emasligini ko‘rsatishdan iborat.

Agar ikkita tenglamalar sistemasining yechimlari to‘plami ustma-ust tushsa, ular **teng kuchli** deyiladi.

Tenglamalar sistemasini yechish uchun uni soddaroq teng kuchli sistemaga keltiriladi. Masalan, berilgan tenglamalar sistemasini unga teng kuchli bo‘lgan quyidagi sistemaga almashtirish mumkin:

$$\begin{cases} F_1(x;y)=0 \\ F_2(x;y)=0 \end{cases}.$$

Bu yerda, $F_1(x,y)=f_1(x,y)-g_1(x,y)$ va $F_2(x,y)=f_2(x,y)-g_2(x,y)$ ga teng.

O‘rniga qo‘yish usuli

1-misol. Quyidagi tenglamani yechish uchun o‘rniga qo‘yish usulidan foydalanamiz:

$$\begin{cases} 3xy=21 \\ x-8y=-1 \end{cases}$$

2-tenglamadan $x-8y=-1 \Rightarrow \boldsymbol{x=8y-1}$.

x ning hosil bo‘lgan bu qiymatini 1-tenglamaga qo‘yib, $3(8y-1)y=21$ tenglamaga kelamiz.

Bu tenglamani yechib,

$$(8y-1)y=7$$

$8y^2-y-7=0 \Rightarrow y_1=-\frac{7}{8}; y_2=1$ qiymatlarni topamiz va ularni $\boldsymbol{x=8y-1}$ ga qo‘yib $\Rightarrow x_1=-8; x_2=7$ ekanligini aniqlaymiz.

Javob: $\left(-8,\ -\frac{7}{8}\right),(7,1)$.

2-misol. O‘rniga qo‘yish usulidan foydalanib, $\begin{cases} 2x^2+y=4 \\ x^4+y^2=16 \end{cases}$ tenglamalar sistemasini yeching.

Yechish.

$$y=4-2x^2.$$

$$x^4+\left(4-2x^2\right)^2=16,$$

$$x^4+16-16x^2+4x^4=16,$$

$$5x^4-16x^2=0,$$

$$x^2\left(5x^2-16\right)=0, \quad \Rightarrow x_1=0,\ x_2=\frac{4}{\sqrt{5}},\ x_3=-\frac{4}{\sqrt{5}},$$

$$\Rightarrow y_1 = 4,\ y_2 = -\frac{12}{5},\ y_3 = \frac{12}{5}$$

Javob: $(0,4),\left(\frac{4}{\sqrt{5}},-\frac{12}{5}\right),\left(-\frac{4}{\sqrt{5}},\frac{12}{5}\right)$.

Algebraik qo‘shish usuli

3-misol. Ushbu tenglamalar sistemasini yeching: $\begin{cases} x^2 + y = 27 \\ x - y = 3 \end{cases}$.

Yechish.

Ikkala tenglamada y noma‘lum qarama-qarshi ishorali koeffitsiyent bilan qatnashgan, shuning uchun bu tenglamalarni qo‘shamiz.

$$+\begin{cases} x^2 + y = 27 \\ x - y = 3 \end{cases}$$
$$x^2 + x = 30$$

$x^2 + x - 30 = 0$ bir noma‘lumli kvadrat tenglamaga keltirib oldik.

$$x_1 = \frac{-1-11}{2} = -6 \Rightarrow y_1 = -9,$$

$$x_2 = \frac{-1+11}{2} = 5 \Rightarrow y_2 = 2.$$

Javob: $(-6,-9),(5,2)$.

4-misol. Tenglamalar sistemasini algebraik qo‘shish usuli yordamida yeching:

$$\begin{cases} x^3 - y^3 - 3x^2 + 3y^2x = -2, \\ x^2 - x^2y = 1. \end{cases}$$

Yechish.

2-tenglamani 3 ga kopaytirib 1-tenglamaga qo‘shsak:

$$+\begin{cases} x^3 - y^3 - 3x^2 + 3y^2x = -2, \\ 3x^2 - 3x^2y = 3, \end{cases}$$

1-tenglama $(a-b)^3$ qisqa ko‘paytirish formulasiga keladi: $x^3 - y^3 - 3x^2y + 3y^2x = 1$.

Bundan,
$$\begin{cases} (x-y)^3 = 1 \\ x^2 - x^2y = 1 \end{cases}$$
$$\begin{cases} x - y = 1 \\ x^2 - x^2y = 1 \end{cases}$$

Endi esa, o‘rniga qo‘yish usulidan foydalanamiz va sistemani yechamiz:

$$\begin{cases} y = x-1 \\ x^2 - x^2(x-1) = 1 \end{cases} \Rightarrow \begin{cases} y = x-1 \\ x^2 - x^3 + x^2 = 1 \end{cases} \Rightarrow \begin{cases} y = x-1 \\ x^3 - 2x^2 + 1 = 0 \end{cases} \Rightarrow \begin{cases} y = x-1 \\ x^3 - 2x^2 + 1 = 0 \end{cases}$$

$\Rightarrow \begin{cases} y = x-1 \\ (x-1)(x^2 - x - 1) = 0 \end{cases} \Rightarrow x_1 = 1,\ x_2 = \frac{1+\sqrt{5}}{2},\ x_3 = \frac{1-\sqrt{5}}{2}$, bu qiymatlarni $y = x-1$ tenglamaga qo‘yib, $y_1 = 0,\ y_2 = \frac{-1+\sqrt{5}}{2},\ y_3 = \frac{-1-\sqrt{5}}{2}$ ekanligini topamiz.

Javob: $(1,0),\left(\frac{1+\sqrt{5}}{2},\frac{-1+\sqrt{5}}{2}\right),\left(\frac{1-\sqrt{5}}{2},\frac{-1-\sqrt{5}}{2}\right)$.

O‘zgaruvchini almashtirish usuli

5-misol. Tenglamalar sistemasini yeching:

$$\begin{cases} x+xy+y=11, \\ x^2y+xy^2=30. \end{cases}$$

Yechish. Yangi o‘zgaruvchilarni kiritamiz:

$$x+y=a,$$
$$xy=b.$$

Shunda sistema quyidagi ko‘rinishga keladi:

$$\begin{cases} a+b=11, \\ ab=30. \end{cases}$$

Bu sistemani yechib, $a_1=6,\ b_1=5$ va $a_2=5,\ b_2=6$ larni aniqlaymiz. Endi quyidagi sistemalarni yehamiz:

$$\begin{cases} x+y=6, \\ xy=5, \end{cases} \text{ va } \begin{cases} x+y=5, \\ xy=6. \end{cases}$$

Ularning ildizlaridan tuzilgan to‘plami tenglamalar sistemasining yechimi bo‘ladi.

Javob: $\{(5;1),\ (1;5),\ (2;3),\ (3;2)\}$.

MISOLLAR

Tenglamalar sistemasini yeching.

1. $\begin{cases} y-x^2+x=1 \\ x=y-4 \end{cases}$

2. $\begin{cases} 4x^2-y=2 \\ 3x-2y=-1 \end{cases}$

3. $\begin{cases} 4x+3y=-1 \\ 2x^2=y+11 \end{cases}$

4. $\begin{cases} xy=20 \\ x-4y=2 \end{cases}$

5. $\begin{cases} x^2+y^2-2xy=1 \\ x+y=3 \end{cases}$

6. $\begin{cases} 3x-y=10 \\ x^2-y^2=20-xy \end{cases}$

7. $\begin{cases} x+y=8 \\ x^2+y^2=36 \end{cases}$

8. $\begin{cases} x\cdot y=300 \\ x+y=35 \end{cases}$

9. $\begin{cases} x^2+y^2=74 \\ x+y=12 \end{cases}$

10 $\begin{cases} x+y=8 \\ xy=15 \end{cases}$

11 $\begin{cases} x+y=1 \\ x^3+y^3=19 \end{cases}$

12 $\begin{cases} x^3+8y^3=35 \\ x^2-2xy+4y^2=7 \end{cases}$

13. $\begin{cases} \dfrac{xy}{x+2y}+\dfrac{x+2y}{xy}=2 \\ \dfrac{xy}{x-2y}+\dfrac{x-2y}{xy}=4 \end{cases}$

14. $\begin{cases} x^2-xy+\dfrac{1}{4}y^2+x-\dfrac{1}{2}y=2 \\ \dfrac{1}{4}x^2+xy+y^2+2y+x=3 \end{cases}$

15. $\begin{cases} x^2-xy+y^2=19 \\ x^2+xy+y^2=49 \end{cases}$

16. $\begin{cases} x^2+y^2=x+y \\ x^4+y^4=\dfrac{1}{2}(x+y)^2 \end{cases}$

17. $\begin{cases} \dfrac{x-y}{x+y}+6\dfrac{x+y}{x-y}=5 \\ xy=-2 \end{cases}$

18. $\begin{cases} \dfrac{2x}{y}+\dfrac{3y}{x}+6=\dfrac{3}{xy} \\ \dfrac{6y}{x}+\dfrac{4x}{y}-1=\dfrac{45}{xy} \end{cases}$

19. $\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=5 \\ \dfrac{1}{x^2}+\dfrac{1}{y^2}=13 \end{cases}$

20. $\begin{cases} \dfrac{x+y}{xy}+\dfrac{xy}{x+y}=2 \\ \dfrac{x-y}{xy}+\dfrac{xy}{x-y}=\dfrac{5}{2} \end{cases}$

21. $\begin{cases} x^3-y^3=61(x-y) \\ (x+1)(y+1)=12 \end{cases}$

22. $\begin{cases} \dfrac{x^2}{y}+\dfrac{y^2}{x}=\dfrac{9}{2} \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{2} \end{cases}$

23. $\begin{cases} x^4+y^4=17(x+y)^2 \\ xy=2(x+y) \end{cases}$

24. $\begin{cases} x^2+y^2=x-y \\ x^4+y^4=\dfrac{1}{2}(x-y)^2 \end{cases}$

25. $\begin{cases} x^2\left(1+y+y^2+y^3\right)=160 \\ x^2\left(1-y+y^2-y^3\right)=-80 \end{cases}$

26. $\begin{cases} 2x^2y^2-3y^2+5xy-6=0 \\ 3x^2y^2-4y^2+3xy-2=0 \end{cases}$

27. $\begin{cases} x^3y+xy^3=\dfrac{10}{9}(x+y)^2 \\ x^4y+xy^4=\dfrac{2}{3}(x+y)^3 \end{cases}$

28. $\begin{cases} \dfrac{x\left(y^2+1\right)}{x^2+y^2}=\dfrac{3}{5} \\ \dfrac{y\left(x^2-1\right)}{x^2+y^2}=\dfrac{4}{5} \end{cases}$

29. $\begin{cases} \dfrac{y^2}{x}+\dfrac{x^2}{y}=12 \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{3} \end{cases}$

30. $\begin{cases} xy=6 \\ yz=15 \\ zx=10 \end{cases}$

RATSIONAL TENGSIZLIKLAR

Ta‘rif:

$f(x)$ va $g(x)$ funksiyalar qatnashgan quyidagi munosabatlarga

$$f(x)<g(x),\ f(x)>g(x),\ f(x)\le g(x),\ f(x)\ge g(x)$$

bir o‘zgaruvchili tengsizliklar deyiladi.

Tengsizlikni yechish deganda x ning shunday qiymatlar to‘plamini topishga aytiladiki, bu qiymatlarni tengsizlikka qo‘yganda to‘g‘ri tengsizlik hosil qiladi.

x ning bunday qiymatlar to‘plami – **tengsizlikning yechimi** deyiladi.

Ratsional tengsizliklarni yechish xuddi ratsional tenglamalarni yechish kabi, avval tengsizlikni sodda teng kuchli tengsizlikka keltirish orqali bajariladi. Bunda quyidagi qoidalarga rioya etiladi:

1-qoida. Tengsizlikning ixtiyoriy hadini tengsizlikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o‘tkazish mumkin.

2-qoida. Tengsizlikning ikkala qismini bir xil musbat songa ko‘paytirish yoki bo‘lish mumkin, bunda tengsizlik belgisi o‘zgarmaydi.

3-qoida. Tengsizlikning ikkala qismini bir xil manfiy songa ko‘paytirish yoki bo‘lish mumkin, bunda tengsizlik belgisi qarama-qarshisiga o‘zgaradi.

Ratsional tengsizlikni yechishda, **intervallar usulidan** foydalaniladi.

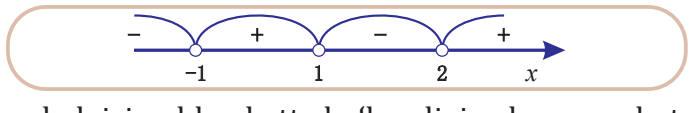
1-misol. Tengsizlikni yeching:

$$(x-1)(x+1)(x-2)>0$$

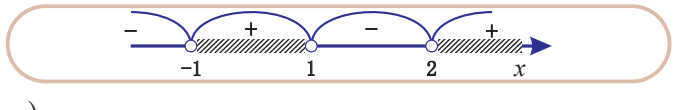
Yechish.

1) Tengsizlikning o‘ng tarafi nolga teng, demak, chap tarafdagi ifodaning nollarini topamiz: $x=1, x=-1, x=2$.

2) x ning bu qiymatlarini son o‘qida belgilaymiz va hosil bo‘lgan intervallarning ishorasini aniqlaymiz.



3) Tengsizligimizning belgisi noldan katta bo‘lganligi uchun, musbat ishorali oraliqlar tengsizlikning yechimi bo‘ladi.



Javob: $x\in(-1;1)\cup(2;\infty)$

2-misol. Tengsizlikni yeching:

$$x^4-3<2x(2x^2-x-2)$$

Yechish.

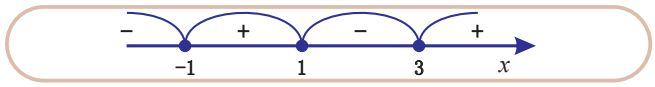
1) Butun ratsional tengsizlik berilgan. Uni yechish uchun avvalambor barcha ifodalarni tengsizlikning chap tarafiga o‘tkazamiz:

$$x^4-4x^3+2x^2+4x-3<0$$

2) Chap tarafdagi hosil bo‘lgan ifodani ko‘paytuvchilarga ajratamiz. Buning uchun uni nollarini topamiz: $x_1=-1,\ x_2=1$ va $x_3=3$.

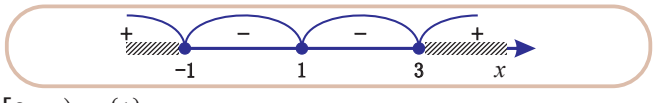
$$(x-1)^2(x+1)(x-3)\geq 0$$

3) Nollarni sonlar o‘qida belgilaymiz va intervallarning ishoralarini belgilaymiz:



4) Chap tarafdagi ifodada $(x-1)$ ikkihad ikkinchi (juft) darajada, shuning uchun son o‘qida 1 sonidan ikki tarafda joylashgan intervallar bir xil ishoraga ega.

5) Tengsizlikning belgisi noldan katta yoki teng bo‘lgani uchun, musbat ishorali oraliqlar va 1 soni tengsizlikning yechimi bo‘ladi:



Javob: $x\in(-\infty;-1]\cup[3;\infty)\cup\{1\}$.

Kasr-ratsional tengsizliklar

$\frac{f(x)}{g(x)}>0$, $\frac{f(x)}{g(x)}<0$, $\frac{f(x)}{g(x)}\geq 0$, $\frac{f(x)}{g(x)}\leq 0$ ko‘rinishiga keltirish mumkin bo‘lgan tengsizliklar ***kasr-ratsional tengsizliklar*** deyiladi.

Kasr-ratsional tengsizliklarni yechish qadamlari:

- Suratining nollari topiladi;
- Aniqlanish sohasi (maxrajining nollari) topiladi;
- Nollar son o‘qida belgilanadi;
- Hosil bo‘lgan intervallarning ishoralari topiladi;
- Tengsizlik belgisiga mos keluvchi oraliq(lar) tengsizlikning yechimi bo‘ladi.

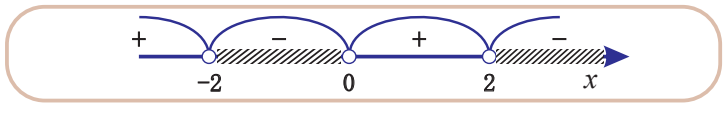
3-misol. Tengsizlikni yeching: $\frac{4}{x}-x<0$.

Yechish.

1) Umumiy maxrajga keltiramiz: $\frac{4-x^2}{x}<0$;

2) Nollarini topamiz: $4-x^2=0\Rightarrow x^2=4\Rightarrow x=\pm 2;\ x\neq 0$;

3) Nollarni son o‘qida belgilaymiz va intervallarning ishoralarini aniqlaymiz:



Tengsizlikning belgisi noldan kichik bo‘lgani uchun, manfiy ishorali oraliqlar tengsizlikning yechimi bo‘ladi.

Javob: $x\in(-2;0)\cup(2;\infty)$.

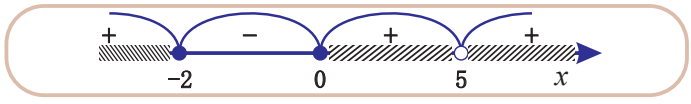
4-misol. Tengsizlikni yeching: $\frac{x(x+2)^3}{(x-5)^2}\geq 0$

Yechish.

1) Suratining nollari: $x=0$ va $x=-2$;

2) Maxrajining nollari (aniqlanish sohasi) $x\neq 5$.

3) Sonlar o‘qida bu qiymatlarni belgilab chiqamiz va intervallarning ishoralarini aniqlaymiz, bunda chap tarafdagi ifodada $(x-5)$ ifoda ikkinchi (juft) darajada qatnashgan, shuning uchun son o‘qida 5 sonidan ikki tarafda joylashgan intervallar bir xil ishoraga ega.



4) Tengsizlikning belgisi noldan katta yoki teng bo‘lgani uchun, musbat ishorali oraliqlar tengsizlikning yechimi bo‘ladi:

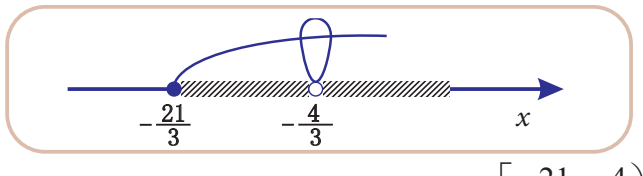
Javob: $x\in(-\infty;-2]\cup[0;5)\cup(5;\infty)$.

Diqqat qiling: $\dfrac{f(x)}{g(x)}<a$ ko‘rinishidagi tengsizliklarni yechishda tengsizlikning ikkala tarafini $g(x)$ ga ko‘paytirishdan boshlamang. Chunki bu noto‘g‘ri javobga olib kelishi mumkin.

Masalan, $\dfrac{2x-1}{3x+4}\le 5$

$$\frac{2x-1}{3x+4}\cdot(3x+4)\le 5\cdot(3x+4)$$
$$2x-1\le 15x+20$$
$$13x\ge -21$$
$$x\ge -\frac{21}{13}$$
$$x\ne -\frac{4}{3}$$

Sonlar o‘qida belgilaymiz:



Ko‘rinib turibdiki, tengsizlikning hosil bo‘lgan yechimi $\left[-\frac{21}{3};-\frac{4}{3}\right)\cup\left(-\frac{4}{3};\infty\right)$ noto‘g‘ridir.

Tog‘ri javobni aniqlash uchun bu tengsizlikni qadamma-qadam mustaqil ravishda ishlab chiqing va nima uchun bunday usul to‘g‘ri javob bermaganligini mulohaza qiling.

MISOLLAR

Tengsizliklarni yeching.

1. $\dfrac{x+4}{(x+5)x}<0$

2. $\dfrac{x-4}{(x-3)x}<0$

3. $\dfrac{5+4x}{(x-2)(x+1)}\ge 0$

4. $\dfrac{4-3x}{(x+2)(x-1)}\ge 0$

5. $\frac{4x+3}{x+2}>5$

6. $\frac{4x-3}{x-5}>5$

7. $\frac{25-16x^2}{x^2+4x+4}>0$

8. $\frac{16-25x^2}{x^2-4x+4}>0$

9. $\frac{6x-1}{4x+3}\le\frac{3x-2}{2x-1}$

10. $\frac{5}{-6x+3}+\frac{6x}{1-2x}\ge 0$

11. $\frac{x^2+3x}{49x^2+70x+25}\le 0$

12. $\frac{6x+1}{4x-3}\le\frac{3x+2}{2x+1}$

13. $\frac{6}{-4x+2}-\frac{5x}{1-2x}\le 0$

14. $\frac{49x^2-70x+25}{x^2-3x}\le 0$

15. $\frac{x^2+3x-2}{(x-1)^2-9}-\frac{3x+1}{3x-12}\le 0$

16. $\frac{x^2+7x+8}{(x+1)^2-9}-\frac{3x+7}{3x-6}\le 0$

17. $\frac{1}{2x^2-5x}-\frac{2}{25+10x}+\frac{4}{25-4x^2}\ge 0$

18. $\frac{6}{-4x-x^2}-\frac{2}{x^2-4x}+\frac{x}{x^2-16}\ge 0$

19. $\left(\frac{4}{x^2+4x}+\frac{32-3x}{x^3+64}\right):\frac{x+8}{x^3-4x^2+16x}\ge\frac{4}{4+x}$

20. $\left(\frac{x^2+2x+4}{4x^2-1}\cdot\frac{2x^2-x}{-x^3+8}-\frac{2-x}{2x^2+x}\right):\frac{4}{x^2-2x}\ge\frac{4-x}{x+2x^2}$

21. $\frac{2x-7}{6}+\frac{7x-2}{3}<3-\frac{1-x}{2}$ tengsizlikning butun sonlardan iborat yechimlaridan eng kattasini ko‘rsating.

22. $\frac{x-4}{2x+6}\le 0$ tengsizlikning barcha butun sonlardagi yechimlari yig‘indisini toping.

23. $\frac{1}{x}<1$ tengsizlikning (-3;3) oraliqdagi butun yechimlari sonini toping.

24. $\frac{(x+3)(x-5)}{x+1}\ge 0$ tengsizlikning butun sonlardan iborat yechimlaridan eng kattasidan eng kichigining ayirmasini toping.

25. $\frac{(x+4)^2-8x-25}{(x-6)^2}\ge 0$ tengsizlikning butun sonlardan iborat yechimlaridan nechtasi [-5; 6] kesmada joylashgan?

RATSIONAL TENGSIZLIKLAR SISTEMASI

Ta'rif:

$$\begin{cases} \dfrac{f_1(x)}{g_1(x)} \vee 0 \\ \dfrac{f_2(x)}{g_2(x)} \vee 0 \end{cases}$$

(bu yerda $\vee -$ ">", "<", "≥", "≤") ko'rinishidagi tengsizliklar sistemasini yechish uchun quyidagi algoritm qo'llaniladi:

Har bir tengsizlikning yechimi alohida topiladi;

Bu yechimlar bitta son o'qida tasvirlanadi;

Ikkala tengsizlikni ham qanoatlantiruvchi intervallar tengsizliklar sistemasining yechimi bo'ladi.

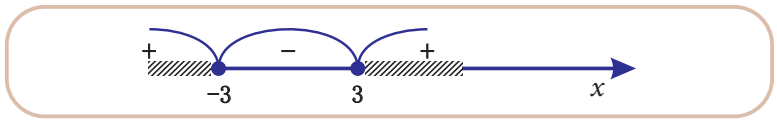
1-misol. Tengsizliklar sistemasini yeching. $\begin{cases} x^2 - 9 \geq 0, \\ 2x - 8 < 0. \end{cases}$

Yechish.

1. Avval $x^2 - 9 \geq 0$ tengsizlikni yechamiz.

Nollarini topamiz, $x_{1,2} = \pm 3$.

Nollarni sonlar o'qida belgilaymiz va hosil bo'lgan oraliqlarni ishoralarini aniqlaymiz:



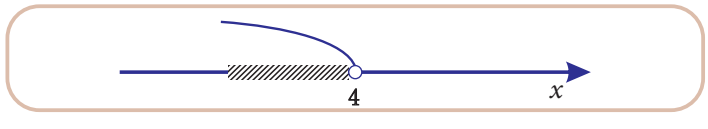
tengsizlikning belgisi noldan katta yoki teng bo'lgani uchun, musbat ishorali oraliqlar yechim bo'ladi: $x \in (-\infty; -3] \cup [3; \infty)$

2. Endi 2-tengsizlikni yechamiz: $2x - 8 < 0$,

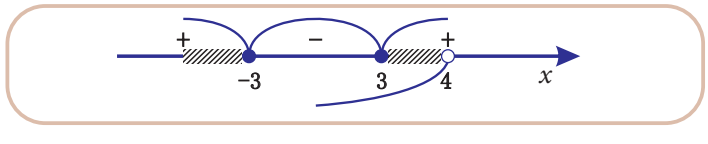
$2x < 8$,

$x < 4$.

Bu tengsizlikning yechimini sonlar o'qida ifodalaymiz:



3. Yakuniy qadamda, ikkala tengsizlikning yechimlarini bitta son o'qida tasvirlaymiz.



4. Ikkala tengsizlikni ham qanoatlantiruvchi oraliqlar berilgan tengsizliklar sistemasining yechimi bo'ladi:

Javob. $x \in (-\infty; -3] \cup [3; 4)$.

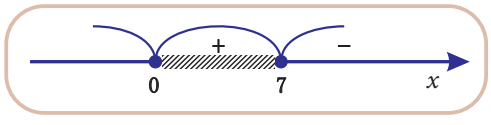
2-misol. Tengsizliklar sistemasini yeching:

$$\begin{cases} 7x - x^2 \geq 0, \\ x^2 - 6x + 5 < 0 \end{cases}$$

Yechish.

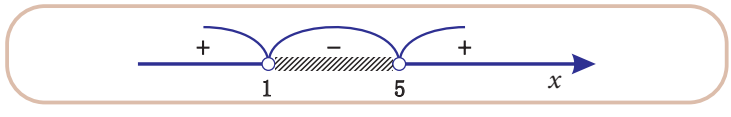
1-tengsizlikni yechamiz: $x(7-x) \geq 0$.

$x = 0$ va $x = 7$ nollarini sonlar o‘qida belgilaymiz va hosil bo‘lgan intervallarning ishoralarini aniqlaymiz:

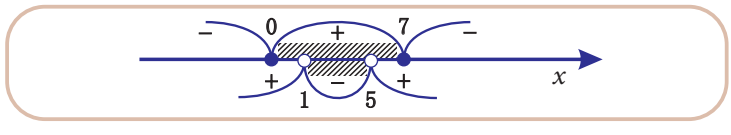


2-tengsizlikni yechamiz: $x^2 - 6x + 5 < 0$.

Nollari $x = 1$ va $x = 5$ ga teng. Ularni son o‘qida belgilaymiz va hosil bo‘lgan intervallarning ishoralarini aniqlaymiz:



3. Ikkala tengsizlikning yechimini bitta son o‘qida belgilaymiz va



ikkala tengsizlikni ham qanoatlantiradigan oraliq sistemaning yechimi bo‘ladi.

Javob: $x \in (1;5)$.

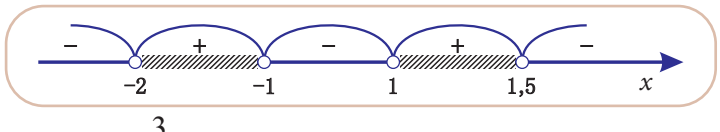
3-misol.

$$\begin{cases} \dfrac{(3-2x)(x+2)}{x^2-1} > 0, \\ 1 + 2x \leq \dfrac{3}{x} \end{cases}$$

1-tengsizlikni yechamiz;

$$\frac{(3-2x)(x+2)}{x^2-1} > 0$$

$x = -2, x = -1, x = 1$ va $x = 1,5$ nollarini sonlar o‘qida belgilaymiz va hosil bo‘lgan intervallarning ishoralarini aniqlaymiz:

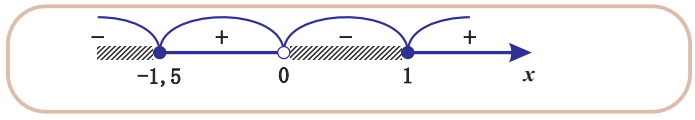


2-tengsizlikni yechamiz: $1 + 2x \leq \dfrac{3}{x}$.

Barcha ifodalarni tengsizlikning chap tarafiga o‘tkazib olamiz va bir xil maxrajga keltiramiz.

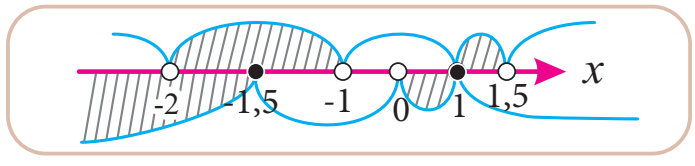
$$\frac{2x^2 + x - 3}{x} \leq 0$$

Kasrning nollari $x = 1$ va $x = -1,5$ ga, aniqlanish sohasi $x \neq 0$ ga teng. Ularni son o‘qida belgilaymiz va hosil bo‘lgan intervallarning ishoralarini aniqlaymiz:



Ikkala tengsizlikning yechimini bitta son o‘qida belgilaymiz va ikkala tengsizlikni ham qanoatlantiradigan oraliq sistemaning yechimi bo‘ladi.

Javob: $x \in (-2;-1,5]$



MISOLLAR

Tengsizliklar sistemasini yeching:

1. $\begin{cases} \dfrac{x+4}{2}-\dfrac{4-3x}{4}<\dfrac{1}{6} \\ 3x^2+7x-6\le 0 \end{cases}$

2. $\begin{cases} \dfrac{x+2}{5}-\dfrac{5-4x}{12}<\dfrac{1}{30} \\ 9x^2-6x-8\le 0 \end{cases}$

3. $\begin{cases} (x+3)(3-x)<(x+4)(5-x) \\ \dfrac{x+4}{14}+\dfrac{2-3x}{4}\ge 1 \end{cases}$

4. $\begin{cases} (x+2)(2-x)<(x+3)(4-x) \\ \dfrac{3+x}{4}+\dfrac{1-2x}{6}\ge 1 \end{cases}$

5. $\begin{cases} \dfrac{x^2-7}{2x-7}>1 \\ \dfrac{3x-2}{5}-\dfrac{6-x}{2}\ge 2x-7 \end{cases}$

6. $\begin{cases} \dfrac{x^2-9}{4x-9}>1 \\ \dfrac{4x-3}{7}-\dfrac{5-x}{5}\le \dfrac{x+3}{2} \end{cases}$

7. $\begin{cases} \dfrac{7x+3}{x+3}\ge \dfrac{-x}{2(x+3)} \\ \dfrac{4+x}{2x-3}\le \dfrac{5+3x}{3-2x} \end{cases}$

8. $\begin{cases} \dfrac{x-5}{x-6}<\dfrac{x+7}{x+6} \\ \dfrac{x+4}{x+8}>\dfrac{x-7}{x-3} \end{cases}$

9. $\begin{cases} \dfrac{x+7}{x-5}+\dfrac{3x+1}{2}\ge 0 \\ (5-x)^2\le 2^2 \end{cases}$

10. $\begin{cases} (x-2)^2\ge 1 \\ \dfrac{2x+3}{x^2-1}<\dfrac{7}{3} \end{cases}$

11. $\begin{cases} \dfrac{x^3-8x^2+12x}{x-6}\le 6x \\ x-2\le \dfrac{x^3-8x^2+12x}{x-6} \end{cases}$

12. $\begin{cases} \dfrac{x^2-5x+11}{x^2-x-2}+\dfrac{7}{x+1}\le 0 \\ \dfrac{2x^2-14x+6}{x^2-4x+3}\ge \dfrac{3x-8}{x-3} \end{cases}$

13. $\begin{cases} \dfrac{3x+5}{x+4}\ge \dfrac{x}{2(x+4)} \\ \dfrac{10+2x}{3x-5}\le \dfrac{6+4x}{5-3x} \end{cases}$

14. $\begin{cases} \dfrac{2x-7}{x-4}<\dfrac{2x+9}{x+4} \\ \dfrac{3x+5}{3x+8}>\dfrac{3x-7}{3x-4} \end{cases}$

IRRATSIONAL TENGLAMALAR

Ta‘rif.

Noma‘lum qatnashgan ifodalari ildiz belgisi ostida bo‘lgan tenglamalar **irratsional tenglamalar** deyiladi.

Masalan, $\sqrt{2x-5}=7$, $2\sqrt{x}+5=8$, $\sqrt[3]{x+3}=-1-x$, $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x}$,

$\sqrt[5]{(x+1)^2}-\sqrt[5]{(x-1)^2}=\sqrt[5]{x^2-1}$ tenglamalar irratsional tenglamalardir.

Ko‘p hollarda irratsional tenglamalar, o‘zining natijasi bo‘lgan ratsional tenglamalarga keltirib yechiladi. Irratsional tenglamani ratsional tenglamaga keltirish uchun berilgan tenglamaning ikki tomonini bir yoki bir necha marta biror darajaga ko‘tarish kerak bo‘ladi.

Ammo tenglikning ikki tomonini darajaga ko‘tarish natijasida chet ildizlar hosil bo‘lishi mumkin. Buni quyidagi teorema tasdiqlaydi.

Teorema. $f_1(x)=f_2(x)$ tenglamaning ikki tomonini kvadratga ko‘tarishdan hosil bo‘lgan $f_1^2(x)=f_2^2(x)$ tenglamaning ildizlari $f_1(x)=f_2(x)$ va $f_1(x)=-f_2(x)$ tenglamalarning ildizlaridan iborat bo‘ladi.

Isbot. $f_1^2(x)=f_2^2(x)$ tenglama $f_1^2(x)-f_2^2(x)=0$ yoki $(f_1(x)-f_2(x)\big(f_1(x)+f_2(x)\big)=0$ tenglama bilan teng kuchlidir. Ammo keyingi tenglama faqat $f_1(x)=f_2(x)$ va $f_1(x)=-f_2(x)$ tengliklarning eng kamida biri bajarilganda o‘rinli bo‘ladi.

Teorema isbotlandi.

Bu isbotlangan teoremadan $f_1(x)=f_2(x)$ tenglamadan $f_1^2(x)=f_2^2(x)$ tenglamaga o‘tishda ildizlar yo‘qolishi ro‘y bermaydi, lekin chet ildizlar hosil bo‘lishi mumkinligi kelib chiqadi. $f_1(x)=-f_2(x)$ tenglama $f_1(x)=f_2(x)$ tenglamaga ***qo‘shma tenglama*** deyiladi.

Agar $f_1(x)=-f_2(x)$ tenglama ildizga ega bo‘lmasa, u holda ravshanki chet ildizlar bo‘lmaydi.

Masalan, $2x=3$ tenglamaning ikki tomonini kvadratga ko‘tarishdan hosil bo‘lgan $4x^2=9$ yoki $4x^2-9=0$ tenglama quyidagi $2x-3=0$ va $2x+3=0$ tenglamalarga teng kuchli bo‘ladi. Bulardan $2x-3=0$ berilgan tenglamaga teng kuchli bo‘lib, $2x+3=0$ unga qo‘shma bo‘ladi. Keyingi tenglamalarning ildizlaridan $x=\frac{3}{2}=1\frac{1}{2}$ berilgan tenglamani qanoatlantiradi, $x=-\frac{3}{2}=-1\frac{1}{2}$ esa berilgan tenglamaga chet ildizdir.

Irratsional tenglamada birgina ildiz qatnashsa, bu ildizni tenglamaning bir tomonida qoldirib, tenglamaning qolgan hadlarini ikkinchi tomonga o‘tkazamiz. Keyin esa tenglamaning ikki tomonini tenglama ildizdan qutiladigan qilib, biror darajaga ko‘taramiz. Natijada ratsional tenglama hosil bo‘ladi. Bu hosil bo‘lgan tenglamani yechib, uning ildizlarini berilgan irratsional tenglamaga qo‘yib tekshirib ko‘rish kerak. Agar topilgan ildizlardan birortasi berilgan tenglamani qanoatlantirmasa, u chet ildiz hisoblanadi.

Irratsional tenglamalarni yechish usullari:

1. $\sqrt{f(x)}=g(x)$ ko‘rinishidagi tenglamani yechishda quyidagi teng kuchli sistemaga o‘tiladi va yechiladi: $\begin{cases} g(x)\ge 0 \\ f(x)=g^2(x) \end{cases}$

1-misol. $3\sqrt{x}-7=5$ tenglamani yeching.

Yechish:

1. Tenglamaning aniqlanish sohasini topamiz: $x \geq 0$.
2. $3\sqrt{x}-7=5, \Rightarrow 3\sqrt{x}=5+7, \Rightarrow 3\sqrt{x}=12, \Rightarrow \sqrt{x}=4$ tenglamaning ikki tomonini kvadratga ko‘tarsak, $x=16$.
3. Tekshirish: $3\sqrt{16}-7=3\cdot 4-7=5$.

Demak, $x=16$ berilgan irratsional tenglamani qanoatlantirar ekan.

Javob: $x=16$.

2-misol. $\sqrt{x^2-x-2}=x-3$ tenglamani yeching.

Yechish:

1. Tenglamaning aniqlanish sohasini topamiz.
$x^2-x-2\geq 0, \Rightarrow (x+1)(x-2)\geq 0, \Rightarrow x\leq -1,\ x\geq 2$.
2. $\sqrt{x^2-x-2}=x-3$ tenglamaning ikki tomonini kvadratga ko‘taramiz: $x^2-x-2=x^2-6x+9$, $\Rightarrow 5x=11, \Rightarrow x=2,2$.
3. Tekshirish: $\sqrt{2,2^2-2,2-2}=2,2-3$, $\sqrt{0,64}=-0,8$; $0.8\neq -0,8$. Demak, $x=2,2$ chet ildiz, tenglama yechimga ega emas.

Javob: $\varnothing$.

3-misol. $\sqrt{3x^2-6x+16}=2x-1$ tenglamani yeching.

Yechish:

1. $\sqrt{3x^2-6x+16}=2x-1 \Rightarrow \begin{cases} 3x^2-6x+16=(2x-1)^2 \\ 2x-1\geq 0 \end{cases} \Rightarrow$

$\begin{cases} 3x^2-6x+16=4x^2-4x+1 \\ 2x\geq 1 \end{cases} \Rightarrow \begin{cases} x^2+2x-15=0 \\ x\geq \dfrac{1}{2} \end{cases}$

2. $x^2+2x-15=0$ tenglamani yechamiz. $x_{1,2}=\dfrac{-2\pm\sqrt{4+60}}{2}=\dfrac{-2\pm 8}{2}, \Rightarrow x_1=3,\ x_2=-5.$
3. $x\geq \dfrac{1}{2}$ bo‘lganligi sababli tenglamaning yechimi $x=3$.
4. Tekshirish: $\sqrt{3\cdot 3^2-6\cdot 3+16}=2\cdot 3-1,\ \sqrt{25}=5$

Javob: $x=3$.

2. $\sqrt{f(x)}\cdot g(x)=0$ ko‘rinishidagi tenglamani yechishda, chap tarafdagi ko‘paytmaning hech bo‘lmasa bitta ko‘paytuvchisi nolga teng bo‘lishi kerakligi inobatga olinadi, va tenglama quyidagi sistemaga keltiriladi.

4-misol. Tenglamani yeching. $(x^2-25)\sqrt{6-2x}=0$

Yechish.

$$\begin{cases} x^2-25=0 \\ 6-2x\geq 0 \end{cases} \Rightarrow \begin{cases} x_{1,2}=\pm 5 \\ x\leq 3 \end{cases} \Rightarrow x_1=-5; x_2=3$$

Javob: $x_1=-5; x_2=3$.

3. $\sqrt{f(x)}=\sqrt{g(x)}$ ko‘rinishidagi tenglamani yechishda quyidagi sistemalarning biriga keltirlib yechiladi:

$$\begin{cases} f(x)=g(x), \\ f(x)\ge 0 \end{cases} \quad \text{yoki} \quad \begin{cases} f(x)=g(x), \\ g(x)\ge 0 \end{cases}$$

5-misol. Tenglamani yeching: $\sqrt{x+1}=\sqrt{2x-3}$.

Yechish.

$$\begin{cases} x+1=2x-3 \\ 2x-3\ge 0 \end{cases} \Rightarrow \begin{cases} x=4 \\ x\ge 1,5 \end{cases} \Rightarrow x=4$$

Javob: $x=4$.

6-misol. Tenglamani yeching; $\sqrt{x^2+4x}=\sqrt{14-x}$.

Yechish.

$$\begin{cases} x^2+4x=14-x \\ 14-x\ge 0 \end{cases} \cdot \Rightarrow \begin{cases} x^2+5x-14=0 \\ x\le 14 \end{cases} \cdot \Rightarrow \begin{cases} x_1=-7; x_2=2 \\ x\le 14 \end{cases}$$

Javob: $x_2=-7$.

4. $\sqrt{f(x)}\cdot\sqrt{g(x)}=0$ ko‘rinishidagi tenglamani yechishda quyidagi sistemalarning biriga keltirlib yechiladi:

$$\begin{cases} f(x)=0, \\ g(x)\ge 0 \end{cases} \quad \text{yoki} \quad \begin{cases} g(x)=0, \\ f(x)\ge 0 \end{cases}$$

Ba‘zi hollarda tenglamada qatnashayotgan funksiyaning aniqlanish sohasini bilish, tenglamaning yechimi mavjud yoki mavjud emasligini bilishga, yoki yechimini topishga yordam beradi.

7-misol. $\sqrt{1-x^2}\cdot\sqrt{x^2-9}=0$ tenglamani yeching.

Yechish: 1. Tenglamaning aniqlanish sohasini topamiz.

$$\begin{cases} 1-x^2\ge 0, \\ x^2-9\ge 0 \end{cases} \Rightarrow \begin{cases} x^2\le 1, \\ x^2\ge 9 \end{cases} \Rightarrow \begin{cases} -1\le x\le 1, \\ x\le -3,\ x\ge 3 \end{cases} \Rightarrow \varnothing$$

2. Tenglamaning aniqlanish sohasi bo‘sh to‘plam bo‘lganligi sababli, tenglama yechimga ega emas.

Javob: $\varnothing$.

5. $\sqrt[3]{f(x)}+\sqrt[3]{g(x)}=V(x)$ ko‘rinishidagi tenglamalar quyidagicha yechiladi:

$\sqrt[3]{f(x)}+\sqrt[3]{g(x)}=V(x)$. Ikkala tarafini kub darajaga ko‘taramiz.

$$f(x)+3\cdot\sqrt[3]{f^2(x)\cdot g(x)}+3\cdot\sqrt[3]{g^2(x)\cdot f(x)}+g(x)=V^3(x)$$

Guruhlab olamiz va qavsdan tashqariga umumiy ko‘paytuvchini chiqarib yozib olamiz:

$$f(x)+g(x)+3\cdot\sqrt[3]{f(x)\cdot g(x)}\left(\sqrt[3]{f(x)}+\sqrt[3]{g(x)}\right)=V^3(x), \quad (1)$$

$$f(x)+g(x)+3\cdot\sqrt[3]{f(x)\cdot g(x)}\cdot V(x)=V^3(x), \quad (2)$$

(1) dan (2) ga o‘tish teng kuchli o‘tish emas, shuning uchun **tekshirish** lozim bo‘ladi.

8-misol. $\sqrt[3]{x+45}-\sqrt[3]{x-16}=1$ tenglamani yeching.

Yechish:

1. Tenglamaning aniqlanish sohasini topamiz: $x \in R$.

2. $(a-b)^3=a^3-3ab(a-b)-b^3$ bo‘lgani uchun berilgan tenglamaning ikki tomonini kubga ko‘taramiz.

$$\left(\sqrt[3]{x+45}-\sqrt[3]{x-16}\right)^3=1^3,$$

$$x+45-3\sqrt[3]{x+45}\cdot\sqrt[3]{x-16}\ (\sqrt[3]{x+45}-\sqrt[3]{x-16})-(x-16)=1,$$

$\sqrt[3]{x+45}\cdot\sqrt[3]{x-16}=20$ tenglama hosil bo‘ladi.

3. $\sqrt[3]{x+45}\cdot\sqrt[3]{x-16}=20$ tenglamani yana kubga ko‘tarsak,

$$(x+45)(x-16)=8000 \text{ yoki } x^2+29x-8720=0.$$

Bu tenglamani yechib, $x_1=80$, $x_2=-109$ ni topamiz.

4. $x_1=80$, $x_2=-109$ ildizlar tenglamaning aniqlanish sohasiga tegishli.

Javob: $x_1=80$, $x_2=-109$.

6. $\sqrt[2k]{f(x)}=g(x)\Rightarrow\begin{cases}f(x)=[g(x)]^{2k}\\ g(x)\geq 0\end{cases}$

7. $\sqrt[2k+1]{f(x)}=g(x)\Rightarrow f(x)=[g(x)]^{2k+1}$

9-misol. $\sqrt{3x+7}-\sqrt{x+1}=2$ tenglamani yeching.

Yechish:

1. Tenglamaning aniqlanish sohasini topamiz: $\begin{cases}x+1\geq 0\\ 3x+7\geq 0\end{cases}\Rightarrow x\geq -1$

2. $\sqrt{3x+7}-\sqrt{x+1}=2$ tenglamaning ikki tomonini kvadratga ko‘taramiz.

$$\left(\sqrt{3x+7}-\sqrt{x+1}\right)^2=2^2,$$

$$3x+7-2\sqrt{(3x+7)(x+1)}+x+1=4,$$

$$\sqrt{(3x+7)(x+1)}=2x+2.$$

3. $\sqrt{(3x+7)(x+1)}=2x+2$ tenglamani yana bir marta kvadratga ko‘tarsak,

$(3x+7)(x+1)=4x^2+8x+4$ tenglama hosil bo‘ladi.

Bundan, $x^2-2x-3=0$ kelib chiqadi.

Bu tenglamaning ildizlari $x_1=-1$, $x_2=3$.

4. Tekshirish:

$x=-1$ da, $\sqrt{3(-1)+7}-\sqrt{-1+1}=2-0=2$;

$x=3$ da $\sqrt{3\cdot 3+7}-\sqrt{3+1}=4-2=2$.

Ikkala ildiz ham berilgan tenglamani qanoatlantiradi.

Javob: $x_1=-1$, $x_2=3$.

10-misol. $\sqrt{1-x}+\sqrt{1+x^2}+\sqrt{x-1}=\sqrt{2}$ tenglamani yeching.

Yechish: 1. Tenglamaning aniqlanish sohasini topamiz.

$$\begin{cases}1-x\ge 0\\ x-1\ge 0\end{cases}\Rightarrow\begin{cases}x\le 1\\ x\ge 1\end{cases}\Rightarrow x=1$$

2. Tenglamaning aniqlanish sohasi faqat bitta x = 1 nuqtadan iborat. x = 1 berilgan tenglamani qanoatlantirishini tekshiramiz.

3. Tekshirish: $\sqrt{1-1}+\sqrt{1+1^2}+\sqrt{1-1}=0+\sqrt{2}+0=\sqrt{2}$, $\sqrt{2}=\sqrt{2}$ tenglik to‘g‘ri. Demak, tenglama faqat x = 1 ildizga ega.

Javob: x = 1.

11-misol. $\sqrt{\dfrac{3x+2}{x}}+\sqrt{\dfrac{x}{3x+2}}=\dfrac{5}{2}$ tenglamani yeching.

Yechish: 1. Tenglamaning aniqlanish sohasini topamiz:

$\dfrac{3x+2}{x}\ge 0$, $x\ne 0$, $3x+2\ne 0$ shartlardan $x<-\dfrac{2}{3}$ va $x>0$.

2. $\sqrt{\dfrac{3x+2}{x}}=a$ belgilash kiritsak, $\sqrt{\dfrac{x}{3x+2}}=\dfrac{1}{a}$ bo‘lib, tenglama $a+\dfrac{1}{a}=\dfrac{5}{2}$ ko‘rinishga keladi. Bu tenglamani yechib, $a_1=2$ va $a_2=\dfrac{1}{2}$ larni topamiz.

3. $\sqrt{\dfrac{3x+2}{x}}=a$ almashtirishdan foydalansak $x_1=2$ va $x_2=-\dfrac{8}{11}$ kelib chiqadi. Demak, tenglamaning ildizlari $x_1=2$ va $x_2=-\dfrac{8}{11}$.

Javob: $x_1=2$ va $x_2=-\dfrac{8}{11}$.

12-misol. $\sqrt{x+6}+x=0$ tenglamani yeching.

Yechish: 1) $\sqrt{x+6}=-x\Rightarrow\begin{cases}x+6=x^2\\ -x\ge 0.\end{cases}\Rightarrow\begin{cases}x^2-x-6=0\\ x\le 0.\end{cases}$

2) $x^2-x-6=0$ tenglamani yechamiz. $x_{1,2}=\dfrac{1\pm\sqrt{1+24}}{2}=\dfrac{1\pm 5}{2}$, $\Rightarrow x_1=3$, $x_2=-2$.

3) $x\le 0$ bo‘lganligi sababli tenglamaning yechimi $x=-2$.

4) Tekshirish: $\sqrt{-2+6}=-(-2)$, $\sqrt{4}=2$

Javob: $x=-2$.

13-misol. $\sqrt[5]{25+\sqrt{x+13}}-2=0$ tenglamani yeching.

Yechish: 1) $\sqrt[5]{25+\sqrt{x+13}}=2\Rightarrow 25+\sqrt{x+13}=2^5\Rightarrow\sqrt{x+13}=7$.

2) $\sqrt{x+13}=7$, $x+13=7^2$, $x=49-13=36$.

3) Tekshirish: $\sqrt[5]{25+\sqrt{36+13}}=\sqrt[5]{25+\sqrt{49}}=\sqrt[5]{25+7}=\sqrt[5]{32}=2$.

Javob: $x=36$.

14-misol. $\sqrt[3]{x^3+4x^2+3x-3}=x+1$ tenglamani yeching.

Yechish: 1) $\sqrt[3]{x^3+4x^2+3x-3}=x+1 \Rightarrow x^3+4x^2+3x-3=(x+1)^3 \Rightarrow$
$\Rightarrow x^3+4x^2+3x-3=x^3+3x^2+3x+1 \Rightarrow x^2-4=0$.

2) $x^2-4=0$ tenglamani yechamiz. $x^2=4$, $x_{1,2}=\pm 2$.

3) $x_{1,2}=\pm 2$ tenglamaning yechimi.

Tekshirish:

$x=2$ da, $\sqrt[3]{2^3+4\cdot 2^2+3\cdot 2-3}=2+1$, $\sqrt[3]{27}=3$

$x=-2$ da, $\sqrt[3]{(-2)^3+4\cdot(-2)^2+3\cdot(-2)-3}=-2+1$, $\sqrt[3]{-1}=-1$.

Javob: $x=\pm 2$.

15-misol. $\sqrt{x^2-3x+5}+x^2=3x+7$ tenglamani yeching.

Yechish:

1) $\sqrt{x^2-3x+5}+x^2=3x+7 \Rightarrow \sqrt{x^2-3x+5}+x^2-3x+5-12=0$

$\sqrt{x^2-3x+5}=a$ belgilash kiritsak, $a^2+a-12=0$ kvadrat tenglama hosil bo'ladi.

2) $a^2+a-12=0$ tenglamani yechamiz. $a_{1,2}=\dfrac{-1\pm\sqrt{1+48}}{2}=\dfrac{-1\pm 7}{2}$, $a_1=3$; $a_2=-4$.

3) $\sqrt{x^2-3x+5}=a$, $a=3$ da $\sqrt{x^2-3x+5}=3$, $x^2-3x+5=9$, $x^2-3x-4=0$ tenglamani yechamiz. $x_{1,2}=\dfrac{3\pm\sqrt{9+16}}{2}=\dfrac{3\pm 5}{2}$, $x_1=4$, $x_2=-1$.

4) $a=-4\notin[0;\infty)$ bo'lganligi uchun $\sqrt{x^2-3x+5}=-4$ tenglama yechimga ega emas.

5) $x_1=4$, $x_2=-1$ tenglamaning yechimlari.

Javob: $x=4$, $x=-1$.

16-misol. $\sqrt{x+9+5\sqrt{2x-7}}+\sqrt{x+1+3\sqrt{2x-7}}=5\sqrt{2}$ tenglamani yeching.

Yechish:

1) $\sqrt{2x-7}=a$ belgilash kiritib yechamiz. $\sqrt{2x-7}=a$ tenglikning ikki tomonini kvadratga ko'tarib, x ni a orqali ifodalaymiz. Bu yerda $a\in[0;\infty)$.

$2x-7=a^2$, $x=\dfrac{a^2+7}{2}$.

2) $x=\dfrac{a^2+7}{2}$ da $\sqrt{\dfrac{a^2+7}{2}+9+5a}+\sqrt{\dfrac{a^2+7}{2}+1+3a}=5\sqrt{2}$ tenglamani soddalashtirib olamiz.

$$\sqrt{\frac{a^2+7+18+10a}{2}}+\sqrt{\frac{a^2+7+2+6a}{2}}=5\sqrt{2},$$

$$\sqrt{\frac{a^2+10a+25}{2}}+\sqrt{\frac{a^2+6a+9}{2}}=5\sqrt{2},$$

$$\sqrt{\frac{(a+5)^2}{2}}+\sqrt{\frac{(a+3)^2}{2}}=5\sqrt{2},$$

$$\frac{a+5}{\sqrt{2}}+\frac{a+3}{\sqrt{2}}=5\sqrt{2}.$$

3) $\frac{a+5}{\sqrt{2}}+\frac{a+3}{\sqrt{2}}==5\sqrt{2}$ tenglamani yechamiz. $2a+8=10,\ a=1.$

4) $a=1$ da $\sqrt{2x-7}=1$ tenglamani yechamiz. $2x-7=1,\ x=4.$

5) Tekshirish: $x=4$ bo‘lganda

$$\sqrt{4+9+5\sqrt{2\cdot4-7}}+\sqrt{4+1+3\sqrt{2\cdot4-7}}=\sqrt{18}+\sqrt{8}=3\sqrt{2}+2\sqrt{2}=5\sqrt{2}.$$

Javob: $x=4.$

17-misol. $\sqrt[3]{x-2}+\sqrt{x+1}=3$ tenglamani yeching.

Yechish: 1) $\sqrt[3]{x-2}=a$ va $\sqrt{x+1}=b$ belgilash kiritib yechamiz.
Bunda, $\boldsymbol{a}\in\boldsymbol{R},\ \boldsymbol{b}\in[0;\infty)$.

$$x-2=a^3,\quad x=a^3+2$$
$$x+1=b^2,\quad x=b^2-1.$$

2) $\sqrt[3]{x-2}+\sqrt{x+1}=3$ tenglama

$\begin{cases}a+b=3\\a^3+2=b^2-1\end{cases}$ tenglamalar sistemasiga teng kuchli bo‘ladi.

Birinchi tenglamadan b ni topib, ikkinchi tenglamaga qo‘yib yechamiz.
$b=3-a,\ a^3+2=(3-a)^2-1,\ a^3+2=9-6a+a^2-1,$

$a^3-a^2+6a-6=0,\ a^2(a-1)+6(a-1)=0$ bundan $a=1$ kelib chqadi.

3) $a=1$ bo‘lganda $b=3-1=2$ bo‘ladi. Demak, $\sqrt[3]{x-2}=1$ va $\sqrt{x+1}=2$. Tenglamalarni yechsak, $x=3$ ekanligi kelib chiqadi.

4) Tekshirish: $x=3$ da $\sqrt[3]{3-2}+\sqrt{3+1}=\sqrt[3]{1}+\sqrt{4}=1+2=3.$

Javob: $x=3.$

18-misol. $\sqrt{2x^2-9x+4}+3\sqrt{2x-1}=\sqrt{2x^2+21x-11}$ tenglamani yeching:

Yechish: 1) Tenglamaning aniqlanish sohasini topamiz.

$$\begin{cases}2x^2-9x+4\ge0,\\2x-1\ge0,\\2x^2+21x-11\ge0.\end{cases}\Rightarrow\begin{cases}(2x-1)(x-4)\ge0,\\2x\ge1,\\(2x-1)(x+11)\ge0.\end{cases}\Rightarrow\begin{cases}x=\frac{1}{2}\\x\ge4\end{cases}$$

2) $\sqrt{2x^2-9x+4}+3\sqrt{2x-1}=\sqrt{2x^2+21x-11}$ tenglamaning ikki tomonini kvadratga oshiramiz.

$2x^2-9x+4+6\sqrt{2x^2-9x+4}\cdot\sqrt{2x-1}+9(2x-1)=2x^2+21x-11,$

$6\sqrt{2x^2-9x+4}\cdot\sqrt{2x-1}=12x-6$ tenglamaning ikki tomonini 6 ga bo‘lamiz.

$\sqrt{2x^2-9x+4}\cdot\sqrt{2x-1} = 2x-1$. Hosil bo'lgan tenglamaning ikki tomonini yana kvadratga oshirsak, quyidagi tenglama hosil bo'ladi:

$$(2x^2-9x+4)(2x-1)=(2x-1)^2,$$

$$(2x-1)(2x^2-9x+4-2x+1)=0,$$

$$(2x-1)(2x^2-11x+5)=0,$$

$$(2x-1)(2x-1)(x-5)=0,$$

$$x_1 = \frac{1}{2},\ x_2 = 5.$$

3) Aniqlanish sohasi $x=\frac{1}{2}$ va $x\ge 4$. Tenglamaning $x_1 = \frac{1}{2}$, $x_2 = 5$ ildizlari aniqlanish sohasiga tegishli.

Javob : $x=\frac{1}{2};\ x=5.$

19-misol. $\frac{3}{3+\sqrt{x}}-\frac{5}{x+3\sqrt{x}} = \frac{1}{4}$ tenglamani yeching.

Yechish: 1) Tenglamaning aniqlanish sohasini topamiz: $x>0$.

2) $\frac{3}{3+\sqrt{x}}-\frac{5}{\sqrt{x}(3+\sqrt{x})} = \frac{1}{4}$ tenglamani umumiy maxrajga keltirib, yechamiz.

$4(3\sqrt{x}-5) = x+3\sqrt{x}$, $x-9\sqrt{x}+20=0$, $\sqrt{x}=4,\ \sqrt{x}=5$. Bundan $x=16,\ x=25$ kelib chiqadi. $x=16,\ x=25$ aniqlanish sohasiga tegishli.

3) Tekshirish: $x=16$ da, $\frac{3}{3+\sqrt{16}}-\frac{5}{16+3\sqrt{16}} = \frac{3}{7}-\frac{5}{28} = \frac{1}{4}$

$x=25$ da, $\frac{3}{3+\sqrt{25}}-\frac{5}{25+3\sqrt{25}} = \frac{3}{8}-\frac{5}{40} = \frac{1}{4}$.

Javob: $x=16,\ x=25$.

20-misol. $\sqrt{5+\sqrt[3]{x}}+\sqrt{5-\sqrt[3]{x}} = \sqrt[3]{x}$ tenglamani yeching.

Yechish: 1) Tenglamaning aniqlanish sohasini topamiz. $\begin{cases} 5+\sqrt[3]{x}\ge 0 \\ 5-\sqrt[3]{x}\ge 0 \\ \sqrt[3]{x}\ge 0 \end{cases} \Rightarrow x\in[0;125]$

2) $\sqrt[3]{x}=a$ belgilash kiritsak, $\sqrt{5+a}+\sqrt{5-a} = a$, bunda $a\in(0;\infty)$.

3) $\sqrt{5+a}+\sqrt{5-a} = a$ tenglamaning ikki tomonini kvadratga oshiramiz.

$5+a+2\sqrt{(5+a)(5-a)}+5-a=a^2 \Rightarrow \quad 10+2\sqrt{25-a^2}=a^2 \Rightarrow \quad 2\sqrt{25-a^2}=a^2-10$

4) $2\sqrt{25-a^2}=a^2-10$ tenglamaning ikki tomonini yana kvadratga oshiramiz.

$4(25-a^2)=(a^2-10)^2 \Rightarrow 100-4a^2=a^4-20a^2+100 \Rightarrow a^4-16a^2=0$,

bundan $a=0,\ a=\pm 4$.

5) $a\in(0;\infty)$ bo'lgani uchun $a=4$ bo'ladi. $\sqrt[3]{x}=4, x=4^3, x=64$.

6) Tekshirish: $x=64$ da $\sqrt{5+\sqrt[3]{64}}+\sqrt{5-\sqrt[3]{64}}=\sqrt[3]{64}$, $\sqrt{5+4}+\sqrt{5-4}=4$, $\sqrt{9}+\sqrt{1}=4$, $4=4$.

Javob: $x=64$.

21-misol. $(x^2-x-6)\sqrt{\dfrac{x^2-1}{2x}}=0$ tenglamani yeching.

Yechish: 1) Tenglamaning aniqlanish sohasini topamiz:

$$\frac{x^2-1}{2x}\ge 0 \Rightarrow \frac{(x+1)(x-1)}{2x}\ge 0 \ \Rightarrow\ x\in[-1;0)\cup[1;\infty).$$

2) $(x^2-x-6)\sqrt{\dfrac{x^2-1}{2x}}=0$ ko'paytma nolga tengligidan, $x^2-x-6=0$ va $\dfrac{x^2-1}{2x}=0$.

$x^2-x-6=0$ tenglamani yechsak, $x_1=-2$, $x_2=3$ kelib chiqadi.

$\dfrac{x^2-1}{2x}=0$ tenglamani yechsak, $x_3=-1$, $x_4=1$, $x_5\ne 0$ kelib chiqadi.

3) aniqlanish sohasi $x\in[-1;0)[1;\infty)$ bo'lganligi uchun tenglamaning yechimlari $x=\pm 1$, $x=3$ bo'ladi.

Javob: $x=\pm 1$, $x=3$.

22-misol. $x^4+4x^3+6x^2+4x+\sqrt{x^2+2x+10}=2$ tenglamani yeching.

Yechish: 1) Tenglamaning aniqlanish sohasini topamiz. $x\in R$.

2) $x^4+4x^3+6x^2+4x=x^4+4x^3+4x^2+2x^2+4x=(x^2+2x)^2+2(x^2+2x)=$

$=(x^2+2x)^2+2(x^2+2x)+1-1=(x^2+2x+1)^2-1$.

3) $(x^2+2x+1)^2-1+\sqrt{x^2+2x+10}=2$, $(x^2+2x+1)^2+\sqrt{x^2+2x+10}=3$.

4) $\sqrt{x^2+2x+10}=a$, $a\in[0;\infty)$, belgilash kiritsak, $x^2+2x+10=a^2$ bo'ladi. Bundan, $x^2+2x+1=a^2-9$.

5) $(a^2-9)^2+a=3$ tenglamani yechamiz. $((a-3)(a+3))^2+(a-3)=0$,

$(a-3)\left((a-3)(a+3)^2+1\right)=0 \Rightarrow a-3=0$ va $(a-3)(a+3)^2+1=0$.

$a-3=0$ va $(a-3)(a+3)^2+1=0$ tenglamalarni yechsak, $a=3$ bo'ladi.

6) $\sqrt{x^2+2x+10}=3$, $x^2+2x+10=9 \Rightarrow x^2+2x+1=0$, $(x+1)^2=0$, $x=-1$.

7) tekshirish: $x=-1$ da

$(-1)^4+4(-1)^3+6(-1)^2+4(-1)+\sqrt{(-1)^2+2(-1)+10}=1-4+6-4+\sqrt{1-2+10}=-1+3=2$.

Javob: $x=-1$.

MISOLLAR

Tenglamalarni yeching.

1. $\sqrt{5x+2}=10$

2. $\sqrt{4x-6}=12$

3. $\sqrt{7x+1}=6$

4. $\sqrt{10-2x}=4$

5. $\sqrt{49-3x}=2$

6. $\sqrt{-27-x}=11$

7. $\sqrt{-41+3x}=7$

8. $\sqrt{51-13x}=5$

9. $\sqrt{x+12}+x=0$

10. $\sqrt{4+3x}=-x$

11. $\sqrt{-12+7x}=x$

12. $-x=\sqrt{15-2x}$

13. $x-6=\sqrt{8-x}$

14. $x-3=\sqrt{9-x}$

15. $\sqrt{5x-3}=\sqrt{2}x$

16. $\sqrt{2-x}=x$

17. $\sqrt{\dfrac{3x-17}{7}}=4$

18. $\sqrt{\dfrac{11}{6-4x}}=\dfrac{1}{2}$

19. $\sqrt{\dfrac{4}{5x-2}}=1$

20. $\sqrt{\dfrac{2x-8}{4}}=3$

21. $\sqrt{5x-3}=\sqrt{2x}$

22. $\sqrt{4-2x}=2\sqrt{x-1}$

23. $\sqrt{x^2-3x+1}=\sqrt{2x-5}$

24. $3x+2\sqrt{2x^2+3x-5}=12$

25. $3+\sqrt{3x^2-8x+14}=2x$

26. $\sqrt{15x^2-7x+8}=4x$

27. $\sqrt{x^2+x}=2-x$

28. $\left(x^2-25\right)\sqrt{6-2x}=0$

29. $\left(4-x^2\right)\sqrt{-1-3x}=0$

30. $\left(x^2-16\right)(x-3)(x-6)\sqrt{5-x}=0$

31. $\left(x^2-9x+14\right)\sqrt{x^2-9}=0$

32. $(x-4)\sqrt{3+2x-x^2}=0$

33. $(x^2-49)\sqrt{10-3x-x^2}=0$

34. $\sqrt{x-2+\sqrt{2x-5}}+\sqrt{x+2+3\sqrt{2x-5}}=7\sqrt{2}$

35. $\sqrt{x+3+4\sqrt{x-1}}+\sqrt{x+8+6\sqrt{x-1}}=17$

36. $\sqrt{x+2+\sqrt{2x+3}}+\sqrt{x+6+3\sqrt{2x+3}}=11\sqrt{2}$

37. $\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x+5-6\sqrt{x-4}}=2$

38. $\sqrt{x-4\sqrt{x-4}}+\sqrt{x-3-2\sqrt{x-4}}=1$

39. $\sqrt{x^2+10+6\sqrt{1+x^2}}+\sqrt{2+x^2-2\sqrt{x^2+1}}=4$

40. $\sqrt{x^2+77}-2\sqrt[4]{x^2+77}-3=0$

41. $\sqrt{x}+\sqrt[4]{x}=12$

42. $x-5\sqrt{x}+4=0$

43. $\sqrt{x^2+32}=2\sqrt[4]{x^2+32}+3$

44. $2\sqrt{x+8}=9\sqrt[4]{x+8}+18$

45. $2\cdot\sqrt{\frac{3x+2}{4+x}}+3\cdot\sqrt{\frac{4+x}{3x+2}}=5$

46. $4\cdot\sqrt{\frac{5x+4}{3x-2}}-5\cdot\sqrt{\frac{3x-2}{5x+4}}=8$

47. $\sqrt{\frac{3-x}{2+x}}+3\cdot\sqrt{\frac{2+x}{3-x}}=4$

48. $\sqrt{\frac{x-2}{2x+1}}+16\cdot\sqrt{\frac{2x+1}{x-2}}=8$

49. $4\cdot\sqrt{3-\frac{1}{x}}-\sqrt{\frac{x}{3x-1}}=3$

50. $\sqrt[3]{\frac{x+3}{5x+2}}+\sqrt[3]{\frac{5x+2}{x+3}}=\frac{13}{6}$

51. $\sqrt{1-7x}-\sqrt{x+6}=\sqrt{15-2x}$

52. $\sqrt{5x+4}-\sqrt{x+3}=1$

53. $\sqrt{x-2}+\sqrt{1-x}=2$

54. $\sqrt{x-13}+\sqrt{10-x}=4$

55. $\sqrt{x}+\sqrt{x-2}=1-x$

56. $\sqrt{3x-2}+\sqrt{x-1}=3$

57. $2\sqrt{x-2}+2=\sqrt{3x+1}$

58. $x^2+5x+\sqrt{x^2+5x-5}=17$

59. $2x^2+5\sqrt{2x^2+3x+9}+3x+3=0$

60. $x^2+5x+4-5\sqrt{x^2+5x+28}=0$

61. $x^2+\sqrt{x^2+2x+8}=12-2x$

62. $x+42-11\sqrt{x^2-x-42}-x^2=0$

63. $\sqrt{x^2+11}+x^2+9=40$

64. $\frac{8}{\sqrt{10-x}}-\sqrt{10-x}=2$

65. $\frac{3}{1+\sqrt{3-x}}+2\sqrt{3-x}=5$

66. $1+\frac{15}{\sqrt[4]{2x+1}}-2\sqrt[4]{2x+1}=0$

67. $\frac{3}{1+\sqrt{x+1}}+2\sqrt{x+1}=5$

68. $\frac{1}{\sqrt{x}-2}-\frac{1}{\sqrt{x}}=\frac{2}{3}$

69. $\sqrt[3]{7-x}=\sqrt{3-x}$

70. $\sqrt{x-2}+\sqrt{x-1}=\sqrt{3x-5}$

71. $\sqrt[3]{24+x}+\sqrt{12-x}=6$

72. $\sqrt{5x-5}+\sqrt{10x-5}=\sqrt{15x-10}$

73. $\sqrt{2x+5}+\sqrt{5x+6}=\sqrt{12x+25}$

74. $\sqrt{x+2}+\sqrt{x+5}=\sqrt{2x+11}$

75. $\sqrt{5x-1}-\sqrt{3x-2}=\sqrt{x-1}$

76. $\sqrt[3]{2-x}=1-\sqrt{x-1}$

77. $\sqrt[3]{1+\sqrt{x}}+\sqrt[3]{1-\sqrt{x}}=2$

78. $\sqrt[3]{12-x}+\sqrt[3]{14+x}=2$

79. $\sqrt[3]{x+34}-\sqrt[3]{x-3}=1$

80. $\sqrt{\sqrt[3]{2x-7}+1}=5-\sqrt[3]{2x-7}$

81. $\sqrt{1-x}-\sqrt{5+2\sqrt{1-x}}+1=0$

82. $2\sqrt{2x+3}+\sqrt[4]{2x^2+17x+21}=3\sqrt{x+7}$

83. $\sqrt{x+3}+4\sqrt[4]{2x^2+x-15}=5\sqrt{2x-5}$

84. $\sqrt[3]{38-4x}-\sqrt[6]{4x^2-46x+76}=2\sqrt[3]{2-x}$

85. $\sqrt[3]{x+2}+2\sqrt[6]{2x^2+3x-2}=3\sqrt[3]{2x-1}$

86. $x^2+10=x\left(2\sqrt{3x+10}-3\right)$

87. $2x\ (x+2)=3(x\sqrt{4x-3}+1)$

88. $x^2+36+3x\left(3+2\sqrt{x+4}\right)=0$

89. $x^2+48=x\left(18-\sqrt{3x-8}\right)$

90. $\sqrt[3]{x+1}+\sqrt[3]{x+2}+\sqrt[3]{x+3}=0$

91. $\sqrt[3]{x}+\sqrt[3]{2x-3}=\sqrt[3]{12(x-1)}$

92. $\sqrt[3]{x}+\sqrt[3]{5x+3}=\sqrt[3]{24x+3}$

93. $\sqrt[3]{8x}+\sqrt[3]{6x-5}=\sqrt[3]{32x-5}$

94. $\sqrt{4x-7}-\sqrt{2x+3}=\sqrt{9x-20}-\sqrt{7x-10}$

95. $3\left(\sqrt{x+5}-\sqrt{x+1}\right)=2(\sqrt{x+17}-\sqrt{x+2})$

96. $\sqrt{8x+1}+\sqrt{3x-5}=\sqrt{7x+4}-\sqrt{2x-2}$

97. $2\left(\sqrt{x+15}-\sqrt{x}\right)=3(\sqrt{x+3}-\sqrt{x-1})$

98. $2\sqrt{x-3}+\sqrt{3x+4}-\sqrt{3x^2-5x-12}=3x-22$

99. $\sqrt{x^2+3x+3}-\sqrt{x^2+x-2}=2x+5$

100. $\sqrt{x^2+2x-3}-\sqrt{x^2+4x-6}=3-2x$

IRRATSIONAL TENGLAMALAR SISTEMASI

Irratsional tenglamalar sistemasini yechish teng kuchli sistemalarga yoki natijalarga o‘tish qoidalariga asoslanadi. Irratsional tenglamalar sistemasini yechishda turli usullar qo‘llaniladi: ko‘paytuvchilarga ajratish, o‘zgaruvchilarni yo‘qotish, algebraik qo‘shish, o‘zgaruvchilarni almashtirish va shu kabilar.

1-misol. $\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{xy}=7\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\ge 0$, $y\ge 0$.

2) $\sqrt{x}=a, \sqrt{y}=b$ belgilash kiritamiz.

$$\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{xy}=7\end{cases}\Leftrightarrow\begin{cases}a+b=8\\ ab=7\end{cases}\Rightarrow\begin{cases}a=8-b\\ (8-b)b=7\end{cases}\Rightarrow\begin{cases}a=8-b\\ b^2-8b+7=0\end{cases}$$

3) $b^2-8b+7=0$ tenglamani yechamiz, $b_{1,2}=\dfrac{8\pm\sqrt{64-28}}{2}=\dfrac{8\pm 6}{2}$, $\Rightarrow b_1=7,\ b_2=1$.

$a_1=8-b_1=8-7=1,\ \Rightarrow\ a_1=1.$

$a_2=8-b_2=8-1=7,\ \Rightarrow\ a_2=7.$

4) $a_1=1,\ b_1=7$ da $\sqrt{x}=1,\ \sqrt{y}=7.\Rightarrow x=1,\ y=49.$

$a_2=7,\ b_2=1$ da $\sqrt{x}=7,\ \sqrt{y}=1.\Rightarrow x=49,\ y=1.$

5) (1;49) va (49;1) sistemaning yechimi.

Tekshirish: $x=1,\ y=49$ da

$$\begin{cases}\sqrt{1}+\sqrt{49}=8\\ \sqrt{49}=7\end{cases}\Rightarrow\begin{cases}1+7=8\\ 7=7\end{cases}$$

$x=49,\ y=1$ da

$$\begin{cases}\sqrt{49}+\sqrt{1}=8\\ \sqrt{49}=7\end{cases}\Rightarrow\begin{cases}7+1=8\\ 7=7\end{cases}$$

Javob: (1;49), (49;1).

2-misol. $\begin{cases}x-y=21\\ \sqrt{x}-\sqrt{y}=3\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\ge 0$, $y\ge 0$.

2) $x-y=(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$ qisqa ko‘paytirish formuladan foydalanib yechamiz.

$$\begin{cases}x-y=21\\ \sqrt{x}-\sqrt{y}=3\end{cases}\Rightarrow\begin{cases}(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})=21\\ \sqrt{x}-\sqrt{y}=3\end{cases}\Rightarrow\begin{cases}3(\sqrt{x}+\sqrt{y})=21\\ \sqrt{x}-\sqrt{y}=3\end{cases}\Rightarrow$$

3) $+\begin{cases}\sqrt{x}+\sqrt{y}=7\\ \sqrt{x}-\sqrt{y}=3\end{cases}\Rightarrow\begin{cases}\sqrt{x}=5\\ \sqrt{y}=2\end{cases}\Rightarrow\begin{cases}x=25\\ y=4.\end{cases}$

4) (25;4) sistemaning yechimi. Tekshirish: $x=25,\ y=4$ da

$\begin{cases}25-4=21\\ \sqrt{25}-\sqrt{4}=3\end{cases} \Rightarrow \begin{cases}21=21\\ 5-2=3\end{cases}$

Javob: $(25;4)$.

3-misol. $\begin{cases}x+y+\sqrt{x+y}=20\\ x^2+y^2=136\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) Avval sistemadagi birinchi tenglamani yechib olamiz.

$x+y+\sqrt{x+y}=20$, $\sqrt{x+y}=a$ belgilash kiritsak, $a^2+a-20=0$ kvadrat tenglama hosil bo'ladi. $\sqrt{x+y}\geq 0$ bo'lganligi uchun $a\geq 0$ bo'ladi. $a\in[0;\infty)$.

2) $a^2+a-20=0$ tenglamani yechamiz, $a_{1,2}=\dfrac{-1\pm\sqrt{1+80}}{2}=\dfrac{-1\pm 9}{2}$, $a_1=4$, $a_2=-5$.

3) $4\in[0;\infty)$, $-5\notin[0;\infty)$. Demak, $\sqrt{x+y}=4$. $x+y=16$.

4) $\begin{cases}x+y+\sqrt{x+y}=20\\ x^2+y^2=136\end{cases} \Rightarrow \begin{cases}x+y=16\\ x^2+y^2=136\end{cases} \Rightarrow \begin{cases}x=16-y\\ (16-y)^2+y^2=136\end{cases}$

$(16-y)^2+y^2=136 \Rightarrow y^2-16y+60=0$ tenglamani yechamiz.

$y_{1,2}=\dfrac{16\pm\sqrt{256-240}}{2}=\dfrac{16\pm 4}{2}$, $\Rightarrow y_1=10$, $y_2=6$.

5) $y_1=10$, $y_2=6$ bo'lsa, $x+y=16$ dan $x_1=6$, $x_2=10$ kelib chiqadi. (10;6) va (6;10) sistemaning yechimi.

6) Tekshirish: $\begin{cases}10+6+\sqrt{10+6}=16+4=20\\ 10^2+6^2=100+36=136\end{cases}$

Javob: (10;6), (6;10).

4-misol. $\begin{cases}\sqrt{x}+\sqrt{y}=10\\ \sqrt[4]{x}+\sqrt[4]{y}=4\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\geq 0$, $y\geq 0$.

2) $\sqrt[4]{x}=a$, $\sqrt[4]{y}=b$ belgilash kiritsak, $\sqrt{x}=a^2$ va $\sqrt{y}=b^2$ bo'ladi. $\sqrt[4]{x}\geq 0$, $\sqrt[4]{y}\geq 0$ ekanligidan $a\geq 0$, $b\geq 0$ kelib chiqadi.

$\begin{cases}\sqrt{x}+\sqrt{y}=10\\ \sqrt[4]{x}+\sqrt[4]{y}=4\end{cases} \Leftrightarrow \begin{cases}a^2+b^2=10\\ a+b=4\end{cases} \Rightarrow \begin{cases}a=4-b\\ (4-b)^2+b^2=10\end{cases}$

$\Rightarrow \begin{cases}a=4-b\\ 2b^2-8b+6=0\end{cases} \Rightarrow \begin{cases}a=4-b\\ b^2-4b+3=0\end{cases}$

3) $b^2-4b+3=0$ tenglamani yechamiz, $b_{1,2}=\dfrac{4\pm\sqrt{16-12}}{2}=\dfrac{4\pm 2}{2}$, $b_1=3$, $b_2=1$.

$a_1=4-b_1=4-3=1$, $\Rightarrow a_1=1$.

$a_2=4-b_2=4-1=3$, $\Rightarrow a_2=3$.

4) $a_1=1,\ b_1=3$ da $\sqrt[4]{x}=1,\ \sqrt[4]{y}=3. \Rightarrow x=1,\ y=81.$

$a_2=3,\ b_2=1$ da $\sqrt[4]{x}=3,\ \sqrt[4]{y}=1. \Rightarrow x=81,\ y=1.$

5) (1;81) va (81;1) sistemaning yechimi. Tekshirish: $x=1,\ y=81$ da

$$\begin{cases}\sqrt{1}+\sqrt{81}=1+9=10\\ \sqrt[4]{1}+\sqrt[4]{81}=1+3=4\end{cases}$$

Javob: (1;81), (81:1).

5-misol. $\begin{cases}x+y=28\\ \sqrt[3]{x}+\sqrt[3]{y}=4\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\in R, y\in R.$

2) $\sqrt[3]{x}=a,\ \sqrt[3]{y}=b$ belgilash kiritamiz: $x=a^3,\ y=b^3.$

$$\begin{cases}x+y=28\\ \sqrt[3]{x}+\sqrt[3]{y}=4\end{cases} \Leftrightarrow \begin{cases}a^3+b^3=28\\ a+b=4\end{cases}$$

$$\Rightarrow \begin{cases}(a+b)\left(a^2-ab+b^2\right)=28\\ a+b=4\end{cases} \Rightarrow \begin{cases}4\left(a^2-ab+b^2\right)=28\\ a+b=4\end{cases}$$

$$\Rightarrow \begin{cases}a^2-ab+b^2=7\\ a+b=4\end{cases} \Rightarrow \begin{cases}(a+b)^2-3ab=7\\ a+b=4\end{cases}$$

$$\Rightarrow \begin{cases}4^2-3ab=7\\ a+b=4\end{cases} \Rightarrow \begin{cases}3ab=9\\ a+b=4\end{cases} \Rightarrow \begin{cases}ab=3\\ a+b=4\end{cases}$$

3) $\begin{cases}ab=3\\ a+b=4\end{cases}$ tenglamalar sistemasidan $a_1=1,\ b_1=3$ va $a_2=3,\ b_2=1$ kelib chiqadi.

4) $\sqrt[3]{x}=a,\ \sqrt[3]{y}=b,\ a_1=1,\ b_1=3$ da $\sqrt[3]{x}=1,\ \sqrt[3]{y}=3 \Rightarrow x=1,\ y=27.$

$a_2=3,\ b_2=1$ da $\sqrt[3]{x}=3,\ \sqrt[3]{y}=1 \Rightarrow x=27, y=1.$

5) Tekshirish: $x=1, y=27$ yoki $x=27, y=1$ da

$$\begin{cases}1+27=28\\ \sqrt[3]{1}+\sqrt[3]{27}=1+3=4\end{cases}$$

Javob: (1;27), (27;1).

6-misol. $\begin{cases}3x-\sqrt{y+2x}=1\\ y+3x=5\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) $\begin{cases}3x-\sqrt{y+2x}=1\\ y+3x=5\end{cases} \Rightarrow \begin{cases}3x-\sqrt{y+2x}=1\\ y=5-3x\end{cases}$

$$\Rightarrow \begin{cases}3x-\sqrt{5-3x+2x}=1\\ y=5-3x\end{cases} \Rightarrow \begin{cases}\sqrt{5-x}=3x-1\\ y=5-3x\end{cases}$$

2) $\sqrt{5-x}=3x-1$ tenglamani yechamiz. $\sqrt{5-x}\ge 0$ bo'lganligi uchun

$3x-1\ge 0, \Rightarrow 3x\ge 1, \Rightarrow x\ge \frac{1}{3}, \Rightarrow x\in\left[\frac{1}{3};\infty\right).$

$\sqrt{5-x}=3x-1$ tenglikning ikki tomonini kvadratga ko‘tarsak,

$5-x=(3x-1)^2$, $\Rightarrow\ 5-x=9x^2-6x+1$, $\Rightarrow\ 9x^2-5x-4=0$ kvadrat tenglama hosil bo‘ladi. Tenglamaning ildizlarini topamiz:

$$x_{1,2}=\frac{5\pm\sqrt{25+144}}{18}=\frac{5\pm13}{18},\Rightarrow x_1=1,\ x_2=-\frac{4}{9}.$$

3) $x\in\left[\frac{1}{3};\infty\right)$ bo‘lganligi uchun $1\in\left[\frac{1}{3};\infty\right)$, $-\frac{4}{9}\notin\left[\frac{1}{3};\infty\right)$.

$x=1$ da $y=5-3x=5-3=2$, $y=2$. (1;2) sistemaning yechimi.

4) Tekshirish: (1;2) da $\begin{cases}3\cdot1-\sqrt{2+2\cdot1}=3-\sqrt{4}=3-2=1\\2+3\cdot1=5\end{cases}$

Javob: (1;2).

7-misol. $\begin{cases}\sqrt{x^2}+y=5\\y^2-x=7\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\in R$.

2) $\sqrt{x^2}=|x|$. Modul ta‘rifidan foydalanib yechamiz:

$$|x|=\begin{cases}x,\text{agar } x\ge0 \text{ bo'lsa},\\-x,\text{ agar } x<0 \text{ bo'lsa}.\end{cases}$$

3) $x\ge0$ bo‘lganda, $\begin{cases}\sqrt{x^2}+y=5,\\y^2-x=7,\end{cases}\Leftrightarrow\begin{cases}x+y=5,\\y^2-x=7.\end{cases}$

$+\begin{cases}x+y=5,\\y^2-x=7,\end{cases}\Rightarrow y^2+y-12=0\Rightarrow\ y_1=-4,\ y_2=3.$

4) $y_1=-4$, $y_2=3$. $x+y=5\Rightarrow\ x_1=9$, $x_2=2$.

$x\ge0$ shartga asosan $x_1=9$, $x_2=2$. Demak, (9; −4), $(2;3)$.

5) $x<0$ bo‘lganda, $\begin{cases}\sqrt{x^2}+y=5\\y^2-x=7\end{cases}\Leftrightarrow\begin{cases}-x+y=5\\y^2-x=7\end{cases}$.

$-\begin{cases}-x+y=5\\y^2-x=7\end{cases}\Rightarrow y^2-y-2=0\Rightarrow\ y_3=-1,\ y_4=2.$

6) $y_3=-1$, $y_4=2$. $-x+y=5\Rightarrow\ x_3=-6$, $x_4=-3$.

$x<0$ shartga asosan $x_3=-6$, $x_4=-3$. Demak, $(-6;-1),(-3;2)$.

7) Tekshirish: (9; −4) da $\begin{cases}\sqrt{9^2}+(-4)=9-4=5\\(-4)^2-9=16-9=7,\end{cases}$

$(2;3)$ da $\begin{cases}\sqrt{2^2}+3=2+3=5,\\3^2-2=9-2=7.\end{cases}$

$(-6;-1)$ da $\begin{cases}\sqrt{(-6)^2}+(-1)=6-1=5\\(-1)^2-(-6)=1+6=7\end{cases}$

$(-3;2)$ da $\begin{cases}\sqrt{(-3)^2}+2=3+2=5\\ 2^2-(-3)=4+3=7\end{cases}$

Javob: $(-6;-1),(-3;2),(9;-4),(2;3)$.

8-misol. $\begin{cases}\sqrt{2x+3y}+\sqrt{2x-3y}=10,\\ \sqrt{4x^2-9y^2}=16\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) Qisqa ko'paytirish formulasiga asosan $a^2-b^2=(a-b)(a+b)\Rightarrow$ $4x^2-9y^2=(2x-3y)(2x+3y)$.

2) $\sqrt{2x+3y}=a$ va $\sqrt{2x-3y}=b$ belgilash kiritsak,

$\sqrt{4x^2-9y^2}=\sqrt{2x+3y}\sqrt{2x-3y}=ab$ bo'ladi. U holda $a\ge 0$, $b\ge 0$ va

$$\begin{cases}\sqrt{2x+3y}+\sqrt{2x-3y}=10\\ \sqrt{4x^2-9y^2}=16\end{cases}\Leftrightarrow\begin{cases}a+b=10\\ ab=16\end{cases}$$

$$\Rightarrow\begin{cases}a=10-b\\ (10-b)b=16\end{cases}\Rightarrow\begin{cases}a=10-b\\ b^2-10b+16=0.\end{cases}$$

3) $b^2-10b+16=0$ tenglamani yechamiz, $b_{1,2}=\dfrac{10\pm\sqrt{100-64}}{2}=\dfrac{10\pm 6}{2}$,

$b_1=8$, $b_2=2$.

$a_1=10-b_1=10-8=2,\quad a_1=2.$

$a_2=10-b_2=10-2=8,\quad a_2=8.$

4) $a_1=2$, $b_1=8$ da $\begin{cases}\sqrt{2x+3y}=2\\ \sqrt{2x-3y}=8\end{cases}\Rightarrow\begin{cases}2x+3y=4\\ 2x-3y=64\end{cases}$

$$+\begin{cases}2x+3y=4\\ 2x-3y=64\end{cases}\Rightarrow x=17,\ y=-10.$$

$a_1=8$, $b_1=2$ da $\begin{cases}\sqrt{2x+3y}=8\\ \sqrt{2x-3y}=2\end{cases}\Rightarrow\begin{cases}2x+3y=64\\ 2x-3y=4\end{cases}$

$$+\begin{cases}2x+3y=64\\ 2x-3y=4\end{cases}\Rightarrow x=17,\ y=10.$$

5) Tekshirish: (17; –10) da

$$\begin{cases}\sqrt{2\cdot 17+3\cdot(-10)}+\sqrt{2\cdot 17-3\cdot(-10)}=\sqrt{4}+\sqrt{64}=2+8=10\\ \sqrt{4\cdot 17^2-9\cdot(-10)^2}=\sqrt{1156-900}=\sqrt{256}=16\end{cases}$$

(17; 10) da

$$\begin{cases}\sqrt{2\cdot 17+3\cdot 10}+\sqrt{2\cdot 17-3\cdot 10}=\sqrt{64}+\sqrt{4}=8+2=10\\ \sqrt{4\cdot 17^2-9\cdot 10^2}=\sqrt{1156-900}=\sqrt{256}=16\end{cases}$$

Javob: (17; –10), (17; 10)

9-misol. $\begin{cases}\sqrt{x-2y+2}=2,\\ \sqrt{y-2x+11}=x-5\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) $\sqrt{y-2x+11}\ge 0$ bo'lganligi uchun $x-5\ge 0$, $x\ge 5$. $x\in[5;\infty)$.

2) $\begin{cases}\sqrt{x-2y+2}=2,\\ \sqrt{y-2x+11}=x-5\end{cases}\Rightarrow\begin{cases}\sqrt{(x-2y+2)^2}=4,\\ \sqrt{(y-2x+11)^2}=(x-5)^2\end{cases}$

$\Rightarrow\begin{cases}x-2y+2=4,\\ y-2x+11=x^2-10x+25\end{cases}\Rightarrow\begin{cases}x-2y=2,\\ y=x^2-8x+14\end{cases}$

$\Rightarrow\begin{cases}x-2(x^2-8x+14)=2,\\ y=x^2-8x+14\end{cases}\Rightarrow\begin{cases}2x^2-17x+30=0,\\ y=x^2-8x+14\end{cases}$

3) $2x^2-17x+30=0$ tenglamani yechamiz, $x_{1,2}=\dfrac{17\pm\sqrt{289-240}}{4}=\dfrac{17\pm 7}{4}$,
$x_1=6$, $x_2=\dfrac{5}{2}$.

4) $x\in[5;\infty)$ shartga asosan $6\in[5;\infty)$, $\dfrac{5}{2}\notin[5;\infty)$.

5) $x_1=6$ da $y_1=6^2-8\cdot 6+14=36-48+14=2$. $\Rightarrow y_1=2$

6) Tekshirish: (6;2) da $\begin{cases}\sqrt{6-2\cdot 2+2}=\sqrt{4}=2,\\ \sqrt{2-2\cdot 6+11}=6-5=1\end{cases}$

Javob: (6;2).

10-misol. $\begin{cases}\sqrt{x+y}+\sqrt{2x+y+2}=7,\\ 3x+2y=23\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) $\sqrt{x+y}=a$ va $\sqrt{2x+y+2}=b$ belgilash kiritsak, $a\ge 0$, $b\ge 0$ bo'ladi.
$x+y=a^2$, $2x+y+2=b^2$

2) $+\begin{cases}x+y=a^2\\ 2x+y+2=b^2\end{cases}\Rightarrow 3x+2y+2=a^2+b^2$.

3) $3x+2y=23$ ekanligini hisobga olsak, $3x+2y+2=a^2+b^2\Rightarrow 25=a^2+b^2$.

4) $\begin{cases}\sqrt{x+y}+\sqrt{2x+y+2}=7\\ 3x+2y=23\end{cases}\Leftrightarrow\begin{cases}a+b=7\\ a^2+b^2=25\end{cases}\Rightarrow$

$a_1=3, b_1=4$.
$a_2=4, b_2=3$.

5) $a_1=3, b_1=4$ da $\begin{cases}\sqrt{x+y}=3,\\ \sqrt{2x+y+2}=4\end{cases}\Rightarrow\begin{cases}x+y=9,\\ 2x+y+2=16\end{cases}$

$\Rightarrow\begin{cases}x+y=9\\ 2x+y=14\end{cases}\Rightarrow x=5, y=4$.

$a_2=4, b_2=3$ da $\begin{cases}\sqrt{x+y}=4,\\ \sqrt{2x+y+2}=3\end{cases} \Rightarrow \begin{cases}x+y=16,\\ 2x+y+2=9\end{cases}$

$\Rightarrow \begin{cases}x+y=16\\ 2x+y=7\end{cases} \Rightarrow x=-9,\ y=25.$

6) Tekshirish: $(5;4)$ da $\begin{cases}\sqrt{5+4}+\sqrt{2\cdot5+4+2}=\sqrt{9}+\sqrt{16}=3+4=7,\\ 3\cdot5+2\cdot4=15+8=23\end{cases}$

$(-9;\ 25)$ da $\begin{cases}\sqrt{(-9)+25}+\sqrt{2\cdot(-9)+25+2}=\sqrt{16}+\sqrt{9}=4+3=7,\\ 3\cdot(-9)+2\cdot25=-27+50=23\end{cases}$

Javob: $(5;4), (-9;\ 25)$.

11-misol. $\begin{cases}\sqrt[3]{x-1}+\sqrt[3]{y+1}=3\\ \sqrt[3]{x^2-2x+1}-\sqrt[3]{x-1}\sqrt[3]{y+1}+\sqrt[3]{y^2+2y+1}=3\end{cases}$ tenglamalar sistemasini yeching.

Yechish:

1) $\sqrt[3]{x-1}=a$, $\sqrt[3]{y+1}=b$ belgilash kiritsak,

$\sqrt[3]{x^2-2x+1}=\sqrt[3]{(x-1)^2}=a^2$ va $\sqrt[3]{(y+1)^2}=b^2$ bo‘ladi.

2) $\begin{cases}\sqrt[3]{x-1}+\sqrt[3]{y+1}=3\\ \sqrt[3]{x^2-2x+1}-\sqrt[3]{x-1}\sqrt[3]{y+1}+\sqrt[3]{y^2+2y+1}=3\end{cases} \Leftrightarrow \begin{cases}a+b=3\\ a^2-ab+b^2=3\end{cases} \Rightarrow \begin{cases}a+b=3\\ (a+b)^2-3ab=3\end{cases}$

$\Rightarrow \begin{cases}a+b=3\\ 3^2-3ab=3\end{cases} \Rightarrow \begin{cases}a+b=3\\ 3ab=6\end{cases} \Rightarrow \begin{cases}a+b=3\\ ab=2\end{cases}$

3) $\begin{cases}a+b=3\\ ab=2\end{cases}$ tenglamalar sistemasining yechimlari $a_1=1, b_1=2$ va $a_2=2, b_2=1$ bo‘ladi.

4) $a_1=1, b_1=2$ da $\sqrt[3]{x-1}=1$ va $\sqrt[3]{y+1}=2$.

$x-1=1,\ x=2.$

$y+1=8,\ y=7.$ Demak, $(2;7)$.

$a_2=2, b_2=1$ da $\sqrt[3]{x-1}=2$ va $\sqrt[3]{y+1}=1$.

$x-1=8,\ x=9.$

$y+1=1,\ y=0.$ Demak, $(9;0)$.

5) Tekshirish: $(2;7)$ da

$\begin{cases}\sqrt[3]{2-1}+\sqrt[3]{7+1}=\sqrt[3]{1}+\sqrt[3]{8}=1+2=3\\ \sqrt[3]{2^2-2\cdot2+1}-\sqrt[3]{2-1}\sqrt[3]{7+1}+\sqrt[3]{7^2+2\cdot7+1}=\sqrt[3]{1}-\sqrt[3]{8}+\sqrt[3]{64}=1-2+4=3.\end{cases}$ $(9;0)$ da

$\begin{cases}\sqrt[3]{9-1}+\sqrt[3]{0+1}=\sqrt[3]{8}+\sqrt[3]{1}=2+1=3\\ \sqrt[3]{9^2-2\cdot9+1}-\sqrt[3]{9-1}\sqrt[3]{0+1}+\sqrt[3]{0+2\cdot0+1}=\sqrt[3]{64}-\sqrt[3]{8}+\sqrt[3]{1}=1-2+4=3.\end{cases}$

Javob: $(2;7), (9;0)$.

12-misol. $\begin{cases}\sqrt{x+y}+\sqrt{7x+y}=y\\ 7x+\sqrt{x+y}=2\end{cases}$ tenglamalar sistemasini yeching.

Yechish: 1) Birinchi tenglamada $\sqrt{x+y}\ge 0$, $\sqrt{7x+y}\ge 0$ bo'lganligi uchun $y\ge 0$.

2) Ikkinchi tenglamadan birinchi tenglamani ayirsak,

$$-\begin{cases}7x+\sqrt{x+y}=2\\ \sqrt{x+y}+\sqrt{7x+y}=y\end{cases} \Rightarrow 7x-\sqrt{7x+y}=2-y,$$

$7x+y-\sqrt{7x+y}-2=0$ irratsional tenglama hosil bo'ladi.

3) $7x+y-\sqrt{7x+y}-2=0$ tenglamani $\sqrt{7x+y}=a$ belgilash kiritib yechamiz.

$\sqrt{7x+y}\ge 0 \Rightarrow a\ge 0$. $a\in[0;\infty)$

$a^2-a-2=0$ kvadrat tenglamaning ildizlarini topamiz. $a_{1,2}=\dfrac{1\pm\sqrt{1+8}}{2}=\dfrac{1\pm 3}{2}$,

$a_1=2$, $a_2=-1$.

4) $a\in[0;\infty)$ shartga asosan $2\in[0;\infty)$, $-1\notin[0;\infty)$. $a=2$ da $\sqrt{7x+y}=2$ yoki $7x+y=4$.

$$\begin{cases}\sqrt{x+y}+\sqrt{7x+y}=y\\ 7x+\sqrt{x+y}=2\end{cases} \Rightarrow \begin{cases}7x+y=4\\ 7x+\sqrt{x+y}=2\end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases}y=4-7x\\ 7x+\sqrt{x+4-7x}=2\end{cases} \Rightarrow \begin{cases}y=4-7x\\ \sqrt{4-6x}=2-7x\end{cases}$$

5) $\sqrt{4-6x}=2-7x$ tenglamani yechamiz. $\sqrt{4-6x}\ge 0$, $2-7x\ge 0$.

$$4-6=(2-7x)^2 \Rightarrow 49x^2-28x+4=4-6x \Rightarrow 49x-22x=0 \Rightarrow x_1=0,\ x_2=\frac{22}{49}.$$

$$0\in\left(-\infty;\frac{2}{7}\right],\quad \frac{22}{49}\notin\left(-\infty;\frac{2}{7}\right].$$

6) $x=0$ da $y=4-7x=4$. Demak, $y=4$.

7) Tekshirish: (0;4) da $\begin{cases}\sqrt{0+4}+\sqrt{7\cdot 0+4}=4\\ 7\cdot 0+\sqrt{0+4}=2\end{cases} \Rightarrow \begin{cases}\sqrt{4}+\sqrt{4}=4\\ \sqrt{4}=2\end{cases}$

Javob: (0;4).

MISOLLAR

Tenglamalar sistemasini yeching.

1. $a)\begin{cases}\sqrt{x}-\sqrt{y}=4\\ 2\sqrt{x}+3\sqrt{y}=18\end{cases}$ $b)\begin{cases}3\sqrt{x}-\sqrt{y}=8\\ \sqrt{x}+2\sqrt{y}=19\end{cases}$

2. $a)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\\ \sqrt[3]{x}+\sqrt[3]{y}=5\end{cases}$ $b)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=1\\ \sqrt[3]{x}+\sqrt[3]{y}=3\end{cases}$

3. $a)\begin{cases}2\sqrt[3]{x}+3\sqrt[3]{y}=-1\\ 2\sqrt[3]{x}-3\sqrt[3]{y}=-7\end{cases}$ $b)\begin{cases}3\sqrt[3]{x}+2\sqrt[3]{y}=3\\ 3\sqrt[3]{x}-2\sqrt[3]{y}=-9\end{cases}$

4. $a)\begin{cases}\sqrt{x}+\sqrt{y}=26\\ \sqrt[4]{x}+\sqrt[4]{y}=6\end{cases}$ $b)\begin{cases}\sqrt{x}-\sqrt{y}=5\\ \sqrt[4]{x}-\sqrt[4]{y}=1\end{cases}$

5. $a)\begin{cases}\sqrt{x}+\sqrt{y}=9\\ \sqrt[6]{x}+\sqrt[6]{y}=3\end{cases}$ $b)\begin{cases}\sqrt{x}-\sqrt{y}=7\\ \sqrt[6]{x}-\sqrt[6]{y}=1\end{cases}$

6. $a)\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{x}\sqrt{y}=15\end{cases}$ $b)\begin{cases}\sqrt{x}+\sqrt{y}=7\\ \sqrt{x}\sqrt{y}=12\end{cases}$

7. $a)\begin{cases}\sqrt{x}+3\sqrt{y}=10\\ \sqrt{x}\sqrt{y}=8\end{cases}$ $b)\begin{cases}2\sqrt{x}-\sqrt{y}=5\\ \sqrt{x}\sqrt{y}=3\end{cases}$

8. $a)\begin{cases}\sqrt{x}-\sqrt{y}=4\\ x-y=32\end{cases}$ $b)\begin{cases}\sqrt{x}+\sqrt{y}=8\\ x-y=16\end{cases}$

$c)\begin{cases}\sqrt{x}+\sqrt{y}=6\\ x-y=12\end{cases}$

9. $a)\begin{cases}\sqrt{6+x}-3\sqrt{3y+4}=-10\\ 4\sqrt{3y+4}-\sqrt{6+x}=14\end{cases}$ $b)\begin{cases}2\sqrt{x-2}+\sqrt{5y+1}=8\\ 3\sqrt{x-2}-2\sqrt{5y+1}=-2\end{cases}$

10. $a)\begin{cases}\sqrt[4]{x+y}-\sqrt[4]{x-y}=2\\ \sqrt{x+y}-\sqrt{x-y}=8\end{cases}$ $b)\begin{cases}\sqrt[4]{x+y}+\sqrt[4]{x-y}=4\\ \sqrt{x+y}+\sqrt{x-y}=10\end{cases}$

11. $a)\begin{cases}\sqrt[3]{x}+\sqrt[3]{y}=5\\ x\cdot y=216\end{cases}$ $b)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\frac{3}{4}\\ x\cdot y=1\end{cases}$

$c)\begin{cases}\sqrt[3]{x}+\sqrt[3]{y}=-3\\ x\cdot y=8\end{cases}$ $d)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=2\\ x\cdot y=27\end{cases}$

12. $a)\begin{cases}y\sqrt{x}+x\sqrt{y}=30\\ \sqrt{x}+\sqrt{y}=5\end{cases}$ $b)\begin{cases}y\sqrt{x}-x\sqrt{y}=-12\\ \sqrt{x}-\sqrt{y}=1\end{cases}$

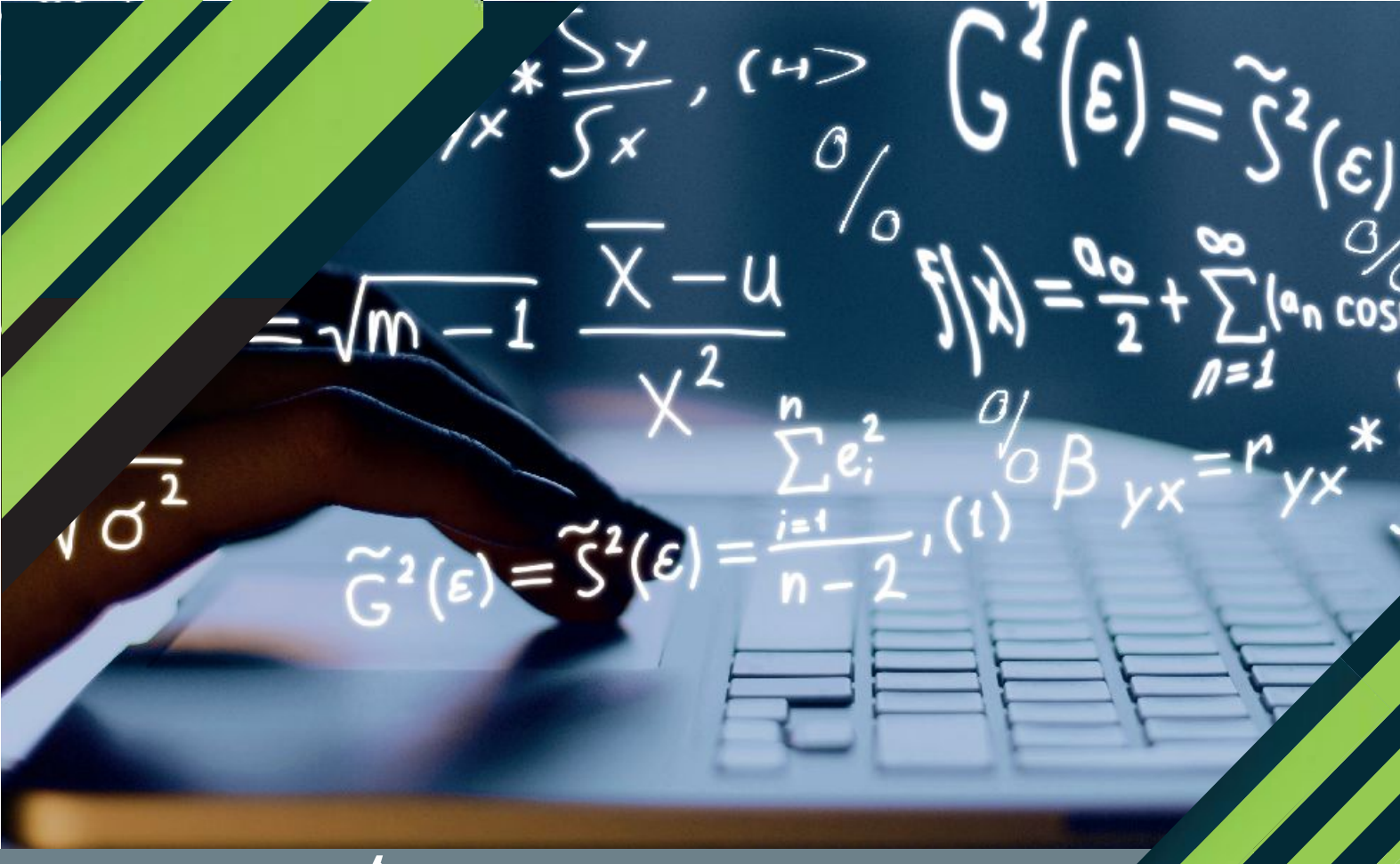
13. $a)\begin{cases}y+x-\sqrt{xy}=7\\ xy=9\end{cases}$ $b)\begin{cases}x-y+\sqrt{xy}=20\\ xy=64\end{cases}$

14. $a)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\\ \sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}=3\end{cases}$ $b)\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=-1\\ \sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}=7\end{cases}$

15. $a)\begin{cases}3\sqrt{\frac{x}{y}}+2\sqrt{\frac{y}{x}}=5\\ 4\sqrt{x}+\sqrt{y}=10\end{cases}$ $b)\begin{cases}4\sqrt{\frac{x}{y}}+2\sqrt{\frac{y}{x}}=9\\ 7\sqrt{x}+2\sqrt{y}=48\end{cases}$

16. $a)\begin{cases}\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3}\\ y^2+x^2=82\end{cases}$ $b)\begin{cases}\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{5}{2}\\ y^2-x^2=15\end{cases}$

17. $a)\begin{cases}4y+5x-\sqrt{xy}=79\\ 5x-4y+\sqrt{xy}=81\end{cases}$ $b)\begin{cases}9y+2x-\sqrt{xy}=71\\ 2x-9y+\sqrt{xy}=73\end{cases}$



3-BOB. KO‘RSATKICHLI VA LOGARIFMIK FUNKSIYALAR

- KO‘RSATKICHLI FUNKSIYA VA UNING XOSSALARI, GRAFIGI
- LOGARIFM HAQIDA TUSHUNCHA. LOGARIFMIK FUNKSIYA VA UNING XOSSALARI
- LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH
- KO‘RSATKICHLI TENGLAMALAR. LOGORIFMIIK TENGLAMALAR
- KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI
- KO‘RSATKICHLI TENGSIZLIKLAR
- LOGARIFMIK TENGSIZLIKLAR
- KO‘RSATKICHLI MODELLASHTIRISH. MURAKKAB FOIZ FORMULASI VA UNING TATBIQLARI. O‘SISH HODISALARI. RADIOAKTIV YEMIRILISH

KO‘RSATKICHLI FUNKSIYA VA UNING XOSSALARI, GRAFIGI

Keyingi vaqtlarda yer sathidan chang-to‘zon ko‘tarilishi tez-tez kuzatilmoqda. Bunda chang miqdori yuqoriga ko‘tarilgan sari kamayib borishi isbotlangan. Chang miqdorining balandlikka bog‘liqligi ko‘rsatkichli funksiya orqali ifodalanar ekan. Undan tashqari, viruslarnig ko‘payishi, radioaktiv moddalarning yemirilishi kabi hodisalar ham ko‘rsatkichli funksiyalar orqali tavsiflanadi.

Ko‘rsatkichli funksiyalarni o‘rganish uchun:

1) $a^0=1$; 2) $a^1=a$; 3) $a^{n+m}=a^n\cdot a^m$; 4) $a^{n-m}=\frac{a^n}{a^m}$;

5) $\left(a^n\right)^m=a^{nm}$; 6) $(ab)^n=a^n\cdot b^n$; 7) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$; 8) $a^{\frac{m}{n}}=\sqrt[n]{a^m}$

xossalarni bilish talab etiladi.

Ma’lumki, kasr ko‘rsatkichli $a^{\frac{m}{n}}$, yoki haqiqiy ko‘rsatkichli a^p ko‘rinishidagi darajalarni ham qarash mumkin. Bunda ko‘rsatkichning ayrim qiymatlarida a^p daraja ma’noga ega bo‘lmay qolishi mumkin. Masalan, $(-3)^{\frac{1}{2}}=\sqrt{-3}$ ifoda haqiqiy sonlar to‘plamida ma’noga ega bo‘lmaydi. Undan tashqari, $0^{-3}=\frac{1}{0^3}=\frac{1}{0}$ ifoda ham aniqlanmagan. Bunday holatlarning oldini olish maqsadida haqiqiy p ko‘rsatkichli a^p daraja uchun $a>0$ tengsizlik bajarilishi talab etiladi. Har qanday p haqiqiy son uchun $1^p=1$ ekanligidan asosi 1 bo‘lgan darajalarni o‘rganish orqali hech qanday yangi ma’lumotga erishilmaydi.

Demak, yuqorida bayon etilganlar asosida quyidagi xulosaga kelish mumkin.

Xulosa. Ixtiyoriy p haqiqiy ko‘rsatkichli a^p daraja aniq qiymat qabul qilishi uchun a asos $a>0$ va $a\neq 1$ shartlarni bajarishi talab etiladi.

$a>0$ va $a\neq 1$ shartlarni qanoatlantiradigan a haqiqiy sonni qaraylik. Ushbu $y=a^x$ ko‘rinishdagi funksiya ***ko‘rsatkichli funksiya*** deyiladi (daraja ko‘rsatkichi – o‘zgaruvchi miqdor).

$y=a^x$ ko‘rsatkichli funksiya quyidagi xossalarga ega:

- $y=a^x$ ko‘rsatkichli funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to‘plamidan iborat:

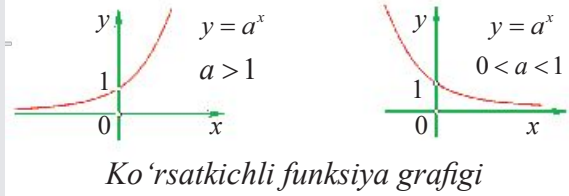
$$D(y)=(-\infty;+\infty);$$

- $y=a^x$ ko‘rsatkichli funksiyaning qiymatlar to‘plami barcha musbat haqiqiy sonlar to‘plamidan iborat:

$$E(y)=(0;+\infty);$$

- $y=a^x$ ko‘rsatkichli funksiya Ox o‘qi bilan kesishmaydi.
- $y=a^x$ ko‘rsatkichli funksiya Oy o‘qi bilan esa $(0,1)$ nuqtada kesishadi.
- Ko‘rsatkichli funksiya davriy bo‘lmaydi, juft ham emas, toq ham emas.
- a asosning $0<a<1$ tengsizliklarni

1-rasm



Ko‘rsatkichli funksiya grafigi

qanoatlantiruvchi qiymatlarida $y=a^x$ funksiya kamayadi: Kamayish oralig‘i $(-\infty,\ +\infty)$ dan iborat.

● a asosning $a>1$ tengsizlikni qanoatlantiruvchi qiymatlarida $y=a^x$ funksiya o‘sadi: O‘sish oralig‘i $(-\infty;\ +\infty)$ dan iborat.

1-misol. Chang miqdori y ning x balandlikka bog‘liqligi $y=p\cdot e^{-qx}$ ko‘rinishdagi funksiya orqali ifodalanar ekan. Bu yerda p, q sonlari parametrlar deb ataluvchi kattaliklar, e esa Eyler soni deb ataluvchi irratsional son. Uning taqribiy qiymati $2,71$ ga teng: ($e\approx 2,71$.)

MISOLLAR

1. Funksiya xossalarini ayting va uning grafigini yasang:

a) $y=3^x$; b) $y=0,4^x$; c) $y=0,8^x$; d) $y=1,5^x$.

2. Funksiyaning qiymatlar sohasini toping:

a) $y=-3^x$; b) $y=\left(\frac{1}{2}\right)^x-1$;

c) $y=-\left(\frac{1}{3}\right)^x$; d) $y=4^x+2$.

3. Sonlarni taqqoslang:

a) $\left(\frac{3}{5}\right)^{-\frac{\sqrt{3}}{2}}$ va 1; b) $5^{-\sqrt{13}}$ va $\left(\frac{1}{5}\right)^{2,1}$; c) $3,2^{-\sqrt{2}}$ va 1; d) $0,7^{\frac{\sqrt{5}}{9}}$ va $0,7^{\frac{1}{6}}$.

4. Hisoblang:

a) $((\sqrt{3})^{\sqrt{3}})^{\sqrt{3}}$; b) $3^{1-2\sqrt{3}}\cdot 9^{1+\sqrt{3}}$; c) $64^{\sqrt{2}}:64^{3\sqrt{2}}$; d) $(5^{\sqrt[5]{16}})^{\sqrt[5]{2}}$.

5. Ifodani soddalashtiring:

a) $b^{\sqrt{2}}\cdot(\frac{1}{b})^{\sqrt{2}-1}$; b) $x^{\pi}\cdot\sqrt[4]{x^2:6x^{4\pi}}$; c) $(c^{\sqrt{3}})^{\sqrt{3}}$; d) $y^{\sqrt{2}}\cdot y^{1,5}:6\sqrt[3]{y^{3\sqrt{2}}}$.

6. Ifodani soddalashtiring:

a) $\frac{a^{2\sqrt{2}}-b^{2\sqrt{3}}}{(a^{\sqrt{2}}-b^{\sqrt{3}})^2}+1$; b) $\frac{\left(a^{2\sqrt{3}}-1\right)\left(a^{2\sqrt{3}}+a^{\sqrt{3}}+a^{3\sqrt{3}}\right)}{a^{4\sqrt{3}}-a^{\sqrt{3}}}$;

c) $\frac{a^{\sqrt{5}}-b^{\sqrt{7}}}{a^{\frac{2\sqrt{5}}{3}}+a^{\frac{\sqrt{5}}{3}}b^{\frac{\sqrt{7}}{3}}+b^{\frac{2\sqrt{7}}{3}}}$; d) $\sqrt{\left(x^{\pi}+y^{\pi}\right)^2-\left(4^{\frac{1}{\pi}}xy\right)^{\pi}}$.

7. Quyidagi funksiyalardan qaysi biri R to‘plamda o‘suvchi, qaysi biri kamayuvchi ekanligini aniqlang:

a) $y=\left(\sqrt{2}\right)^x$, $y=\left(\frac{1}{\sqrt{2}}\right)^x$; b) $y=\left(\sqrt{5}-2\right)^x$, $y=\frac{1}{\left(\sqrt{5}-2\right)^x}$;

c) $y=\left(\frac{\pi}{3}\right)^x$, $y=\left(\frac{3}{\pi}\right)^x$; d) $y=\left(3-\sqrt{7}\right)^x$, $y=\frac{1}{\left(3-\sqrt{7}\right)^x}$.

8. Funksiyaning qiymatlar sohasini toping:

a) $y=3^{x+1}-3$; b) $y=\left|2^x-2\right|$; c) $y=\left(\frac{1}{2}\right)^{x-1}+2$; d) $y=4^{|x|}$.

9. Funksiyaning (R sonlar to‘plamida) eng katta va eng kichik qiymatini toping:

a) $y=\left(\frac{1}{2}\right)^{\sin x}$; b) $y=5+3^{|\cos x|}$; c) $y=4^{\cos x}$; d) $y=\left(\frac{1}{3}\right)^{|\sin x|}-2$.

10. Tenglama ildizining ishorasini aniqlang:

a) $\left(\frac{1}{6}\right)^{x}=10$; b) $0,3^{x}=0,1$; c) $10^{x}=4$; d) $0,7^{x}=5$.

11. Tenglamalarni grafik usulda yeching:

a) $3^{x}=4-x$; b) $\left(\frac{1}{2}\right)^{x}=x+3$; c) $\left(\frac{1}{3}\right)^{x}=x+1$; d) $4^{x}=5-x$.

12. Tenglamalarni yeching:

a) $3^{1-x}=2x-1$; b) $4^{x}+1=6-x$; c) $2^{x}-2=1-x$; d) $3^{-x}=-\frac{3}{x}$.

Ko‘rsatkichli funksiyaning hayotda qo‘llanilishi



Qaynayotgan choynak olovdan olinsa, u avval tez soviydi, so‘ngra esa sovish tezligi pasayadi. Gap shundaki, sovish tezligi choynak temperaturasi va tashqi muhit temperaturasining ayirmasiga proporsional. Bu ayirma qancha kamaysa, choynak shuncha sekin soviydi. Choynakning dastlabki temperaturasi T_0, havo temperaturasi T_1 bo‘lsa, u holda t sekunddan keyin choynak temperaturasi $T=(T_1-T_0)e^{-kt}+T_1$ formula bilan aniqlanadi.

Fizikada qo‘llanilishi.



Havosiz bo‘shliqda (vakuum) jismning erkin tushishida uning tezligi ortib boradi. Havoda ham jismlarning tushish tezligi ortib boradi, ammo ma’lum bir qiymatdan ortib ketmaydi. Agar, havoning qarshilik kuchi parashutchining tushish tezligiga to‘g‘ri proporsional bo‘lsa, ya’ni $F=kv$ bo‘lsa, u holda t sekunddan keyin uning tushish tezligi $v=\frac{mg}{k}\left(1-e^{-\frac{kt}{m}}\right)$ ga teng bo‘ladi, bu yerda m parashutchining tezligi.

Aholining o‘sishi.



Mamlakatda aholi sonining ma’lum vaqt oralig‘ida o‘zgarishi $N=N_0e^{kt}$ formula bilan ko‘rsatiladi, bu yerda N_0 – $t=0$ vaqtdagi aholi soni, N esa t vaqtdagi aholi soni, $k-$ konstanta.

Biologiyada qo‘llanilishi.



Organik olamning ko‘payish qonuni: organizm uchun qulay muhitda (yirtqichlar soni kamligi, oziq miqdori yetarli bo‘lishi) tirik organizmlar ko‘rsatkichli funksiya qonuni bo‘yicha ko‘payardi. Masalan, bitta uy pashshasi yoz davomida $8\cdot10^{14}$ miqdorida yangi avlod hosil qiladi. Ularning og‘irligi bir necha million tonnani tashkil qilardi (ikkita uy passhshasining nasli esa sayyoramiz massasidan ortardi), ular juda katta maydonni egallab olardi. Agarda ularni zanjir qilib joylashtirsa, u holda bu zanjir uzunligi Yerdan Quyoshgacha bo‘lgan masofadan ham katta bo‘lardi. Ammo, tabiatda pashshaning tabiiy “dushmani” hisoblangan ko‘plab jonivor va o‘simliklar mavjudligi, pashshalarning sonini bu darajada ortishiga yo‘l qo‘ymaydi.

LOGARIFM HAQIDA TUSHUNCHA. LOGARIFMIK FUNKSIYA VA UNING XOSSALARI, GRAFIGI

Logarifmlar kundalik hayotning turli xil jabhalarida keng qo‘llaniladi. Masalan, bankka qo‘yilgan mablag‘ biror miqdorga qancha vaqtda ko‘payishini topishda logarifmdan foydalaniladi. Yoki tovush balandligini baholashda logarifmik bog‘lanish ishlatiladi. Undan tashqari, kungaboqar pallasidagi urug‘lar logarifmik spiral deb ataluvchi chiziqqa o‘xshash yoylar bo‘ylab joylashar ekan.

Logarifm tuchunchasini va logarifmik funksiyani o‘rganish uchun:
1) ko‘rsatkichli funksiyani;
2) ko‘rsatkichli funksiyalarning xossalarini **bilish talab etiladi.**

Logarifm haqida tushuncha

Darajaga ko‘tarish amaliga teskari amalni qarab chiqamiz. a va b sonlar berilgan bo‘lsin. Bunda $a>0$, $a\neq 1$, $b>0$ shartlar bajarilishi talab etiladi. Aytaylik, b ni hosil qilish uchun a ni ko‘tarish kerak bo‘lgan ko‘rsatkich x bo‘lsin, ya’ni $a^x=b$ bo‘lsin. $a^x=b$ ifodada x noma’lumni topish ko‘rsatkichni topish amali deyiladi. Masalan, $3^x=27$ bo‘lsa, $x=3$, yoki $10^x=0,01$ bo‘lsa, $x=-2$ bo‘ladi.

b sonning a asosga ko‘ra logarifmi deb, b ni hosil qilish uchun a ni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi. b ning a asosga ko‘ra logarifmi $\log_a b$ orqali belgilanadi. Bu tenglik ***asosiy logarifmik ayniyat*** deyiladi.

$\log_a b$	a – logarifm asosi. b – logarifm osti ifodasi.

Logarifmik funksiya va uning xossalari, grafigi

$a>0$ va $a\neq 1$ shartlarni qanoatlantiradigan a haqiqiy sonni qaraylik. Ushbu

$$y=\log_a x$$

ko‘rinishdagi funksiya ***logarifmik funksiya*** deyiladi.

Logarifmik funksiyalar quyidagi ***xossalar***ga ega:

- $y=\log_a x$ logarifmik funksiyaning aniqlanish sohasi barcha musbat haqiqiy sonlar to‘plamidan iborat:

$$D(f)=(0,\ +\infty);$$

- $y=\log_a x$ logarifmik funksiyaning qiymatlar to‘plami esa barcha haqiqiy sonlar to‘plamidan iborat:

$$E(f)=(-\infty,\ +\infty).$$

- $y=\log_a x$ logarifmik funksiya Ox o‘qini (1,0) nuqtada kesadi.
- $y=\log_a x$ logarifmik funksiya Oy o‘qi bilan esa kesishmaydi.
- Logarimik funksiya davriy bo‘lmaydi.
- Logarifmik funksiya juft ham emas, toq ham emas.

● a asosning $0<a<1$ tengsizliklarni qanoatlantiruvchi qiymatlarida $y=\log_a x$ funksiya kamayadi: Kamayish oralig‘i $(0;+\infty)$ dan iborat.

● a asosning $a>1$ tengsizlikni qanoatlantiruvchi qiymatlarida $y=\log_a x$ funksiya o‘sadi: O‘sish oralig‘i $(0;+\infty)$ dan iborat.

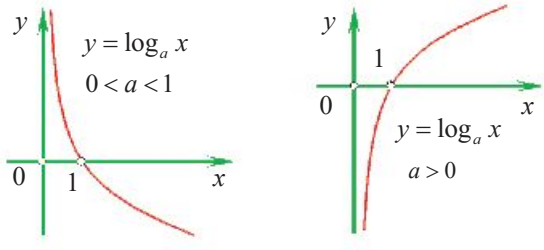
Natural va o‘nli logarifm

$\log_e a=\ln a;$ $\log_{10} a=\lg a;$

Asosi e bo‘lgan logarifmik funksiya $y=\ln x$ orqali, asosi 10 bo‘lgan logarifmik funksiya $y=\lg x$ orqali belgilanadi.

Logarifmlashga teskari amalni potensirlash deyiladi. Boshqacha aytganda, $\log_a x=b$ tenglikdan x noma’lumni topish amali deyiladi. Bu holda potensirlash natijasi $x=a^b$ bo‘ladi.

1-rasm



Logarifmik funksiya grafigi

MISOLLAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y=\log_4 x$ b) $y=\log_2(x-1)$ c) $y=\log_3(x^2-2x-3)$

d) $y=\log_4(x^2-4)$ e) $y=\lg(3-x)$ f) $y=-\log_2(x^2+5x-6)$

g) $y=\log_{x^2}(4-x)$ h) $f(x)=\log_{x-2}(x-1)+\sqrt{2-x}$

i) $f(x)=\sqrt{9-x^2}+\lg(x-1)-\sqrt{x}$ j) $f(x)=\sqrt{x+4}+\log_2(x^2-4)$

k) $f(x)=\dfrac{\log_{x^2+1}(6-x)}{\sqrt{x+2}}$ l) $y=\sqrt{2+\log_{\frac{1}{2}}(3-x)}$

2. Funksiyaning grafigini chizing.

a) $y=\log_3 x$ b) $y=\log_2(5x+1)$ c) $y=\log_2(x^2-2x+1)$

d) $y=\log_4(x-1)$ e) $y=\lg(3-x)$ f) $y=-\log_2(x^2+5x-6)$

i) $y=\log_4(3x-2)$ j) $y=\log_2(x-1)$ k) $y=\log_3(x^2-2x-3)$

l) $y=\log_4(x^2-4)$ m) $y=\lg(3-x)$ n) $y=-\log_2(x^2+5x-6)$

LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH

Logarifmik ifodalar ustida amallar bajarishda va ularni soddalashtirishda quyidagi ayniy almashtirishlardan foydalaniladi. Bu xossalarda qatnashadigan ifodalar logarifm aniqlangan bo‘lishi uchun talab etiladigan shartlarni qanoatlantiradi deb olamiz.

Logarifmning ta’rifidan uning quyidagi ***xossalari*** kelib chiqadi:

1°. $\log_a 1 = 0$.

2°. $\log_a a = 1$.

3°. Ko‘paytmaning logarifmi ko‘paytuvchilar logarifmlarining yig‘indisiga teng:

$$\log_a (bc) = \log_a b + \log_a c.$$

4°. Bo‘linmaning logarifmi bo‘linuvchi va bo‘luvchi logarifmlarining ayirmasiga teng:

$$\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c.$$

5°. Darajaning logarifmi daraja ko‘rsatkichi bilan asos logarifmining ko‘paytmasiga teng:

$$\log_a b^{\alpha} = \alpha \log_a b.$$

6°. Bir asosdan boshqa asosga o‘tish formulasi: $\log_a b = \dfrac{\log_c b}{\log_c a}$.

1 va 6 xossalardan quyidagi tenglik kelib chiqadi:

7°. $\log_a b = \dfrac{1}{\log_b a}$.

5 va 7 xossalardan quyidagi tenglik kelib chiqadi:

8°. $\log_{a^{\beta}} b = \dfrac{1}{\beta} \log_a b$.

5 va 8 xossalarni quyidagicha umumlashtirish mumkin:

9°. $\log_{a^{\beta}} b^{\alpha} = \dfrac{\alpha}{\beta} \log_a b$.

MISOLLAR

Ko‘rsatkichli va logarifmik ifodalarni soddalashtirish

Logarifmning va logarifmik funksiyaning, shuningdek, darajaning va ko‘rsatkichli funksiyaning xossalari bilan tanishgan edik. Bu xossalardan logarifmik va ko‘rsatkichli ifodalarni shakl almashtirishlarda foydalaniladi.

1-misol. $3^{2+\log_3 2}$ ni hisoblang.

Yechish. Bu misolni yechishda $a^{m+n} = a^n \cdot a^m$ va $a^{\log_a b} = b$ tengliklardan foydalaniladi. Natijada ushbu natijaga ega bo‘lamiz:

$$3^{2+\log_3 2} = 3^2 \cdot 3^{\log_3 2} = 9 \cdot 2 = 18.$$

Ko‘rsatkichli va logarifmik ifodalarni soddalashtirishda keng qo‘llaniladigan

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$

tenglikni isbotlaymiz. Bu yerda $a>0$, $c>0$, $b>0$ va $b\neq 1$ shartlar bajarilishi talab etiladi. Mazkur shartlar bajarilganda $\log_b c$ va $\log_b a$ ifodalar ma’noga ega bo‘ladi. Ravshanki,

$$\log_b c \log_b a = \log_b a \log_b c$$

tenglik o‘rinli. Bu ayniyatdan logarifmning $n\log_p q = \log_p q^n$ xossasiga ko‘ra

$$\log_b a^{\log_b c} = \log_b c^{\log_b a}$$

tenglik kelib chiqadi. Uning ikkala tomonini potensirlab,

$$a^{\log_b c} = c^{\log_b a}$$

tenglikka erishamiz.

2-misol. $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}}$ ifodani soddalashtiring. Bu yerda $a>0$, $a\neq 1$, $b>0$, $b\neq 1$.

Yechish. $a^{\log_a b} = b$ va $\log_a b = \dfrac{1}{\log_b a}$ tengliklarni qo‘llab, quyidagi ayniy almashtirishlarni bajaramiz:

$$a^{\sqrt{\log_a b}} = a^{\frac{\log_a b}{\sqrt{\log_a b}}} = \left(a^{\log_a b}\right)^{\frac{1}{\sqrt{\log_a b}}} = b^{\frac{1}{\sqrt{\log_a b}}} = b^{\sqrt{\log_b a}},$$

ya’ni $a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$. Demak, $a^{\sqrt{\log_a b}} - b^{\sqrt{\log_b a}} = 0$.

3-misol. $f(x) = \log_4 \dfrac{x^2}{4} - 2\log_4\left(4x^4\right)$ ifodani soddalashtiring va uning $x=-2$ dagi qiymatini toping.

Yechish. Berilgan ifoda ma’noga ega bo‘lishi uchun $x\neq 0$ bo‘lishi talab etiladi. Quyidagi tengliklar logarifmning xossalaridan kelib chiqadi:

$$f(x) = \log_4 \frac{x^2}{4} - 2\log_4\left(4x^4\right) = \log_4 x^2 - \log_4 4 - 2\left(\log_4 4 + \log_4 x^4\right) =$$

$$= 2\log_4 |x| - 1 - 2\left(1 + 4\log_4 |x|\right) = -6\log_4 |x| - 3;$$

$$f(-2) = -6\log_4 |-2| - 3 = -6\log_{2^2} 2 - 3 = -\frac{6}{2}\log_2 2 - 3 = -3 - 3 = -6.$$

4-misol. Ifodani soddalashtiring:

$$A = \frac{2^{\log_2 (\lg b)^{\frac{1}{4}}} \lg^{\frac{1}{4}} b^4}{\sqrt{\dfrac{\lg^2 b + 1}{2\lg b} + 1} - 10^{0{,}5\lg\left(\lg b^{\frac{1}{2}}\right)}}.$$

Yechish. Musbat sonlargina logarifmga ega bo‘lgani uchun $\lg b > 0$ yoki $b>1$ munosabatga ega bo‘lamiz. Demak, berilgan ifoda $b>1$ bo‘lganda ma’noga ega. Darajaning va logarifmning tegishli xossalaridan foydalanib, quyidagi almashtirishlarni bajaramiz:

$$A=\frac{(\lg b)^{\frac{1}{4}}\left(\lg b^4\right)^{\frac{1}{4}}}{\sqrt{\frac{\lg^2 b+2\lg b+1}{2\lg b}}-\sqrt{\lg b^{\frac{1}{2}}}}=\frac{(\lg b)^{\frac{1}{4}}(4\lg b)^{\frac{1}{4}}}{\sqrt{\frac{(\lg b+1)^2}{\sqrt{2\lg b}}}-\sqrt{\frac{1}{2}\lg b}}=\frac{4^{\frac{1}{4}}(\lg b)^{\frac{1}{2}}}{\frac{\lg b+1}{\sqrt{21gb}}-\sqrt{\frac{\lg b}{2}}}=$$

$$=\frac{\sqrt{2}(\lg b)^{\frac{1}{2}}}{\frac{\lg b+1}{\sqrt{2}\sqrt{\lg b}}-\frac{\sqrt{\lg b}}{\sqrt{2}}}=\frac{\sqrt{2}(\lg b)^{\frac{1}{2}}}{\frac{\lg b+1-\lg b}{\sqrt{2}\sqrt{\lg b}}}=\frac{2\lg b}{1}=2\lg b.$$

5-misol. Agar $a=\log_{98}112$ bo'lsa, $\log_7 2$ ni a orqali ifodalang.

Yechish.

$$a=\log_{98}112=\frac{\log_7 112}{\log_7 98}=\frac{\log_7\left(7\cdot 2^4\right)}{\log_7\left(7^2\cdot 2\right)}=\frac{\log_7 7+\log_7 2^4}{\log_7 7^2+\log_7 2}=\frac{1+4\log_7 2}{2+\log_7 2};$$

$$\frac{1+4\log_7 2}{2+\log_7 2}=a;$$

$$1+4\log_7 2=2a+a\log_7 2;$$

$$4\log_7 2-a\log_7 2=2a-1;$$

$$(4-a)\log_7 2=2a-1;$$

$$\log_7 2=\frac{2a-1}{4-a}.$$

MISOLLAR

1. Hisoblang:

1) $\log_2 8+\log_2 4$; 2) $\log_3 6+\log_3\frac{3}{2}$; 3) $\log_2 15-\log_2\frac{15}{16}$;

4) $\log_{\frac{1}{3}} 54-\log_{\frac{1}{3}} 2$; 5) $\log_{0,2} 75-\log_{0,2} 3$; 6) $\log_{36} 9+\log_{36} 4$.

2. Sonlardan qaysi biri 2 dan kichik?

a) $M=\log_5 100-\log_5 4$;

b) $N=4\log_2 3-\log_2 9$;

c) $P=\log_6 72-\log_6 2$;

d) $Q=\log_4 16+\log_4\frac{1}{8}$.

3. Uchbu sonlaridan qaysi biri qolgan uchtasiga teng emas?

a) $m=2\log_2 8-\log_2 4$;

b) $n=\log_2 400-2\log_2 5$;

c) $p=\log_5 125+\log_5 5$;

d) $q=\ln 12e-\ln 2$.

4. Quyidagi funksiyalarning aniqlanish sohalarini toping:

a) $y=\log_2(x+3)$, b) $y=\log_{0,2}(x^2-4x)$, c) $y=\log_{0,7}\left(2x-\frac{1}{8}\right)$, d) $y=\log_2(5-3x)$.

5. Funksiyalarning grafigini chizing.

a) $f(x)=\log_3(x)$; b) $g(x)=\log_{10}(x)$;

c) $y=\log_4(x)$; d) $y=\log_5(x)$.

6. Logarifmik ifodalarning qiymatini toping:

a) $\log_2 4$; b) $\log_2 1$; c) $\log_2 16$; d) $\log_4 16$;

e) $\log_2 64$; f) $\log_8 64$; g) $\log_4 64$; h) $\log_{64} 64$.

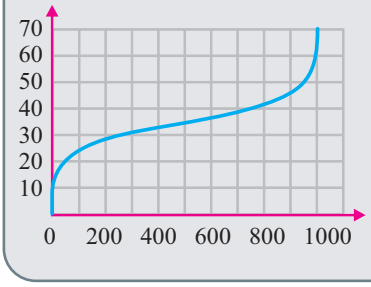
7. Logarafmik ifodalarning qiymatini toping:

a) $\log_5(25)$; b) $\log_2(18)$; c) $\log_{12}(4)$; d) $\log_{10}(0{,}001)$;

e) $\log_9(3)$; f) $\lg(1000)$; g) $\ln(e)$; h) $\lg\left(\frac{1}{100}\right)$.

8. Agar chorvadorning 1000 bosh podasidagi bitta sigiri yuqumli kasallikka chalingan bo‘lsa, u holda t va qt kunda n ta sigirning kasallanish ko‘rsatkichi $t=-5\cdot\ln\left(\frac{1000-n}{999n}\right)$ formula bilan modellashtirilgan. 100 ta, 800 ta, 1000 ta sigir necha kunda kasallanishini toping. Chizma asosida xulosa tayyorlang (2-rasm).

2-rasm





9. Jadvallar asosida funksiya grafigini chizing.

a)

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$g(x)=log_2(x)$	-2	-1	0	1	2	3

b)

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$f(x)=log_{1/2}(x)$	2	1	0	-1	-2	-3

10. Quyidagi funksiyalarga teskari funksiyalarni aniqlang.

a) $f(x)=10^x$; b) $f(x)=\log_3(x)$;

c) $f(x)=2+e^{x+4}$; d) $f(x)=5+\log_2(x-3)$

11. Noma’lumlarni toping.

a) $\log_3(x)=2$; b) $\log_x(8)=3$; c) $\log(x^2)=\lg(4)$; d) $\log_x(36)=2$.

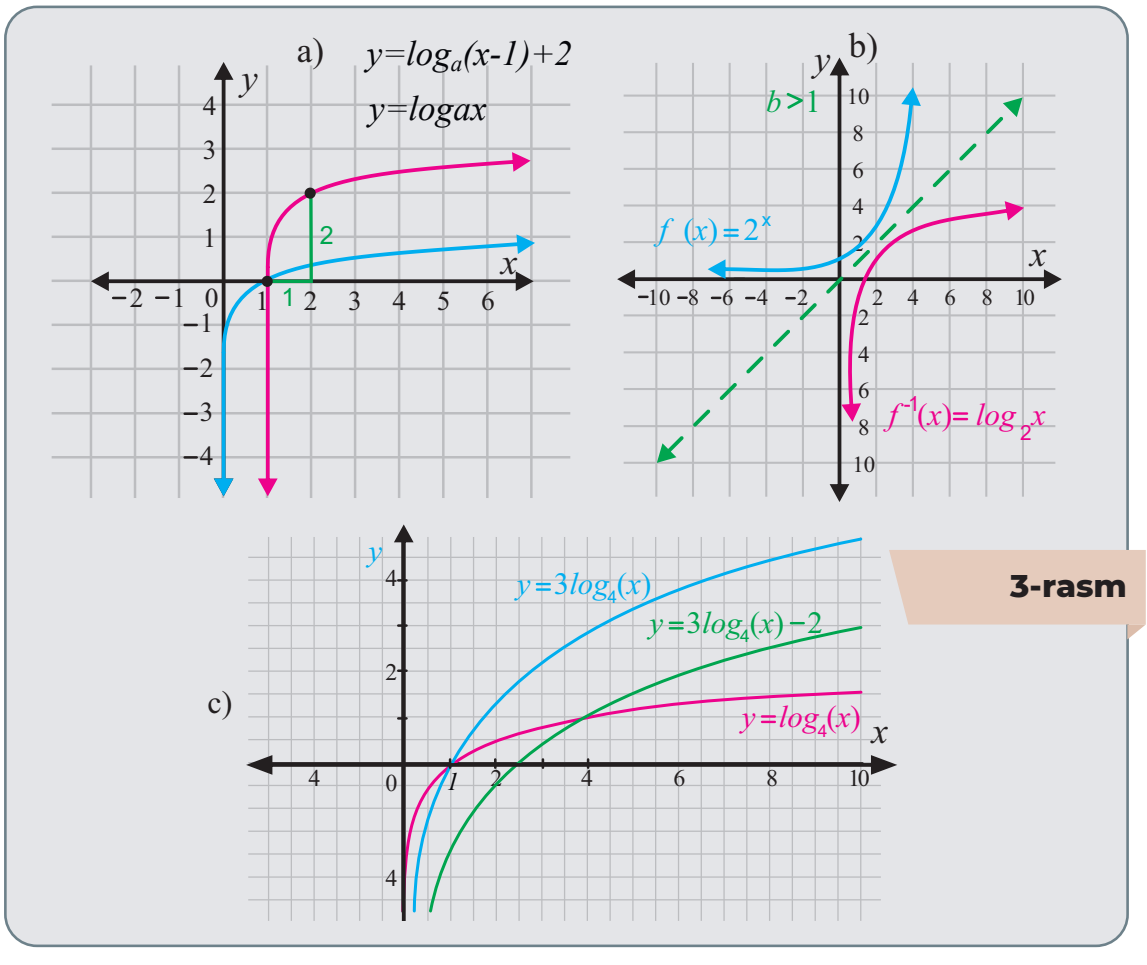
12. $a=\log_{36}108$ bo‘lsa, $\log_2 3$ ni a orqali ifodalang.

13. $a=\log_{75}135$ bo‘lsa, $\log_5 3$ ni a orqali ifodalang.

14. $a = \log_{147} 189$ bo‘lsa, $\log_7 3$ ni a orqali ifodalang.

15. $a = \log_{50} 80$ bo‘lsa, $\log_5 2$ ni a orqali ifodalang.

16. Berilgan rasmdan qanday xulosa chiqarish mumkin? Fikringizni bayon qiling.



3-rasm

17. $y = \ln(e^x)$ va $y = e^{\ln(x)}$ funksiyalarining grafigini yasang. O‘xshashlik va farqlarini tushuntiring.

Logarifmik funksiyaning hayotda qo‘llanilishi

Tovush intensivligi darajasi.

Yuza birligi orqali vaqt birligida tovush to‘lqini olib o‘tayotgan energiya tovushning intensivligi deb ataladi. Elastik muhit bo‘ylab tovush tarqalganda u tarqalmagan paytdagiga nisbatan ortiqcha bosim hosil bo‘ladi, uni tovushning bosimi deyiladi. Tovushning intensivligi tovush bosimining amplitudasiga hamda muhit xossasiga va to‘lqin shakliga bog‘liq. Ovoz darajasining intensivligi detsibelda (dB) o‘lchanadi.



$$L = 10 \log_{10} \left(\frac{I}{I_0} \right) dB$$

Smartfon quloqliklariga uzatadigan tovush intensivligi 100 detsibeldan oshadi. Odam qulog‘i uchun 80 detsibeldan yuqori bo‘lmagan tovush balandligi xavfsiz hisoblanadi. Bu balandlikdan ortiqcha tovush eshitish qobiliyatining buzilishiga yoki yo‘qolib borishiga sabab bo‘ladi.

KO‘RSATKICHLI TENGLAMALAR

Ko‘rsatkichli tenglamalar

Noma’lumi daraja ko‘rsatkichida qatnashgan tenglama ***ko‘rsatkichli tenglama*** deyiladi. Masalan, $a^x + 2b^x - c = 0$ ko‘rsatkichli tenglama bo‘ladi.

Noma’lumning berilgan ko‘rsatkichli tenglamani ayniyatga aylantiradigan qiymati bu ko‘rsatkichli tenglamaning ***ildizi*** deyiladi.

Ko‘rsatkichli sodda tenglamalar va ularni yechish

Ushbu $a^x = a^p$ tenglama ko‘rsatkichli eng sodda tenglama bo‘ladi. Bu tenglamaning yechimi $x = p$ bo‘ladi.

Ko‘rsatkichli tenglamalarni yechishda ushbu qoida ishlatiladi:

$a > 0$, $a \neq 1$ bo‘lganda $a^{f(x)} = a^{g(x)}$ tenglamaning ildizlari $f(x) = g(x)$ tenglamaning ildizlaridan iborat bo‘ladi.

1-misol. Tenglamani yeching: $3^{2x} \cdot 3^{x^2} = 3^{15}$

Yechish:

1. Tenglamani $3^{2x+x^2} = 3^{15}$ ko‘rinishda yozib olamiz, bu yerdan $2x + x^2 = 15$.

2. $2x + x^2 = 15$ ni $x^2 + 2x - 15 = 0$ kvadrat tenglamaga keltirib yechamiz. Viyet teoremasiga ko‘ra $x_1 = -5; x_2 = 3$.

Javob: $x_1 = -5;\ x_2 = 3$

2-misol. Tenglamani yeching: $\left(5^{x+1}\right)^x = \left(\frac{5^x}{5^{24}}\right)^{-1}$

Yechish:

1. $(a^n)^m = a^{nm}; \frac{a^n}{a^m} = a^{n-m}; a^{-1} = \frac{1}{a}$ xossalaridan foydalanib, $5^{x^2+x} = 5^{24-x}$ tenglamaga ega bo‘lamiz. Bu yerdan, $x^2 + x = 24 - x$ kelib chiqadi.

2. $x^2 + 2x - 24 = 0$ hosil bo‘lgan kvadrat tenglamani yechamiz. Viyet teoremasiga ko‘ra $x_1 = -6; x_2 = 4.$

Javob: $x_1 = -6; x_2 = 4.$

3-misol. $6^{x^2} + 36 = 2^{1-x^2} \cdot 12^{x^2}$ tenglamaning ildizlari ko‘paytmasini toping.

Yechish. Bu tenglamani yechishni quyidagiga soddalashtirishdan boshlaymiz:

$$12^{x^2} = (6 \cdot 2)^{x^2} = 6^{x^2} \cdot 2^{x^2};$$

Shunda tenglama quyidagi ko‘rinishga keladi:

$$6^{x^2}+36=2^{1-x^2}\cdot 6^{x^2}\cdot 2^{x^2};$$

$$6^{x^2}+36=2^{1-x^2+x^2}\cdot 6^{x^2};$$

$$6^{x^2}+36=2\cdot 6^{x^2};$$

$$6^{x^2}=36;$$

$$6^{x^2}=6^2;$$

$$x^2=2;$$

$$x_{1,2}=\pm 2.$$

Tenglamaning 2ta ildizi bor, ularning ko‘paytmasi $x_1\cdot x_2=-2\cdot 2=-4;$

Javob: -4.

4-misol. $3^{2x-1}=7^{2x-1}$ tenglamani yeching.

Yechish. Berilgan tenglamada tenglikning ikkala tarafidagi ko‘rsatkichli ifodalarning daraja ko‘rsatkichlari bir xil. Bu tenglamani yechish uchun tenglikning ikkala tomonini 7^{2x-1} ifodaga bo‘lamiz:

$$\frac{3^{2x-1}}{7^{2x-1}}=\frac{7^{2x-1}}{7^{2x-1}};$$

Chap tarafdagi ifodaning surati va maxrajining ko‘rsatkichi bir xil bo‘lgani uchun bu daraja ko‘rsatkichini umumiy qilib qavsdan tashqariga chiqaramiz, o‘ng taraf esa 1 ga teng:

$$\left(\frac{3}{7}\right)^{2x-1}=1;$$

$$\left(\frac{3}{7}\right)^{2x-1}=\left(\frac{3}{7}\right)^{0};$$

Tenglikning ikki tarafidagi ifodada asoslari bir xil bo‘lgani uchun daraja ko‘rsatkichlarini tenglashtiramiz va hosil bo‘lgan chiziqli tenglamani yechamiz:

$$2x-1=0;$$

$$x=\frac{1}{2}.$$

Javob: $x=\frac{1}{2}.$

MISOLLAR

1. Ko‘rsatkichli tenglamalarni yeching:

a) $3^x \cdot 3 = 81$; b) $4^{3x} \cdot 2^x = 128$; c) $5^{x+1} - 4 \cdot 5^x = 25$; d) $7^x \cdot 8^x = 1$;

e) $4^{x^2-3x-4} = 1$; f) $0,3^{2x-1} = 0,09$; g) $2^{2x} = 4^{2\sqrt{3}}$; h) $\left(\frac{1}{3}\right)^{3x} = 9$;

i) $27^x = \frac{1}{3}$; j) $400^x = \frac{1}{20}$; k) $\left(\frac{1}{3}\right)^x = \frac{1}{81}$; l) $0,6^{x+3} = 0,6^{2x-5}$;

m) $2 \cdot 4^x = 64$; n) $3 \cdot 9^x = 81$.

2. Tenglamani yeching:

a) $3^x = 81$ b) $\left(\frac{1}{2}\right)^x = \frac{1}{1024}$ c) $7^x = -49$ d) $13^x = -169$

e) $5^x = 0$ f) $8^{2x} = 0$ g) $3^{6-x} = 3^{3x-2}$ h) $\left(\frac{3}{7}\right)^{3x+1} = \left(\frac{7}{3}\right)^{5x-9}$

i) $2^{7x-15} = 2^{9-4x}$ j) $13^{5-2x} = 13^{6x+1}$ k) $2^{x^2+x-0,5} = 4\sqrt{2}$ l) $\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$

3. $\left(\frac{21}{6}\right)^{29x^2-8x} = \left(\frac{6}{21}\right)^{8x^2-29x}$

4. $\sqrt[3]{5^{2x-3}} = \frac{5}{\sqrt[4]{5}}$

5. $\left(\frac{37}{5}\right)^{71\sqrt{x}-3} = \left(\frac{5}{37}\right)^{3\sqrt{x}-293}$

6. $\left(\frac{1}{\sqrt{2}}\right)^{x^2-9x} = 1$

7. $2^{x^2-3} \cdot 5^{x^2-3} = 0,01\left(10^{x-1}\right)^3$

8. $2^{x+1} = 5^{x+1}$

9. $7^{x+2} + 4 \cdot 7^{x-1} = 347$

9. $2 \cdot 3^{x+1} - 4 \cdot 3^{x-2} = 150$

10. $5^{2x} + 5^{2x+2} + 5^{2x+4} = 651$

11. $4 \cdot 7^{x+3} - 7^{x+2} - 3 \cdot 7^{x+1} = 1302$

12. $6 \cdot 2^{x+4} - 4 \cdot 2^{x+3} + 3 \cdot 2^{x+2} = 152$

13. $7^{3x} - 7^{3x-1} = 6$

14. $4^x - 6 \cdot 2^x + 8 = 0$

15. $5 \cdot 25^x - 6 \cdot 5^x + 1 = 0$

16. $9^x + 3 \cdot 3^x - 18 = 0$

17. $3^{2x+3} - 4 \cdot 3^{x+1} + 1 = 0$

18. $(0,25)^{2-x} = \frac{256}{2^{x+3}}$

19. $3^{4\sqrt{x}} - 4 \cdot 3^{2\sqrt{x}} + 3 = 0$

20. $9 \cdot 16^x + 2 \cdot 12^x - 32 \cdot 9^x = 0$

21. $64 \cdot 9^x - 84 \cdot 12^x + 27 \cdot 16^x = 0$

22. $3 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$

23. $4^{x^2} + 6^{x^2} = 2 \cdot 9^{x^2}$

24. $\sqrt{10^{2x+6}} = \frac{10}{\sqrt[4]{10}}$

25. $8^x - 6 \cdot 12^x + 11 \cdot 18^x = 2 \cdot 27^{x+\frac{1}{3}}$

26. $x \cdot 3^{x-1} + 3 \cdot 3^{\sqrt{3-x}} = 3^x + x \cdot 3^{\sqrt{3-x}}$

27. $x^2 \cdot 4^{\sqrt{6-x}} + 4^{2+x} = 16 \cdot 2^{2\sqrt{6-x}} + x^2 \cdot 2^{2x}$

28. $\left(\frac{1}{2}\right)^{2-x} + 2^{x-3} = 80 + \sqrt{4^{x-4}}$

SODDA LOGARIFMIK TENGLAMALAR

Logarifmik tenglamalar

Noma'lumi logarifmosti ifodada qatnashgan tenglama ***logarifmik tenglama*** deyiladi. Masalan,

$$\log_a x - \log_b 2x + c = 0$$

logarifmik tenglama bo'ladi.

Noma'lumning berilgan logarifmik tenglamani to'g'ri tenglikka aylantiradigan qiymati bu logarifmik ***tenglamaning yechimi*** deyiladi.

Sodda logarifmik tenglamalarni yechish

$a>0$, $a\neq 1$ bo'lganda ushbu $\log_a x = b$ tenglama eng sodda logarifmik tenglama bo'ladi. Bu tenglamaning yechimi $x = a^b$ bo'ladi.

Logarifmik tenglamalarni yechishda ushbu qoida ishlatiladi:

$a>0$, $a\neq 1$ bo'lganda $\log_a f(x) = \log_a g(x)$ tenglamaning ildizlari $f(x) = g(x)$ tenglamaning $f(x)>0$ (yoki $g(x)>0$) shartni qanoatlantiruvchi ildizlaridan iborat bo'ladi.

Quyida logarifmik tenglamalarni yechishning namunalarini keltiramiz.

1-misol. $\log_3(x^2-4) = \log_3(5x-8)$ logarifmik tenglamani yeching.

Yechish. Aniqlanish sohasini topamiz:

$$\begin{cases} x^2-4>0 \\ 5x-8>0 \end{cases} \Rightarrow \begin{cases} (x-2)(x+2>0) \\ 5x>8 \end{cases} \Rightarrow \begin{cases} x\in(-\infty;-2)\cup(2;\infty) \\ x>1,6 \end{cases} \Rightarrow x\in(2;\infty)'$$

Endi $x^2-4 = 5x-8$ tenglamani yechamiz:

$$x^2-5x+4=0,$$
$$(x-1)(x-4)=0,$$
$$x_1=1,\ x_2=4.$$

Noma'lumning $x_1=1$ qiymati $(-\infty,\ -2)\cup(2,\ +\infty)$ to'plamga tegishli emas, $x_2=4$ qiymati esa bu to'plamga tegishli bo'ladi. Demak, $x_1=1$ qiymat berilgan tenglamaning chet ildizi bo'ladi, $x_2=4$ qiymat esa berilgan tenglamaning ildizi bo'ladi.

2-misol. $\log_5^2 x - 3\log_5 x - 4 = 0$ logarifmik tenglamani yeching.

Yechish. Avvalo $x>0$ aniqlanish sohasi bo'lishini aniqlaymiz va $\log_5 x = t$ belgilash kiritib, quyidagilarga ega bo'lamiz:

$$t^2-3t-4=0,$$
$$(t+1)(t-4)=0,$$
$$t_1=-1,\quad t_2=4.$$

Demak, $\log_5 x=-1$ va $\log_5 x=4$. Bundan $x_1=\frac{1}{5}=0,2$; $x_2=5^4=625$.

3-misol. $\log_5 \log_2 \log_7 x = 0$ tenglamani yeching.

Yechish. Tenglamani yechishda logarifm ta’rifidan foydalanamiz:

$$\log_2 \log_7 x = 5^0;$$
$$\log_2 \log_7 x = 1;$$
$$\log_7 x = 2^1;$$
$$x = 7^2 = 49.$$

Javob: $x = 49$.

4-misol. $\lg\left(x^2 - 3\right) \cdot \lg x = 0$ tenglamani yeching

Yechish. 1. Aniqlanish sohasini topamiz $\begin{cases} x^2 - 3 > 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} x < -\sqrt{3} \ va \ x > \sqrt{3} \\ x > 0 \end{cases} \Rightarrow x > \sqrt{3}$:

2. Har bir ko‘paytuvchini 0 ga tenglashtiramiz:

$\lg\left(x^2 - 3\right) = 0 \Rightarrow x^2 - 3 = 1 \Rightarrow x^2 = 4 \Rightarrow x_{1,2} = \pm 2;$

$\lg x = 0 \Rightarrow x_3 = 1$

$x = 2$ ildiz aniqlanish sohani qanoatlantiradi.

Javob: $x = 2$.

5-misol. $\log_4 \log_2 x + \log_2 \log_4 x = 2$ tenglamani yeching.

Yechish. Tenglamani yechish uchun logarifmning quyidagi xossasidan foydalanamiz:

$\log_{a^p} x = \frac{1}{p} \log_a x$

$\frac{1}{2} \log_2 \log_2 x + \log_2 \log_4 x = 2;$ tenglamaning ikkala tomonini 2 ga ko‘paytiramiz:

$\log_2 \log_2 x + 2 \log_2 \log_4 x = 4;$ $\log_a x^m = m \log_a x$ xossasidan foydalanamiz:

$\log_2 \log_2 x + \log_2 \left(\log_4 x\right)^2 = 4;$ $\log_a x + \log_a y = \log_a x \cdot y$ xossasidan foydalanamiz:

$\log_2 \left(\log_2 x \cdot \left(\log_4 x\right)^2\right) = 4;$

logarifm ta’rifidan foydalanamiz:

$\log_2 x \cdot \left(\log_4 x\right)^2 = 2^4;$ $\log_{a^p} x = \frac{1}{p} \log_a x$ xossasidan yana bir bor foydalanamiz:

$\log_2 x \cdot \left(\log_{2^2} x\right)^2 = 16;$

$\log_2 x \cdot \left(\frac{1}{2} \log_2 x\right)^2 = 16;$

$\log_2 x \cdot \frac{1}{4} \log_2^2 x = 16;$

$\log_2^3 x = 64;$

$\log_2 x = 4;$

$x = 2^4 = 16.$

Javob: $x = 16$.

MISOLLAR

Logarifmik tenglamalarni yeching:

1. $\log_3(3x-1)=2$;

2. $\log_4(2x-3)=4$;

3. $\log_{\frac{1}{2}}(7-8x)=2$;

4. $\log_7=(x+3)=2$;

5. $\log_2 x-2\log_{\frac{1}{2}} x=9$;

6. $\log_2 x+\log_8 x=0$;

7. $\log_9 x^3+\log_{\sqrt{3}} x=3$;

8. $\log_5 x=2\log_5 3+4\log_{25} 2$;

9. $\log_3 x=9\log_{27} 8-3\log_3 4$.

10. $\log 2x=-3$

11. $\log_4 2x=\frac{1}{2}$

12. $\lg\frac{5x}{2}=1$

13. $\log_{\frac{1}{4}} x=-2$

14. $\log_{0,5}(3x+1)=-2$

15. $\log_{0,2}(x+3)=-1$

16. $\log_{0,25}(x+30)=-2$

17. $\log_{\sqrt{3}}(1-2x)=4$

18. $\log_2\sqrt{x-1}=1$

19. $\log_3(x^2-4x+3)=\log_3(3x+21)$

20. $\log_3(2x-5)=\log_3(20-3x)$

21. $\log_7(9x-1)=\log_7 x$

22. $\log_3(2x^2-3x)=2\log_3 x$

23. $\lg(2x)=2\lg(4x-15)$

24. $\lg(3x-11)+\lg(x-27)=3$

25. $\log_{81} x-2\log_3 x+5\log_9 x=1,5$

26. $\log_3((x-1)(2x-1))=0$

27. $3\lg x^2-\lg^2 x=9$

28. $\log_{\frac{1}{3}}\frac{x^2+4x}{2x-3}=1$

29. $\log_{\frac{3}{4}}\frac{2x-1}{x+2}=1$

30. $\log_\pi(\log_2(\log_3 3x))=0$

31. $\log_2^2 x+3=\log_2 x^2$

32. $(x^2-6x-7)\log_2(3x-1)=0$

33. $(x^2-2x-15)\lg(4x-3)=0$

34. $\log_5(x+4)-\log_5(1-2x)=-\log_5(2x+3)$

35. $\log_2^2 x-5\log_2 x=4$

36. $\log_3 x+\log_x 9=3$

37. $\log_7(6+7^{-x})=1+x$

38. $\log_3 x\cdot\log_9 x\cdot\log_{27} x\cdot\log_{81} x=\frac{2}{3}$

39. $\log_5\sqrt{x-9}+\log_5\sqrt{2x-1}=\log_5 10$

KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI

Ko‘rsatkichli tenglamalar sistemasi va uni yechish

Ko‘rsatkichli ifoda qatnashgan tenglamalarni o‘z ichiga olgan tenglamalar sistemasi ko‘rsatkichli tenglamalar sistemasi deyiladi. Ko‘rsatkichli tenglamalar sistemasi turli xil ko‘rinishda bo‘ladi. Bunday sistemaning har birini yechishda o‘ziga xos yondashuv talab etiladi. Bunda ko‘rsatkichli va logarifmik ifodalarning xossalari keng qo‘llaniladi.

1-misol. $\begin{cases} 3^x = 9^{y+1}, \\ 4y = 5 - x \end{cases}$ sistemani yeching.

Yechish. $9 = 3^2$ ekanligidan foydalanamiz. U holda $3^x = 3^{2(y+1)}$ bo‘lib, bu yerdan $x = 2y + 2$ kelib chiqadi. Sistemadagi ikkinchi tenglikda x o‘rniga $2y+2$ ifodani qo‘yib, y ning qiymatini topamiz: $4y = 5 - (2y + 2) \Rightarrow y = \frac{1}{2}$. Endi $x = 2y + 2$ tenglikdagi y o‘rniga uning qiymatini qo‘yib, x ning qiymatini topamiz: $x = 2 \cdot \frac{1}{2} + 2 \Rightarrow x = 3$.

Javob: $\left(3;\ \frac{1}{2}\right)$.

2-misol. $\begin{cases} 9^{x+y} = 729, \\ 3^{x-y-1} = 1 \end{cases}$ sistemani yeching.

Yechish. $9 = 3^2$, $729 = 9^3$ va $1 = 3^0$ ekanligidan foydalanamiz. U holda

$$\begin{cases} 9^{x+y} = 9^3, \\ 3^{x-y-1} = 3^0 \end{cases} \Rightarrow \begin{cases} x + y = 3, \\ x - y - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 2, \\ y = 1 \end{cases} \Rightarrow (2;\ 1).$$

Javob: (2; 1).

3-misol. $\begin{cases} x^{y+1} = 27, \\ x^{2y-5} = \frac{1}{3} \end{cases}$ sistemani yeching.

Yechish. Misolning berilishidan

$x > 0$, $x \neq 1$, $y + 1 \neq 0$, $2y - 5 \neq 0$

shartlar bajarilishi kelib chiqadi. Shuning uchun birinchi va ikkinchi tenlamalarining chap va o‘ng tomonidagi ifodalarni logarifmlash mumkin. Bu ifodalarni 3 asosga ko‘ra logarifmlaymiz va quyidagilarga ega bo‘lamiz:

$$\begin{cases} (y+1)\log_3 x = 3, \\ (2y-5)\log_3 x = -1 \end{cases} \Rightarrow \begin{cases} \log_3 x = \frac{3}{y+1}, \\ (2y-5)\frac{3}{y+1} = -1 \end{cases} \Rightarrow \begin{cases} x = 3, \\ y = 2. \end{cases}$$

Javob. (3; 2).

4-misol. $\begin{cases} 2^x + 2^y = 5, \\ 2^{x+y} = 4 \end{cases}$ sistemani yeching.

Yechish. Birinchi tenglikdan $2^y = 5 - 2^x$ bog‘lanishni topamiz. $2^{x+y} = 2^x \cdot 2^y$ tenglikni e’tiborga olib, Ikkinchi tenglikni $2^x \cdot 2^y = 4$, bu yerdan $2^x\left(5 - 2^x\right) = 4$, undan esa $2^{2x} - 5 \cdot 2^x + 4 = 0$ tenglamaga ega bo ‘lamiz. $t = 2^x$ belgilash kiritgach, ushbu kvadrat tenglamaga ega bo‘lamiz:

$$t^2 - 5t + 4 = 0, \text{ bu yerda } t > 0.$$

Bu kvadrat tenglamaning yechimi $t_1 = 1$, $t_2 = 4$ bo‘lib, x va y noma’lumlarning ularga mos qiymatlari

$t_1 = 1$: $1 = 2^{x_1} \Rightarrow x_1 = 0$; $2^{y_1} = 5 - 2^0 = 4, \Rightarrow y_1 = 2$;

$t_2 = 4$: $4 = 2^{x_2} \Rightarrow x_2 = 2$; $2^{y_2} = 5 - 2^2 = 1, \Rightarrow y_2 = 0$ bo‘ladi.

Javob: (0; 2) va (2; 0).

Izoh. Ishlab ko‘rsatilgan yuqoridagi misollar shuni ko‘rsatadiki, har bir ko‘rsatkichli tenglamalar sistemasini yechish uchun ijodiy yondashish kerak ekan.

Logarifmik tenglamalar sistemasi va uni yechish

Logarifmik ifoda qatnashgan tenglamalarni o‘z ichiga olgan sistema ***logarifmik tenglamalar sistemasi*** deyiladi. Logarifmik tenglamalar sistemasi ham ko‘rsatkichli tenglamalar sistemasi kabi turli xil ko‘rinishda bo‘ladi. Ularning har birini yechishda ko‘rsatkichli va logarifmik ifodalarning xossalari keng qo‘llaniladi hamda o‘ziga xos yondashuv talab etiladi.

5-misol. $\begin{cases} \log_9 \dfrac{x^2}{\sqrt{y}} = \dfrac{1}{2}, \\ \log_3 xy = 3 \end{cases}$ sistemani yeching.

Yechish. Sistemadagi logarifmik ifodalar ma’noga ega bo‘lishi uchun

$$\begin{cases} \dfrac{x^2}{\sqrt{y}} > 0, \\ xy > 0 \end{cases}$$

tengsizliklar bajarilishi talab etiladi. $\dfrac{x^2}{\sqrt{y}} > 0$ tengsizlik $y > 0$ va $x \neq 0$ bo‘lgandagina o‘rinli. U holda sistemadagi ikkinchi $xy > 0$ tengsizlikdan $x > 0$ va $y > 0$ bo‘lishi zarurligi kelib chiqadi.

Endi logarifm xossalaridan foydalanib, $x > 0$ va $y > 0$ bo‘lganda berilgan sistemani

$$\begin{cases} \log_9 x^2 - \log_9 \sqrt{y} = \dfrac{1}{2}, \\ \log_3 x + \log_3 y = 3 \end{cases}$$

kabi qayta yozish mumkin. $\log_9 x^2 = \log_{3^2} x^2 = \log_3 x$, hamda $\log_9 \sqrt{y} = \log_{3^2} y^{\frac{1}{2}} = \dfrac{1}{4}\log_3 y$ tengliklardan foydalansak, sistema ushbu ko‘rinishga keladi $\begin{cases} \log_3 x - \dfrac{1}{4}\log_3 y = \dfrac{1}{2}, \\ \log_3 x + \log_3 y = 3. \end{cases}$

Ikkinchi tenglamadan birinchi tenglamani ayirib, $\dfrac{5}{4}\log_3 y = \dfrac{5}{2}$ tenglikka, undan $\log_3 y = 2$ tenglikka ega bo‘lamiz. Oxirgi tenglikdan $y = 9$ ekanligi kelib chiqadi. Endi sistemaning ikkinchi tenglamasiga y ning bu qiymatini qo‘yib, x noma’lumni topamiz:

$$\log_3 x+\log_3 9=3 \Rightarrow \log_3 x+2=3 \Rightarrow \log_3 x=1 \Rightarrow x=3.$$

Javob: (3; 9).

6-misol. $\begin{cases} x^{\lg y}=1000, \\ \log_y x=3 \end{cases}$ sistemani yeching.

Yechish. Sistemadagi ifodalar ma’noga ega bo‘lishi uchun $x>0$, $x\neq 1$, $y>0$, $y\neq 1$ shartlar bajarilishi zarur. $x^{\lg y}=1000$ tenglikni 10 asosga ko‘ra logarifmlaylik:

$$\lg x^{\lg y}=\lg 1000 \Rightarrow \lg y\lg x=3.$$

$\log_y x=3$ tenglikning chap tomonidagi logarifmning asosini almashtiramiz:

$$\log_y x=\frac{\lg x}{\lg y} \Rightarrow \frac{\lg x}{\lg y}=3 \Rightarrow \lg x=3\lg y.$$

Natijada soddalashtirishlardan so‘ng, $\lg^2 y=1$ tenglamaga ega bo ‘lamiz.

Uni yechaylik: $\lg y=-1 \Rightarrow y=\frac{1}{10}$,

$$\lg y=1 \Rightarrow y=10.$$

$\lg x=3\lg y$ tenglikdan $x=y^3$ bog‘lanishni hosil qilib, x ning mos qiymatlarini topamiz:

$$y=\frac{1}{10} \Rightarrow x=\frac{1}{1000},$$
$$y=10 \Rightarrow x=1000.$$

Javob: $\left(\frac{1}{1000};\frac{1}{10}\right)$ va (1000; 10).

7-misol. $\begin{cases} \log_2(x-y)=1, \\ 2^x\cdot 3^{y+1}=72 \end{cases}$ sistemani yeching.

Yechish. Sistema aniqlangan bo‘lishi uchun $x-y>0$, ya’ni $x>y$ bo‘lishi kerak. U holda sistemaning birinchi tenglamasidan

$$x-y=2 \Rightarrow y=x-2$$

bog‘lanish kelib chiqadi. Sistemaning ikkinchi tenglamasida y o‘rniga $x-2$ ifodani qo‘yib,

$$2^x\cdot 3^{x-2+1}=72 \Rightarrow 2^x\cdot 3^x=3\cdot 72 \Rightarrow 6^x=216 \Rightarrow 6^x=6^3 \Rightarrow x=3$$

topiladi. U holda $y=3-2$, ya’ni $y=1$ qiymat topiladi.

Javob: (3; 1).

MISOLLAR

Tengsizliklar sistemasini yeching: 1–26.

1. $\begin{cases} 3^x\cdot 7^y=63 \\ 3^x+7^y=16 \end{cases}$

2. $\begin{cases} 9^x-3\cdot 5^y=3 \\ 9^x\cdot 5^y=18 \end{cases}$

3. $\begin{cases} 3^x\cdot 2^y=\frac{1}{9} \\ \frac{1}{9}\cdot 3^y=3^x \end{cases}$

4. $\begin{cases} 3^y\cdot 2^x=972 \\ y-x=3 \end{cases}$

5. $\begin{cases} 4^{x+y}=128 \\ 5^{3y-2x-3}=1 \end{cases}$

6. $\begin{cases} 2^y\cdot 8^{-x}=8\sqrt{2} \\ y+3x=\frac{1}{2} \end{cases}$

7. $\begin{cases} 3\cdot 2^{x}-2^{x+y}=-2 \\ 5\cdot 2^{x+1}-2^{x+y+1}=4 \end{cases}$

8. $\begin{cases} 9^{x}-3\cdot 2^{y}=3 \\ 9^{x}\cdot 5^{y}=18 \end{cases}$

9. $\begin{cases} \log_2 x-\log_4 y=0 \\ \log_4 x+\log_2 y=5 \end{cases}$

10. $\begin{cases} 3^{x}\cdot 7^{y}=63 \\ 3^{x}+7^{y}=16 \end{cases}$

11. $\begin{cases} 3^{-x}\cdot 2^{y}=1152 \\ x+y=5 \end{cases}$

12. $\begin{cases} 4^{y-1}\cdot 5^{x}=6400 \\ y-x=3 \end{cases}$

13. $\begin{cases} \log_7 7x+\log_7 y=2 \\ y-5x=2 \end{cases}$

14. $\begin{cases} \log_2 x-\log_2 y=4 \\ y-x=6 \end{cases}$

15. $\begin{cases} 2^{x}\cdot 9^{y}=648 \\ 3^{x}\cdot 4^{y}=432 \end{cases}$

16. $\begin{cases} \log_2\left(x^2+y^2\right)=5 \\ \log_2 x+\log_2 y=4 \end{cases}$

17. $\begin{cases} \log_3 2x-\log_3\left(\dfrac{2}{y}\right)=1 \\ 4x-y=1 \end{cases}$

18. $\begin{cases} \log_2 2x+\log_2\left(\dfrac{y}{2}\right)=-1 \\ x-y=-\dfrac{7}{4} \end{cases}$

19. $\begin{cases} lgx\,(lgx+lgy)=2 \\ lgx-lgy=3 \end{cases}$

20. $\begin{cases} 3^{x}\cdot 25^{y}=5625 \\ 5^{x}\cdot 9^{y}=2025 \end{cases}$

21. $\begin{cases} 3^{x}-2^{y^2}=77 \\ 3^{\frac{x}{2}}-2^{\frac{y^2}{2}}=7 \end{cases}$

22. $\begin{cases} 9^{x+y}=729, \\ 3^{x-y-1}=1. \end{cases}$ x va y tenglamalar sitemasi ildizlari bo‘lsa, x^2+y^2 ni toping.

23. $\begin{cases} x^{\sqrt{y}}=y \\ y^{\sqrt{y}}=x^4 \end{cases}$ tenglamalar sitemasi ildizlarini ifodalovchi nuqtalar orasidagi masofani toping.

24. $\begin{cases} 3^{x}\cdot 2^{y}=972, \\ \log_{\sqrt{3}}(x-y)=2 \end{cases}$ x va y tenglamalar sitemasi ildizlari bo‘lsa, xy ni toping.

25. $\begin{cases} x^{y+1}=27, \\ x^{2y-5}=\dfrac{1}{3} \end{cases}$ x va y tenglamalar sitemasi ildizlari bo‘lsa, $x+y$ ni toping.

KO‘RSATKICHLI TENGSIZLIKLAR

Ko‘rsatkichli sodda tengsizliklar

$a>0$ va $a\neq 1$ bo‘lsin. U holda

$$a^x<b,\qquad a^x>b,\qquad a^x\le b,\qquad a^x\ge b$$

tengsizliklar eng sodda ko‘rsatkichli tengsizliklar bo‘ladi. Ularni yechishda $y=a^x$ funksiyaning monotonligidan, ya’ni

$a>1$ bo‘lganda, $y=a^x$ funksiyaning o‘suvchi ($x_2>x_1\Rightarrow a^{x_2}>a^{x_1}$) va

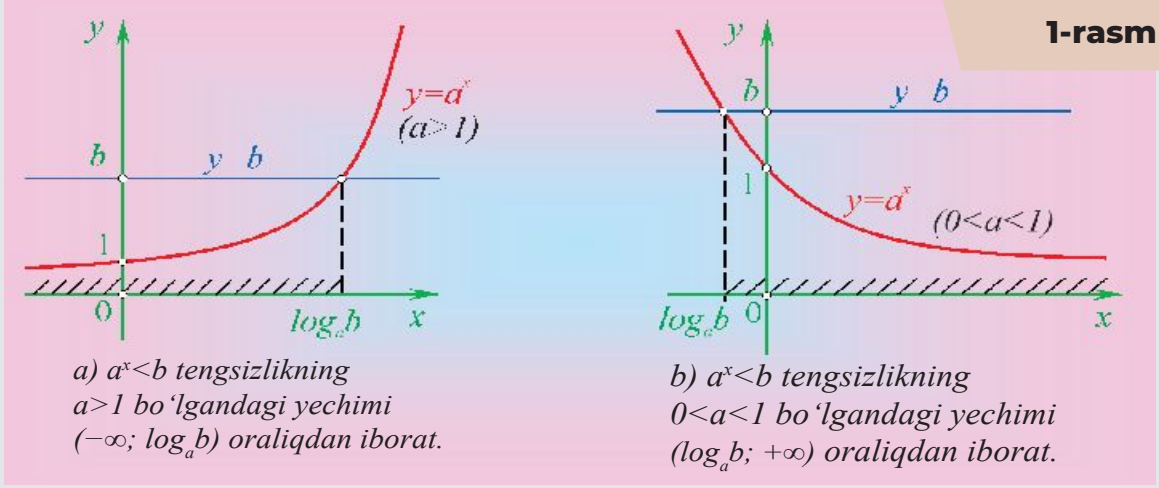
$0<a<1$ bo‘lganda, $y=a^x$ funksiyaning kamayuvchi ($x_2>x_1\Rightarrow a^{x_2}<a^{x_1}$) bo‘lishidan foydalaniladi.

$a^x<b$ tengsizlikni qaraylik. Bu tengsizlikning yechimi x o‘zgaruvchining shunday qiymatlari to‘plamiki, bu qiymatlarda $y=a^x$ funksiyaning Oxy koordinatalar sistemasidagi grafigi $y=b$ to‘g‘ri chiziqdan pastda joylashgan bo‘ladi.

$a^x<b$ tengsizlikni yechish uchun avvalo $y=a^x$ funksiya grafigining $y=b$ to‘g‘ri chiziq bilan kesishish nuqtasining abssissasini bilish, ya’ni $a^x=b$ tenglamaning ildizini bilish muhim. Bu ildiz abssissa o‘qidagi $x=\log_a b$ qiymatdan iborat.

Endi $a^x<b$ tengsizlik yechimini topishning geometrik bayonini keltiramiz.

1-rasm



a) $a^x<b$ tengsizlikning $a>1$ bo‘lgandagi yechimi $(-\infty;\log_a b)$ oraliqdan iborat.

b) $a^x<b$ tengsizlikning $0<a<1$ bo‘lgandagi yechimi $(\log_a b;+\infty)$ oraliqdan iborat.

$a^x<b$ tengsizlik yechimining geometrik talqini.

Topshiriq. $a^x>b$, $a^x\le b$, $a^x\ge b$ tengsizliklar yechimlarining geometrik talqinlarini keltiring.

1-misol. $0{,}5^x<4$ tengsizlikni yeching.

Yechish. Avvalo, $0{,}5^x=4$ tenglikdan, ya’ni $x=\log_{0,5}4$ ekanligidan foydalanib,

$$x=\log_{2^{-1}}2^2=\frac{2}{-1}\log_2 2=-2$$

qiymatni topamiz. Berilgan tengsizlikdagi asos 1 dan kichik bo‘lgani uchun uning yechimi $\left(\log_{0,5}4,\ +\infty\right)$, ya’ni $(-2,\ +\infty)$ oraliqdan iborat bo‘ladi.

Javob: $(-2,\ +\infty)$.

Ko‘rsatkichli sodda tengsizliklarning turlari
Bir xil asosli ko‘rsatkichli sodda tengsizliklarni yechish

Ushbu $a^{f(x)} < a^{g(x)}$ ko‘rsatkichli sodda tengsizlikning yechimi:
$0 < a < 1$ bo ‘lganda $f(x) > g(x)$ tengsizlikning yechimidan; $a > 1$ bo ‘lganda $f(x) < g(x)$ tengsizlikning yechimidan iborat bo‘ladi.

$a^{f(x)} \le a^{g(x)}$, $a^{f(x)} > a^{g(x)}$ va $a^{f(x)} \ge a^{g(x)}$ tengsizliklarning yechilishi quyidagi jadvalda keltirigan.

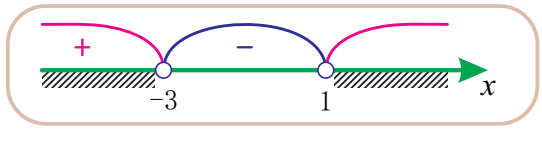
Ko‘rsatkichli tengsizliklar turlari	$a^{f(x)} \le a^{g(x)}$	$a^{f(x)} < a^{g(x)}$	$a^{f(x)} > a^{g(x)}$	$a^{f(x)} \ge a^{g(x)}$
	tengsizlikning yechimi			
$0 < a < 1$ bo‘lganda	$f(x) \ge g(x)$	$f(x) > g(x)$	$f(x) < g(x)$	$f(x) \le g(x)$
$a > 1$ bo‘lganda	$f(x) \le g(x)$	$f(x) < g(x)$	$f(x) > g(x)$	$f(x) \ge g(x)$

2-misol. $3^{x^2+2x} > 3^3$ tengsizlikni yeching.

Yechish. Berilgan tengsizlikda asos 1 dan katta. Shuning uchun $x^2 + 2x > 3$ tengsizlikning yechimini topish kifoya qiladi. $x^2 + 2x > 3$ tengsizlikni intervallar usulidan foydalanib yechamiz:

$$x^2 + 2x - 3 > 0,$$

$$(x+3)(x-1) > 0,$$



$$x \in (-\infty, -3) \cup (1, +\infty).$$

Javob: Berilgan ko‘rsatkichli tengsizlikning yechimi $(-\infty, -3) \cup (1, +\infty)$ oraliqlar birlashmasidan iborat.

Har xil asosli ko‘rsatkichli sodda tengsizliklarni yechish

$a > 0$, $a \ne 1$ va $b > 0$, $b \ne 1$ bo‘lganda ushbu $a^{f(x)} < b^{f(x)}$ ko‘rsatkichli tengsizlik $\left(\frac{a}{b}\right)^{f(x)} < 1$ yoki $\left(\frac{b}{a}\right)^{f(x)} > 1$ ko‘rsatkichli sodda tengsizlik ko‘rinishiga keltirilib yechiladi.

MISOLLAR

Tengsizliklarni yeching:

1. $4^x > 256$

2. $\left(\frac{1}{3}\right)^x \le \frac{1}{729}$

3. $7^x < -49$

4. $13^x > -169$

5. $5^x < 0$

6. $8^{2x} > 0$

7. $10^x \le 0$

8. $\left(\frac{1}{3}\right)^{-\frac{x}{2}} > \sqrt{3}$

9. $\left(\frac{1}{6}\right)^{\frac{2x}{15}} < \sqrt[5]{6}$

10. $2^x > \left(\frac{1}{2}\right)^{x+1}$

11. $3 \cdot 9^{2x-2} > \left(\frac{1}{27}\right)^{3x-1}$

12. $\left(\frac{1}{4}\right)^{2x-3} > 4^{1-2x}$

13. $2 \cdot 8^{4-5x} < \left(\frac{1}{16}\right)^{x+2}$

14. $\left(\frac{3}{4}\right)^{\frac{x+1}{x+2}} > \frac{\sqrt{3}}{2}$

15. $6 \cdot 2^{x+3} - 5 \cdot 2^{x+2} + 4 \cdot 2^x > 128$

16. $7 \cdot 3^{x+4} + 2 \cdot 3^{x+3} - 5 \cdot 3^{x+2} \le 192$

17. $10 \cdot 3^{x+2} - 4 \cdot 10^{x+2} < 3^{x+4} - 3 \cdot 10^{x+2}$

18. $5^{x+2} - 5^{x+1} > 2^{x+2} + 2^{x+4}$

19. $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$

20. $\left(\frac{1}{25}\right)^{2x} < \left(\sqrt{5}\right)^{x^2+2.75}$

21. $\left(\frac{1}{16}\right)^{x^2} < 8 \cdot \sqrt{2}^{16-2x}$

22. $2^{x^2} > \left(\frac{1}{2}\right)^{2x-3}$

23. $0{,}04^x - 26 \cdot (0{,}2)^x + 25 \le 0$

24. $25^x - 4 \cdot 5^x - 5 \ge 0$

25. $4^x - 10 \cdot 2^x + 16 < 0$

26. $3^{2x+1} + 1 < 4 \cdot 3^x$

27. n ning nechta natural qiymati $9 \le 3^n \le 79$ qo‘sh tengsizlikni qanoatlantiradi?

28. x ning qanday qiymatlarida $y = 5x - 5$ funksiya musbat qiymatlarni qabul qiladi?

29. $3^{8x} - 4 \cdot 3^{4x} \le -3$ tengsizlikning butun yechimlari yig‘indisini toping.

30. $x^2 \cdot 3^x - 3^{x+1} \le 0$ tengsizlikning butun sonlardan iborat yechimlari nechta?

31. $\left(\frac{4}{9}\right)^x \left(\frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{-2x}$ tengsizlikning eng katta butun yechimini toping.

LOGARIFMIK TENGSIZLIKLAR

Logarifmik tengsizliklar

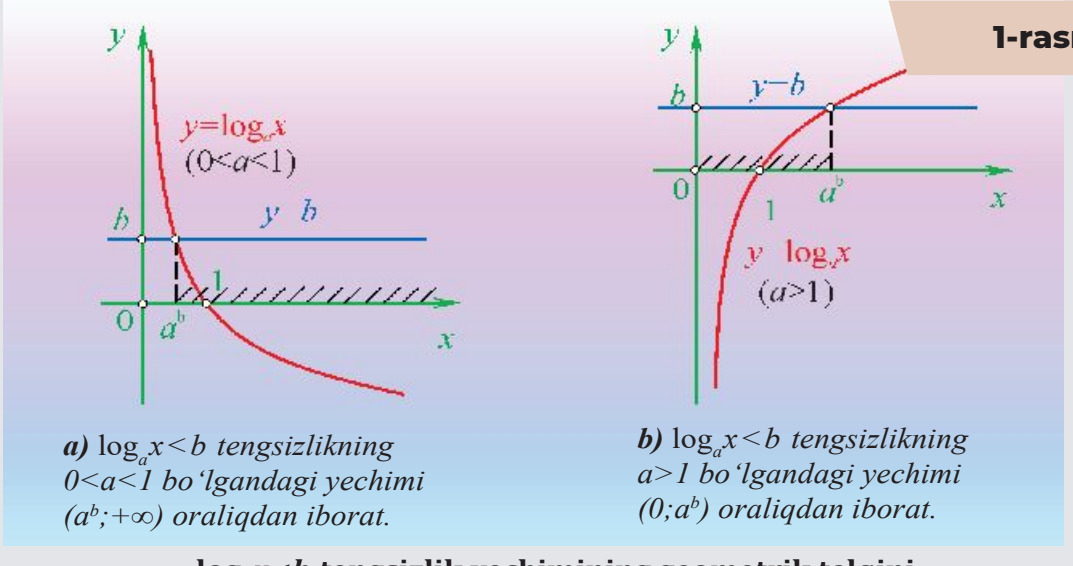
$a>0$ va $a\neq 1$ bo‘lsin. U holda

$\log_a x<b$, $\log_a x>b$, $\log_a x\le b$, $\log_a x\ge b$

tengsizliklar logarifmik eng sodda tengsizliklar bo‘ladi. Ularni yechishda $y=\log_a x$ funksiyaning monotonligidan foydalaniladi.

$\log_a x<b$ tengsizlikni qaraylik. Bu tengsizlikning yechimi x o‘zgaruvchining shunday qiymatlari to‘plamiki, bu qiymatlarda $y=\log_a x$ funksiyaning Oxy koordinatalar sistemasidagi grafigi $y=b$ to‘g‘ri chiziqdan pastda joylashgan bo‘ladi.

1-rasm



a) $\log_a x<b$ *tengsizlikning* $0<a<1$ *bo‘lgandagi yechimi* $(a^b;+\infty)$ *oraliqdan iborat.*

b) $\log_a x<b$ *tengsizlikning* $a>1$ *bo‘lgandagi yechimi* $(0;a^b)$ *oraliqdan iborat.*

$\log_a x<b$ tengsizlik yechimining geometrik talqini.

$\log_a x>b$, $\log_a x\le b$, $\log_a x<b$ $\log_a x\ge b$ tengsizliklar yechimlarining geometrik talqinlarini keltiring.

Logarifmik sodda tengsizliklarni yechish

Ushbu $\log_a f(x)<\log_a g(x)$ logarifmik sodda tengsizlikning yechimi:

$0<a<1$ bo ‘lganda $\begin{cases} f(x)>g(x), \\ g(x)>0 \end{cases}$

sistemaning yechimidan;

$a>1$ bo ‘lganda $\begin{cases} f(x)<g(x), \\ f(x)>0 \end{cases}$ sistemaning yechimidan iborat bo‘ladi.

$\log_a f(x)\le\log_a g(x)$, $\log_a f(x)>\log_a g(x)$ va $\log_a f(x)\ge\log_a g(x)$ tengsizliklarning yechilishi quyidagi jadvalda keltirigan.

Logarifmik tenglamalar turi	$\log_a f(x) \le \log_a g(x)$	$\log_a f(x) < \log_a g(x)$	$\log_a f(x) > \log_a g(x)$	$\log_a f(x) \ge \log_a g(x)$
$0 < a < 1$ bo‘lganda	$\begin{cases} f(x) \ge g(x), \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) < g(x), \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) < g(x), \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) \le g(x), \\ f(x) > 0 \end{cases}$
$a > 1$ bo‘lganda	$\begin{cases} f(x) \le g(x), \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) < g(x), \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) > g(x), \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) \ge g(x), \\ g(x) > 0 \end{cases}$
sistemaning yechimidan iborat bo‘ladi.				

MISOLLAR

1. $\log_2 x > 3$

2. $\log_{\frac{1}{5}}(x-5) > -2$

3. $\log_3(x+20) < 3$

4. $\log_4(5-x^2) > 1$

5. $\log_3(4x+2) - \log_3 10 < 0$

6. $\log_5(3x-2x^2) > 0$

7. $\log_2(x-1) < \log_2(3x-1)$

8. $\log_7(2-x) > \log_7(x+6)$

9. $\log_{\frac{1}{3}}(2x-4) \ge \log_{\frac{1}{3}}(x+1)$

10. $\log_{\frac{1}{2}}(x^2-5x-6) \ge -3$

11. $\lg^2 x + 11\lg x + 10^3 < 0$

12. $\log_2^2 x - 6\log_2 x + 8 \le 0$

13. $\log^2_{\frac{1}{2}} x - 9 \le 0$

14. $5^{x-2} > 3$

15. $\log_2 \log_{\sqrt{2}}(x+1) < 1$

16. $\log_8 64 > \log_{\frac{1}{5}} x$

17. $\log_{\frac{4}{3}}(x+6) - \log_{\frac{4}{3}} 9 < \log_{\frac{4}{3}} 2 - \log_{\frac{4}{3}} 6$

18. $5^{\log_5(x-7)} < 4$

19. $2\log_{\frac{1}{5}}(x-2) + 3\log_5(x-2) < 1$

20. $\log_3 x > 4$

21. $\log_{0,5} x > 2$

22. $\log_5 x > 3$

23. $\log_2 8 > x$

24. $\log_3 x \ge \log_6 36$

25. $\log_2 x < \log_{49} 7$

26. $\log_x x^2 + x > 1$

27. $\lg(x+2) + \lg(x-3) \le \lg x^2$

28. $\log_2(4-x) - \log_2 7 < 0$ tengsizlikni qanoatlantiradigan butun sonlar nechta?

29. $\log_{0,2}(x^4+2x^2+1) > \log_{0,2}(6x^2+1)$ tengsizlikning barcha manfiy yechimlari to‘plamini toping.

KO‘RSATKICHLI MODELLASHTIRISH. MURAKKAB FOIZ FORMULASI VA TATBIQLARI. O‘SISH HODISALARI. RADIOAKTIV YEMIRILISH

Murakkab foiz formulasi va uning tatbiqlari

Aytaylik, biror Q_0 miqdordagi pul qarz olinmoqchi. Qarz beruvchi belgilangan muddatda dastlabki miqdorni biror P foyda bilan qaytarishni talab etishi mumkin. Demak, ko‘rsatilgan muddatda qarz oluvchi qaytaradigan miqdor

$$Q_1 = Q_0 + P$$

bo‘ladi. Muddat sifatida bir kun, ikki kun, ..., bir hafta, ikki hafta, ..., bir oy, ikki oy va hokazo olinishi mumkin. Bunda ushbu

$$p = \frac{P}{Q_0} \cdot 100\% .$$

kattalik olingan qarzni o‘z muddatida qaytarish foizi deyiladi.

1. Oddiy foiz formulasi. Agar foiz faqatgina olingan Q_0 miqdorga qo‘llanilsa, birinchi muddat oxirida qarz miqdori

$Q_1 = Q_0 + \frac{p}{100} \cdot Q_0 = \left(1 + \frac{p}{100}\right) Q_0$ bo‘ladi. Bunda

$$P_1 = \frac{p}{100} \cdot Q_0 = P$$

– qarz beruvchining birinchi muddat oxiridagi foydasi. Ikkinchi muddat oxirida qarz miqdori

$$Q_2 = P_1 + \left(1 + \frac{p}{100}\right) Q_0 = \frac{p}{100} \cdot Q_0 + \left(Q_0 + \frac{p}{100} \cdot Q_0\right) = \left(1 + \frac{2p}{100}\right) Q_0$$

bo‘lib, qarz beruvchining ikkinchi muddat oxiridagi foydasi

$$P_2 = \frac{2p}{100} \cdot Q_0 \quad (\text{ravshanki, } P_2 = 2P)$$

bo‘ladi. Uchinchi muddat oxirida qarz miqdori

$$Q_3 = P_2 + \left(1 + \frac{p}{100}\right) Q_0 = \frac{2p}{100} \cdot Q_0 + \left(Q_0 + \frac{p}{100} \cdot Q_0\right) = \left(1 + \frac{3p}{100}\right) Q_0$$

bo‘lib, qarz beruvchining uchinchi muddat oxiridagi foydasi

$$P_3 = \frac{3p}{100} \cdot Q_0 \quad (\text{ravshanki, } P_3 = 3P)$$

bo‘ladi. Bu jarayonni n marta takrorlab, n-chi muddat oxirida qarz miqdori

$$Q_n = \left(1 + \frac{np}{100}\right) Q_0$$

bo‘lishi, qarz beruvchining n-chi muddat oxiridagi foydasi

$$P_n = \frac{np}{100} \cdot Q_0 \quad (\text{ravshanki, } P_n = nP)$$

bo‘lishi topiladi. Bunday hisoblanadigan foiz oddiy foiz,

$$Q_n = \left(1 + \frac{np}{100}\right) Q_0$$

formula esa ***oddiy foiz formulasi*** deyiladi.

2. Murakkab foiz formulasi. Foizni olingan qarzga hosil bo‘lgan foydani qo‘shib qo‘llash mumkin. Bunda birinchi muddat oxirida qarz miqdori

$$Q_1 = Q_0 + \frac{p}{100}\cdot Q_0 = \left(1+\frac{p}{100}\right)Q_0$$

bo‘ladi. Bunda

$$P_1 = \frac{p}{100}\cdot Q_0 = P$$

– qarz beruvchining birinchi muddat oxiridagi foydasi. Ikkinchi muddat oxirida qarz miqdori

$$Q_2 = \left(1+\frac{p}{100}\right)Q_1 = \left(1+\frac{p}{100}\right)^2 Q_0$$

bo‘lib, qarz beruvchining ikkinchi muddat oxiridagi foydasi

$$Q_2 = \left(1+\frac{p}{100}\right)Q_1 = \left(1+\frac{p}{100}\right)^2 Q_0$$

bo‘ladi. Uchinchi muddat oxirida qarz miqdori

$$Q_3 = \left(1+\frac{p}{100}\right)Q_2 = \left(1+\frac{p}{100}\right)^3 Q_0$$

bo‘lib, qarz beruvchining uchinchi muddat oxiridagi foydasi

$$P_3 = Q_3 - Q_0 = \left(\left(1+\frac{p}{100}\right)^3 - 1\right)Q_0$$

bo‘ladi. Bu jarayonni n marta takrorlab, n-chi muddat oxirida qarz miqdori

$$Q_n = \left(1+\frac{p}{100}\right)^n Q_0$$

bo‘lishi, qarz beruvchining n-chi muddat oxiridagi foydasi

$$P_n = Q_n - Q_0 = \left(\left(1+\frac{p}{100}\right)^n - 1\right)Q_0$$

bo‘lishi topiladi. Bunday hisoblanadigan foiz murakkab foiz,

$$Q_n = \left(1+\frac{p}{100}\right)^n Q_0$$

formula esa ***murakkab foiz formulasi*** deyiladi.

Oddiy va murakkab foiz formulalari ishlatilishiga oid ko‘plab amaliy misol va masalalar uchraydi. Hozirgi kunda kredit, ipoteka qarzi kabi iboralar ko‘plab uchraydi. Ipoteka qarzini hisoblash bo‘yicha masala yechish namunasini keltiramiz.

Odatda qarz beruvchi bank, qarz oluvchi esa mijoz shaklida namoyon bo‘ladi. Banklar uy-joy olishda (ipoteka), transport vositasi, yoki xo‘jalik mollari (televizor, sovutgich, uyali aloqa telefoni va boshqalar) olishda (kredit) qarzni bir necha yilgacha bo‘lgan uzoq muddatga beradi va mijozdan har oyda qarzning ma’lum miqdorini to‘lab borishni talab etadi.

1-misol. Dastlabki narxi 360 000 000 so‘m bo‘lgan xonadonni yosh oila yillik 20% bilan 15 yilga ipoteka qarzi orqali oldi. 15 yil davomida bankka qancha mablag‘ qaytariladi? Bunda bank qancha foyda ko‘radi?

Yechish. Dastlabki 360 000 000 so‘m mablag‘ bank tilida “ona” pul deyiladi. 1 yil 12 oydan iborat ekanligini ta’kidlab o‘tamiz. Shuning uchun mijoz bankka har oyda ona pulning

$\frac{360\ 000\ 000}{15\cdot 12}=2\ 000\ 000$ so‘m miqdorini qaytarishi kerak. Qaytarishning birinchi oyida paydo bo‘ladigan foiz quyidagicha topiladi:

$$a_1=360\ 000\ 000\cdot\frac{20\%}{100\%}\cdot\frac{1}{12}=6\ 000\ 000 \text{ so‘m.}$$

Demak, mijoz birinchi oy oxirida jami $2\ 000\ 000+6\ 000\ 000=8\ 000\ 000$ so‘m qaytarishi kerak. Shundan so‘ng qolgan pul $360\ 000\ 000-2\ 000\ 000=358\ 000\ 000$ so‘m bo‘ladi. Ikkinchi oy oxirida mijoz "ona" pulning 2 000 000 so‘m miqdorini va paydo bo‘lgan ushbu $360\ 000\ 000-2\ 000\ 000=358\ 000\ 000$ so‘m foiz miqdorini, jami esa

$$2\ 000\ 000+5\ 966\ 667=7\ 966\ 667 \text{ so‘m mablag‘ni qaytarishi lozim.}$$

$(n-1)$ chi oyning mablag‘i to‘langach, $360\ 000\ 000-2\ 000\ 000\cdot(n-1)$ so‘m miqdorda "ona" pul qoladi. n-chi oy oxirida mijoz bankka $2\ 000\ 000+a_n$ miqdorda pul to‘laydi. Bu yerda a_n n-chi oyning foizi bo‘lib, $a_n=\left(360\ 000\ 000-2\ 000\ 000\cdot(n-1)\right)\cdot\frac{20\%}{100\%}\cdot\frac{1}{12}=(181-n)\cdot\frac{100\ 000}{3}$ tenglik orqali topiladi. Ko‘rinib turibdiki, a_1, a_2, ... , a_n kamayuvchi arifmetik progressiya bo‘lib, uning ayirmasi $d=a_n-a_{n-1}=(181-n)\cdot\frac{100\ 000}{3}-\left(181-(n-1)\right)\cdot\frac{100\ 000}{3}=-\frac{100\ 000}{3}$, ya’ni $d=-\frac{100\ 000}{3}$ ga teng.

Ravshanki, 15 yil 180 oydan iborat, shuning uchun $1\le n\le 180$ bo‘ladi. Demak, $a_1=6\ 000\ 000$, $a_{180}=\frac{100\ 000}{3}$ bo‘lib, arifmetik progressiyaning 180 ta hadining yig‘indisi

$$S_{180}=\frac{a_1+a_{180}}{2}\cdot 180=\frac{6\ 000\ 000+\frac{100\ 000}{3}}{2}\cdot 180=$$

$$=\frac{18\ 000\ 000+100\ 000}{3}\cdot 90=18\ 100\ 000\cdot 30=543\ 000\ 000$$

bo‘ladi. Demak, mijoz bankka jami $360\ 000\ 000+543\ 000\ 000=903\ 000\ 000$ so‘m qaytarishi kerak ekan. Bunda bankning foydasi 543 000 000 so‘m bo‘ladi.

Radioaktiv yemirilish

Yarim yemirilish davri. Ayrim kimyoviy elementlar o‘z yadrolaridan zarralar chiqarib turishadi. Bunday elementlar radioaktiv elementlar deb yuritiladi, ularning o‘z yadrolaridan zarra chiqarish jarayoni ***radioaktiv yemirilish*** deyiladi. Radioaktiv yemirilish natijasida dastlabki kimyoviy element boshqa kimyoviy elementga aylanib qoladi.

Uzoq yillik tajriba natijasida $T_1=T_2=T_3=\ldots=T_n=T_{n+1}=\ldots$ bo‘lishi isbotlangan. Demak, aynan bitta element massasining yarmi yemirilishi uchun ketadigan vaqt o‘zgarmas miqdor ekan. Bu miqdor elementning yarim yemirilish davri deyiladi va T orqali belgilanadi: $T=T_1=T_2=T_3=\ldots=T_n=T_{n+1}=\ldots$.

Natijada

$$t_1 = T_1 = T,$$
$$t_2 = T_1 + T_2 = 2T,$$
$$t_3 = T_1 + T_2 + T_3 = 3T, \ldots,$$
$$t_n = T_1 + T_2 + T_3 + \ldots + T_n = nT$$

tengliklar hosil bo‘ladi. Demak, dastlabki massasi m_0 bo‘lgan elementning $t_n = nT$ vaqtdan keyin yemirilmay qolgan qismining massasi $m_n = \frac{m_0}{2^n} = 2^{-n} m_0$ bo‘lar ekan. Bu yerda $n = \frac{t_n}{T}$ ekanligini e’tiborga olsak, $m_n = 2^{-\frac{t_n}{T}} m_0$ tenglikka ega bo‘lamiz. Bu formula ixtiyoriy t moment uchun ham o‘rinli:

$$m(t) = 2^{-\frac{t}{T}} m_0.$$

Shunday qilib, radioaktiv elementning yemirilmay qolgan qismining massasi vaqtning ko‘rsatkichli funksiyasi ekan.

2-misol. Sutkaning dastlabki 8 soatida radioaktiv moddaning aktivligi 4 marta kamaydi. Sutka davomida moddaning aktivligi necha marta kamayadi?

Yechish. Qaralayotgan moddaning dastlabki massasi m_0 bo‘lib, yarim yemirilish davri T bo‘lsin. 8 soatdan keyin uning massasi $m(8) = \frac{m_0}{4}$ bo‘lgan. Bu berilganlar uchun $m(t) = 2^{-\frac{t}{T}} m_0$ formulani qo‘llab, moddaning yarim yemirilish davri topiladi:

$$m(8) = 2^{-\frac{8}{T}} m_0, \quad \frac{m_0}{4} = 2^{-\frac{8}{T}} m_0, \quad 2^{\frac{8}{T}} = 2^2, \quad \frac{8}{T} = 2, \quad T = 4 \text{ soat.}$$

Endi $m(t) = 2^{-\frac{t}{T}} m_0$ formulani yana bir bor $t = 24$ (bir sutka = 24 soat) uchun ishlatib, radioaktiv moddaning aktivligi sutka davomida necha marta kamayganligi topiladi:

$$m(24) = 2^{-\frac{24}{T}} m_0 = 2^{-6} m_0 = \frac{m_0}{64}.$$

Demak, sutka davomida radioaktiv moddaning aktivligi 64 marta kamayadi.

MISOLLAR

1. Dastlabki narxi 360 000 000 so‘m bo‘lgan xonadonni yillik 18% bilan 20 yilga ipoteka qarzi orqali olgan oila muddat oxirida bankka qancha mablag‘ qaytargan bo‘ladi? Bank foydasi qancha bo‘ladi?

2. Yillik 8% bilan 3 yil muddatga 5000 AQSh dollari bo‘yicha murakkab foizlarni toping.

3. Vohid 50 mln so‘m qarz oldi va birinchi, ikkinchi va uchinchi yil uchun mos ravishda 10%, 12% va 14% stavkada foiz to‘lashga rozi bo‘ldi. 3 yildan keyin to‘lashi kerak bo‘lgan umumiy miqdorni toping.

4. Bir kishi bankka 100 mln qo‘ygan. Buning evaziga u 1,331 mln so‘m oldi. Bank yiliga 10% foiz berdi. U pulni qancha vaqt bankda saqlagan?

5. Omonatchi 26 million so‘mni bank hisobiga o‘tkazdi. 18 oydan keyin uning hisobida 32 mln so‘m bo‘ldi. Yillik foiz stavkasi nimaga teng?

6. Menda 400 dollar bor. Do‘stim menga bankka sarmoya kiritishni taklif qildi. Men yillik 13% xorijiy valyutadagi hisob raqamiga va har oyda 1% to‘ldiriladigan summa hisobiga sarmoya kiritdim.

A) Agar valyuta hisob raqamiga pul kiritsam, bir yilda qancha olaman?

B) Agar men bu pulni so‘mga to‘liq aylantirib, summa hisobiga qo‘ygan bo‘lsam, bir yilda qanday miqdorda dollar olaman? Dollar va so‘m kursi o‘zgarishini hisobga oling.

Qo‘shilgan qiymat solig‘i

Qo‘shilgan qiymat solig‘i qisqacha QQS deb nomlanadi.

Siz soliq tushunchasi bilan tanishsiz. Tovarlarni ishlab chiqaruvchi yoki import qiluvchi (ulgurji sotuvchi yoki chakana sotuvchi) davlatga savdo solig‘ini to‘lashi kerak. Qo‘shilgan qiymat solig‘i – ishlab chiqaruvchidan tortib to chakana sotuvchiga qadar ta’minot zanjirining ko‘p nuqtalarida hukumat tomonidan amalga oshiriladigan soliq. Har bir bosqichda faqat tovarga qo‘shilgan qiymat savdo solig‘iga tortiladi. Savdo solig‘ining yakuniy holati iste'molchida qoladi.

Bu asl ishlab chiqaruvchidan sotuvchiga tovarlarning har bir o‘tkazilishida qo‘shilgan qiymatga soliq.

Aytaylik, QQS stavkasi 10% va tadbirkor 8 000 000 so‘mga mahsulot sotib oldi, u to‘laydigan soliq = 8 000 000 so‘mning 10 foizi = 800 000 so‘m.

Endi, agar u xuddi shu mahsulotni 11 500 000 so‘mga sotsa, undan undiradigan soliq = 11 500 000 ning 10% = 1 150 00 so‘m.

Tadbirkor uchun QQS = 1 150 000 – 800 000 = 350 000 so‘m bo‘ladi.

MISOLLAR

7. Murod 10 mln so‘mga tovar sotib olsa 7% soliq to‘laydi. U xuddi shu tovarni 13 mln so‘mga sotsa 9% soliq oladi. Murod tomonidan to‘lanadigan QQSni toping.

8. Tadbirkor buyumni 7 500 000 ga sotsa xaridordan 12% stavkada savdo solig‘i oladi. Agar u 180 000 so‘m miqdorida QQS to‘lasa, tadbirkor tomonidan to‘langan soliqni hisobga olgan holda dastlabki narxni hisoblang.

9. Ishlab chiqaruvchi o‘z mahsulotining narxini har biri uchun 12 000 000 deb e’lon qildi. U ulgurji sotuvchiga 30% chegirmaga ruxsat berdi, ulgurgi sotuvchi esa, o'z navbatida chakana sotuvchiga e’lon qilingan narxdan 20% chegirmaga ruxsat berdi. Agar tovar uchun belgilangan savdo solig‘i stavkasi 10% bo‘lsa va chakana sotuvchi uni iste’molchiga e’lon qilingan narxda sotsa, ulgurji va chakana sotuvchi tomonidan to‘langan qo‘shilgan qiymat solig‘ini toping.

10. Chakana sotuvchi ulgurji sotuvchidan buyumni 80 000 so‘mga sotib oldi va ulgurji sotuvchi belgilangan 8% miqdorida savdo solig‘ini oladi. Chakana sotuvchi narxni 100 000 so‘m qilib belgilab qo‘ydi va xuddi shu stavkada savdo solig‘ini iste’molchidan undiradi. Chakana sotuvchi davlatga qancha QQS to‘laydi?



4-BOB TRIGONOMETRIK FUNKSIYALAR

- Trigonometrik funksiyalar. y=sinx, y=cosx, y=tgx, y=ctgx funksiyalar va ularning xossalari, grafigi. Davriy jarayonlar

- Teskari trigonometrik funksiyalar. arcsina, arccosa, arctga, arcctga ning qiymatlari. y=arcsinx, y=arccosx, y=arctgx, y=arcctgx funksiyalar va ularning xossalari, grafigi

TRIGONOMETRIK FUNKSIYALAR VA XOSSALARI, GRAFIGI. DAVRIY JARAYONLAR

Trigonometrik funksiyalar. Davriy jarayonlar

Tabiatda, texnikada, ishlab chiqarishda va boshqa sohalarda vaqt o‘tishi bilan takrorlanadigan hodisa va jarayonlar ko‘plab uchraydi. Masalan, quyosh chiqishi, fasllar almashinuvi, ichki yonuv dvigatelida porshen harakati va boshqalar vaqt o‘tishi bilan takrorlanadi. Bunday jarayonlar ***davriy jarayonlar*** deb ataladi. Davriy jarayonlar trigonometrik funksiyalar orqali tavsiflanadi.

Trigonometrik funksiyalarni o‘rganishda:

1) burchak kattaligining gradus o‘lchovini;

2) 1° burchakning 60 dan bir qismi 1 *minut* (belgilanishi $1'$), $1'$ ning 60 dan bir qismi 1 *sekund* (belgilanishi $1''$) ekanligini, ya’ni

$$1' = \frac{1^\circ}{60},\ 1'' = \frac{1'}{60} = \frac{1^\circ}{3600}$$

tengliklarni;

3) burchak kattaligining radian o‘lchovini;

4) radian birliksiz kattalik ekanligini;

5) burchakning radian o‘lchovidan gradus o‘lchoviga o‘tish

$$\boxed{\alpha^\circ = \frac{180^\circ}{\pi} \cdot \alpha}$$

formulasini;

6) burchakning gradus o‘lchovidan radian o‘lchoviga o‘tish

$$\boxed{\alpha = \frac{\pi}{180^\circ} \cdot \alpha^\circ}$$

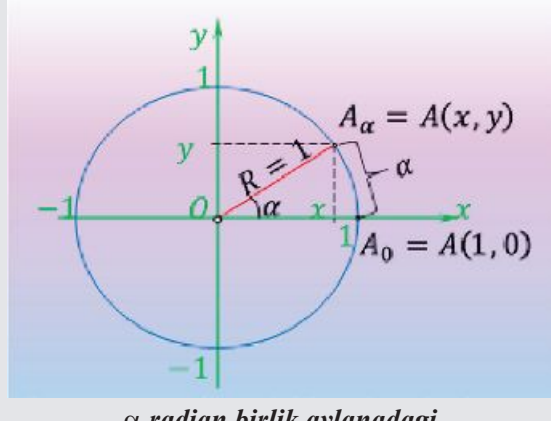
formulasini;

7) keltirish formulalarini bilish talab etiladi.

Burchakning sinusi, kosinusi, tangensi va kotangensi

Oxy Dekart koordinatalar sistemasi kiritilgan tekislikda markazi koordinatalar boshida bo‘lgan **birlik aylana** (ya’ni radiusi 1 ga teng aylana)ni qaraymiz. $A_0 = A(1,0)$ nuqtani tayinlab olamiz. Aylanada A_0 nuqtadan soat strelkasi harakatiga qarshi (ya’ni **musbat**) yo‘nalishda uzunligi α ga teng yoy ajratib olamiz va uning oxirini A_α orqali belgilaymiz (1-rasm). Burchak kattaligining radian o‘lchovi aniqlanishiga ko‘ra A_0OA_α burchakning kattaligi α radianga teng bo‘ladi:

1-rasm



α radian birlik aylanadagi uzunligi α bo‘lgan $\overset{\frown}{A_0A_\alpha}$ yoyning markaziy burchagining burchak kattaligidir.

$$\alpha = \angle A_0 O A_\alpha.$$

Diqqat qiling: A_α nuqta Oxy tekisligida biror koordinatalarga ega bo‘ladi.

Aytalik, A_α nuqtaning Oxy tekisligidagi koordinatalari (x, y) bo‘lsin.

Ta’rif:

1) x kattalik α burchakning *kosinusi* deyiladi va $\cos\alpha$ orqali belgilanadi;

2) y kattalik α burchakning *sinusi* deyiladi va $\sin\alpha$ orqali belgilanadi;

3) $\frac{y}{x}$ nisbat α burchakning *tangensi* deyiladi va $\operatorname{tg}\alpha$ orqali belgilanadi;

4) $\frac{x}{y}$ nisbat α burchakning *kotangensi* deyiladi va $\operatorname{ctg}\alpha$ orqali belgilanadi.

Demak, ta’rifga ko‘ra:

$$\cos\alpha = x, \quad \sin\alpha = y, \quad \operatorname{tg}\alpha = \frac{y}{x}, \quad \operatorname{ctg}\alpha = \frac{x}{y} \qquad (1)$$

bo‘ladi.

Eslatma! Agarda birlik aylana o‘rniga ixtiyoriy R radiusli aylana qaralsa, u holda

$$\cos\alpha = \frac{x}{R}, \quad \sin\alpha = \frac{y}{R}, \quad \operatorname{tg}\alpha = \frac{y}{x}, \quad \operatorname{ctg}\alpha = \frac{x}{y} \qquad (1')$$

tengliklar hosil bo‘ladi.

Ravshanki, aylanadagi A_0 nuqtani berilgan burchakka quyidagicha ikkita yo‘nalishda markaziy burish mumkin:



Musbat burish: burilish soat strelkasi harakatiga qarshi yo‘nalishda bajariladi.



Manfiy burish: burilish soat strelkasi harakati yo‘nalishi bo‘ylab bajariladi.

$y = \sin x$, $y = \cos x$, $y = tg\,x$, $y = ctg\,x$ funksiyalar va ularning xossalari, grafigi

Har bir x songa birlik aylanadagi A_0 nuqtadan boshlab x burchakka burishda hosil bo‘ladigan A_x nuqtani mos qo‘yaylik. U holda, aylanadagi A_x nuqta uchun $\sin x$, $\cos x$, $\operatorname{tg} x$, $\operatorname{ctg} x$ qiymatlarni hisoblash mumkin. Natijada x songa $\sin x$, $\cos x$, $\operatorname{tg}x$, $\operatorname{ctg}x$ qiymatlarni mos qo‘yuvchi va ***trigonometrik funksiyalar*** deb ataluvchi ushbu

$$y = \sin x, \quad y = \cos x, \quad y = \operatorname{tg} x, \quad y = \operatorname{ctg} x$$

funksiyalarga ega bo‘lamiz.

Bu funksiyalar davriy, ya’ni har bir $n \in \mathbb{Z}$ uchun quyidagi tengliklar o‘rinli bo‘ladi:

$$f(x + 2n\pi) = \sin(x + 2n\pi) = \sin x = f(x),$$
$$f(x + 2n\pi) = \cos(x + 2n\pi) = \cos x = f(x),$$
$$f(x + n\pi) = \operatorname{tg}(x + n\pi) = \operatorname{tg}x = f(x),$$

$$f(x+n\pi)=\operatorname{ctg}(x+n\pi)=\operatorname{ctg}x=f(x).$$

Demak, $y=\sin x$ va $y=\cos x$ funksiyalarning asosiy davri $T_0=2\pi$, hamda $y=\operatorname{tg}x$ va $y=\operatorname{ctg}x$ funksiyalarning asosiy davri $T_0=\pi$ ekan.

$y=\cos x$ funksiya juft:

$$f(-x)=\cos(-x)=\cos x=f(x).$$

$y=\sin x$, $y=\operatorname{tg}x$ va $y=\operatorname{ctg}x$ funksiyalar esa toq:

$$f(-x)=\sin(-x)=-\cos x=-f(x),$$
$$f(-x)=\operatorname{tg}(-x)=-\operatorname{tg}x=-f(x),$$
$$f(-x)=\operatorname{ctg}(-x)=-\operatorname{ctg}x=-f(x).$$

Ravshanki,

$D(\sin x)=(-\infty,+\infty)$, $E(\sin x)=[-1,1]$,

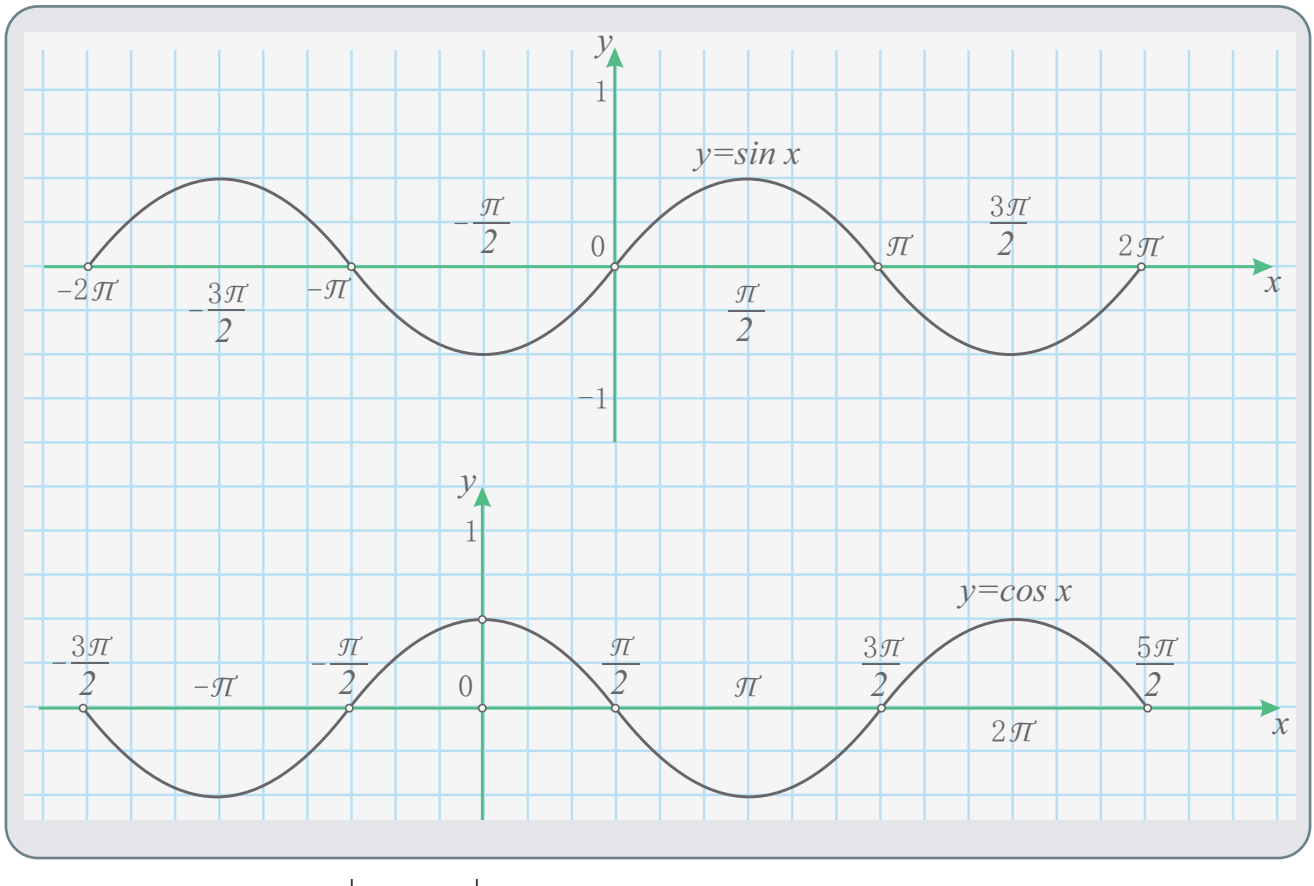
$D(\cos x)=(-\infty,+\infty)$, $E(\cos x)=[-1,1]$,

$D(\operatorname{tg}x)=\bigcup_{k=-\infty}^{+\infty}\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right)$, $E(\operatorname{tg}x)=(-\infty,+\infty)$,

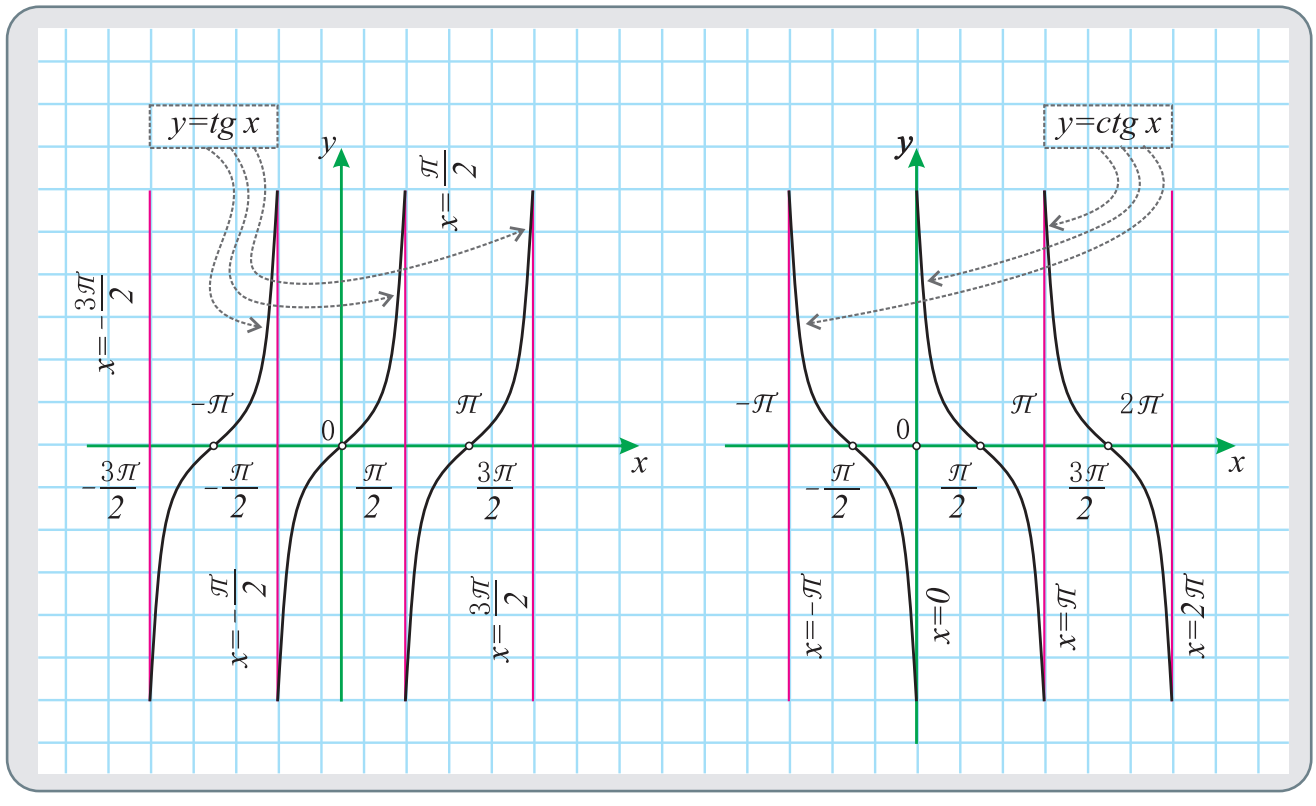
$D(\operatorname{ctg}x)=\bigcup_{k=-\infty}^{+\infty}(k\pi,(k+1)\pi)$, $E(\operatorname{ctg}x)=(-\infty,+\infty)$ bo‘ladi.

Quyidagi rasmlarda trigonometrik funksiyalar grafiklari keltirilgan.

Bu grafiklardan quyidagi muhim xulosalar kelib chiqadi:



1) $y=\sin x$ funksiya $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning



$[-1;1]$ kesmadagi yagona qiymati mos keladi;

2) $y=\cos x$ funksiya $[0;\pi]$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $[-1;1]$ kesmadagi yagona qiymati mos keladi;

3) $y=\operatorname{tg} x$ funksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning $(-\infty;+\infty)$ oraliqdagi yagona qiymati mos keladi;

4) $y=\operatorname{ctg} x$ funksiya $(0;\pi)$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $(-\infty;+\infty)$ oraliqdagi yagona qiymati mos keladi.

MISOLLAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y=\cos 3x$ b) $y=\sin\frac{2x-1}{5}$ c) $y=\sin\frac{1}{x+2}$ d) $y=\sin\sqrt{\frac{1-x}{x+3}}$

e) $y=tg3x$ f) $y=ctg\frac{2x}{5}$ g) $y=tg\frac{1}{x}$

2. Funksiyaning qiymatlar to‘plamini toping.

a) $y=-1+\cos x$ b) $y=2+\cos x$ c) $y=-3\sin 2x+2$ d) $y=3-4\cos 5x$

e) $y=-6\sin 3x\cos 3x$ f) $y=-5+\frac{1}{3}\cos x\sin x$ g) $y=5tg4x$

3. Funksiyaning juft yoki toqligini aniqlang.

a) $y=2xtgx$ b) $y=x^3-tg^3x$ c) $y=tgx\sin^2 x$ d) $y=tg2x+2\sin x$

e) $y=x^2+tg^2x$ f) $y=tg10|x|$ g) $y=\frac{x^2+\cos x}{2}$ h) $y=\frac{\sin x+\cos x}{x+5}$

4. $y=\sin x$ funksiya grafigidan foydalanib, quyidagi funksiyalarning grafiklarini yasang.

a) $y=-\sin x$ b) $y=2\sin x$ c) $y=-0,5\sin x$ d) $y=|\sin x|$

e) $y=\sin(x-\frac{\pi}{3})$ f) $y=|\sin|x||$ g) $y=1+\sin x$ h) $y=\sin 2x$

5. $y=\cos x$ funksiya grafigidan foydalanib, quyidagi funksiyalarning grafiklarini yasang.

a) $y=-\cos x$ b) $y=0,5\cos x$ c) $y=2\cos x$ d) $y=|\cos x|$

e) $y=\cos(x+\frac{\pi}{6})$ f) $y=|\cos|x||$ g) $y=2-\cos x$ h) $y=\cos 4x$

6. Funksiya grafigini yasang.

a) $y=tg2x$ b) $y=ctg\frac{x}{2}$ c) $y=2tgx$ d) $y=\frac{1}{3}ctgx$

7. Funksiyaning juft yoki toqligini aniqlang.

a) $y=\frac{\cos 2x-\sin^2 x}{x^2}$ b) $y=ctg3x+5\sin x$ c) $y=\sin 5x$

d) $y=2\sin^2 x$ e) $y=\sin^2 x+\sin x$ f) $y=5\sin^3 x-2\sin x$

8. $f(x)$ funksiya $(-\infty;\infty)$ oraliqda aniqlangan bo‘lsin:

a) $f(x)+f(-x)$ juft funksiya ekanini ko‘rsating;

b) $f(x)-f(-x)$ juft funksiya ekanini ko‘rsating.

9. Funksiyaning eng kichik musbat davrini toping.

a) $f(x)=\cos(3x+1)$ b) $f(x)=\sin(\frac{x}{4}-3)$ c) $f(x)=tg(2x+1)$

d) $f(x)=\sin 2\pi x$ e) $f(x)=\cos\sqrt{3x}$ f) $f(x)=tg(4\pi x-3)$

10. Berilgan $f(x)$ funksiyaning eng kichik musbat davrini toping:

a) $f(x)=\sin\frac{3x}{2}+tg7x$ b) $f(x)\cos x+2\sin(\frac{3x}{5}+\frac{\pi}{6})$

c) $f(x)=ctg(x-1)-3\sin 3x$ d) $f(x)=\sin 3x+\cos\frac{3x}{4}+\frac{1}{2}tg\frac{9x}{5}$

11. $T=-5\pi$ soni $f(x)=\sin 6x$ funksiyaning davri bo‘lishini ko‘rsating.

12. $T=\pi$ soni $f(x)=\sqrt{\sin 2x-1}$ funksiyaning davri bo‘lishini ko‘rsating.

13. Quyidagi funksiyalardan qaysilarini eng kichik musbat davri π ga teng:

a) $y=\sin x$ b) $y=\cos x$ c) $y=tgx$ d) $y=ctgx$

14. Funksiya grafigini yasang:

a) $y=\cos x+|\cos x|$ b) $y=\frac{\sin x}{\sin x}$ c) $y=\frac{|x|}{x}\cdot\cos x$ d) $y=\frac{1}{\sin x}$

TESKARI TRIGONOMETRIK FUNKSIYALAR VA XOSSALARI, GRAFIGI

Teskari trigonometrik funksiyalar

Kundalik hayotimizda inshootlarni, ko‘priklarni, transport vositalari, elektr stansiyalari, samolyot va boshqa qurilmalarni vayronaga aylantiruvchi rezonans hodisasi uchrab turadi. Rezonans hodisasi davriy jarayonlarning o‘zaro uyg‘unlashuvi natijasida kuzatiladi. Bunday hollarning oldini olish uchun trigonometrik funksiyalar berilgan qiymatni argumentning qanday qiymatida qabul qilishini, ya’ni teskari trigonometrik funksiyalarni bilish lozim.

Teskari trigonometrik funksiyalarni o‘rganishda:

1) trigonometrik funksiyalarning davriyligini va ularning asosiy davrlarini;

2) $y=\sin x$ funksiya $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda o‘sishi va bu oraliqdan olingan har bir x ga y ning $[-1;1]$ kesmadagi yagona qiymati mos kelishini;

3) $y=\cos x$ funksiya $[0;\pi]$ oraliqda kamayishi va bu oraliqdan olingan har bir x ga y ning $[-1;1]$kesmadagi yagona qiymati mos kelishini;

4) $y=\operatorname{tg} x$ funksiya $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda o‘sishi va bu oraliqdan olingan har bir x ga y ning $(-\infty;+\infty)$ oraliqdagi yagona qiymati mos kelishini;

5) $y=\operatorname{ctg} x$ funksiya $(0;\pi)$ oraliqda kamayishi va bu oraliqdan olingan har bir x ga y ning $(-\infty;+\infty)$ oraliqdagi yagona qiymati mos kelishini **bilish talab etiladi.**

$y=\arcsin x$ funksiya va uning xossalari, grafigi

$$y=\sin x$$

tenglama $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu ildiz

$$x=\arcsin y$$

ko‘rinishda yoziladi. Bu tenglik bilan $[-1;1]$ to‘plamning har bir y elementiga $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ to‘plamning yagona x elementini mos qo‘yuvchi arksinus funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

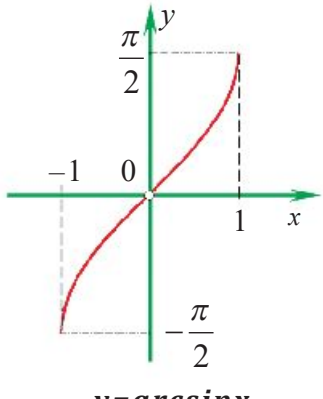
$$y=\arcsin x$$

ko‘rinishda yozamiz (1-rasm).

$y=\arcsin x$ funksiya $y=\sin x$ funksiyaga teskari funksiya bo‘ladi:

$$\sin(\arcsin a)=a,\ a\in[-1;1],$$

1-rasm



***y=arcsinx* funksiyaning grafigi.**

$$\arcsin(\sin\alpha)=\alpha,\quad \alpha\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right].$$

y = arcsin x funksiya quyidagi xossalarga ega:

- $D(y)=[-1;1]$;
- $E(y)=\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;
- $y=\arcsin x$ – o‘suvchi funksiya;
- $y=\arcsin x$ funksiyaning eng katta qiymati $\frac{\pi}{2}$ ga, eng kichik qiymati $-\frac{\pi}{2}$ ga teng;
- $y=\arcsin x$ funksiya grafigi koordinata boshidan o‘tadi;
- $y=\arcsin x$ – toq funksiya, ya’ni $\arcsin(-x)=-\arcsin x$;
- $y=\arcsin x$ funksiya davriy emas.

1-misol. $\arcsin\frac{\sqrt{3}}{2}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $\arcsin\frac{\sqrt{3}}{2}=x$ bo‘lsin. U holda berilgan topshiriqni boshqacha qo‘yish mumkin: $\sin x=\frac{\sqrt{3}}{2}$ tenglikni qanoatlantiruvchi x ning $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqdagi qiymatini toping. Ma’lumki, $\sin x=\frac{\sqrt{3}}{2}$ tenglik $x=\frac{\pi}{3}$ bo‘lganda bajariladi. Demak, $\arcsin\frac{\sqrt{3}}{2}=\frac{\pi}{3}$.

Quyidagi jadvalda $\arcsin a$ ifodaning ayrim qiymatilari keltirilgan.

a	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arcsin a$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

y = arccos x funksiya va uning xossalari, grafigi

$$y=\cos x$$

tenglik $[0;\pi]$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim

$$x=\arccos y$$

ko‘rinishda yoziladi. Bu tenglik bilan $[-1;1]$ to‘plamning har bir y elementiga $[0;\pi]$ to‘plamning yagona x elementini mos qo‘yuvchi arkkosinus funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$y=\arccos x$$

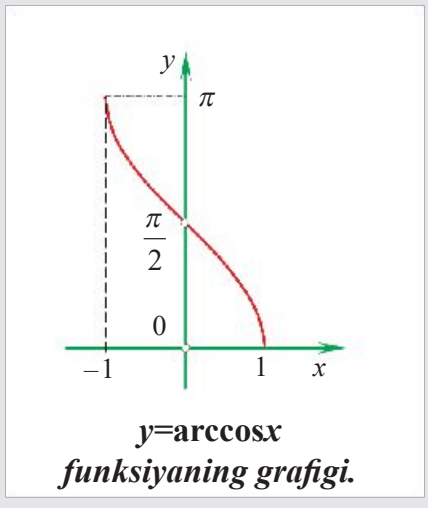
ko‘rinishda yozamiz (2-rasm).

$y=\arccos x$ funksiya $y=\cos x$ funksiyaga teskari funksiya bo‘ladi:

$\cos(\arccos a)=a,\qquad a\in[-1;1],$

$\arccos(\cos\alpha)=\alpha,\qquad \alpha\in[0;\pi].$

2-rasm



y*=arccos*x* *funksiyaning grafigi.

y = arccos x funksiya quyidagi xossalarga ega:

- $D(y)=[-1;1]$;
- $E(y)=[0;\pi]$;
- $y=\arccos x$ – kamayuvchi funksiya;
- $y=\arccos x$ funksiyaning eng katta qiymati π ga, eng kichik qiymati 0 ga teng;
- $y=\arccos x$ funksiya grafigi Ox o‘qini abssissasi $x=1$ bo‘lgan $(1;\ 0)$ nuqtada, Oy o‘qini esa ordinatasi $y=\frac{\pi}{2}$ bo‘lgan $\left(0;\frac{\pi}{2}\right)$ nuqtada kesib o‘tadi;
- $y=\arccos x$ – toq ham emas, juft ham emas. Bu yerda $\arccos(-x)=\pi-\arccos x$ tenglik o‘rinli bo‘ladi;
- $y=\arccos x$ funksiya davriy emas (aniqlanish sohasi davriy emas).

2-misol. $\arccos\frac{\sqrt{2}}{2}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $\arccos\frac{\sqrt{2}}{2}=x$ bo‘lsin. U holda, berilgan vazifani quyidagicha ifodalash mumkin: $\cos x=\frac{\sqrt{2}}{2}$ tenglikni qanoatlantiruvchi x ning $[0;\pi]$ oraliqdagi qiymatini toping. Ma’lumki, $\cos x=\frac{\sqrt{2}}{2}$ tenglik $x=\frac{\pi}{4}$ bo‘lganda bajariladi. Demak,

$$\arccos\frac{\sqrt{2}}{2}=\frac{\pi}{4}.$$

Quyidagi jadvalda $\arccos a$ ifodaning ayrim qiymatilari keltirilgan.

a	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arccos a$	π	$\frac{5\pi}{6}$	$\frac{3\pi}{2}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0

y = arctgx funksiya va uning xossalari, grafigi

$$y=\mathrm{tg}x$$

tenglik $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim $x=\mathrm{arctg}y$ ko‘rinishda yoziladi. Bu tenglik bilan $\mathbb{R}=(-\infty;+\infty)$ to‘plamning har bir y

elementiga $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ to‘plamning yagona x elementini mos qo‘yuvchi arktangens funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$y=\operatorname{arctg} x$$

ko‘rinishda yozamiz (3-rasm).

$y=\operatorname{arctg} x$ funksiya $y=\operatorname{tg} x$ funksiyaga teskari funksiya bo‘ladi:

$$\operatorname{tg}(\operatorname{arctg} a)=a,\quad a\in(-\infty;+\infty),$$

$$\operatorname{arctg}(\operatorname{tg}\alpha)=\alpha,\quad \alpha\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right).$$

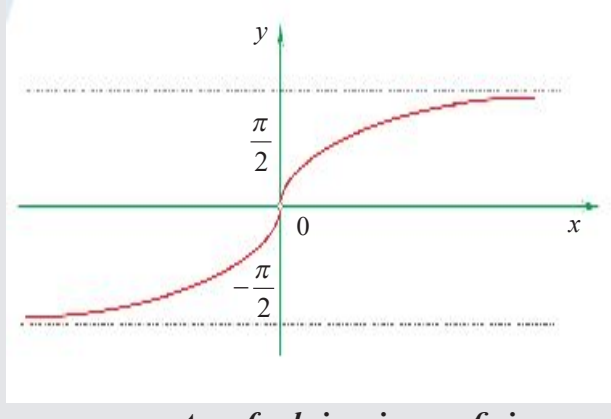
y = arctgx funksiya quyidagi xossalarga ega:

- $D(y)=(-\infty;+\infty)$;
- $E(y)=\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$;
- $y=\operatorname{arctg} x$ – o‘suvchi funksiya;
- $y=\operatorname{arctg} x$ funksiya eng katta va eng kichik qiymatlarga erishmaydi;
- $y=\operatorname{arctg} x$ funksiya grafigi koordinata boshidan o‘tadi;
- $y=\operatorname{arctg} x$ – toq funksiya, ya’ni

$$\operatorname{arctg}(-x)=-\operatorname{arctg} x;$$

- $y=\operatorname{arctg} x$ funksiya davriy emas.

3-rasm



***y*=arctg *x* funksiyaning grafigi.**

3-misol. $\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)$ ifodaning qiymatini toping.

Yechish. Aytaylik, $\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)=x$ bo‘lsin. U holda, $\operatorname{tg} x=-\frac{\sqrt{3}}{3}$ tenglikni qanoatlantiruvchi x ning qiymatini topish talab etiladi.

Ma’lumki, $\operatorname{tg}\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{3}$ bo‘ladi. Demak, $\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{6}$.

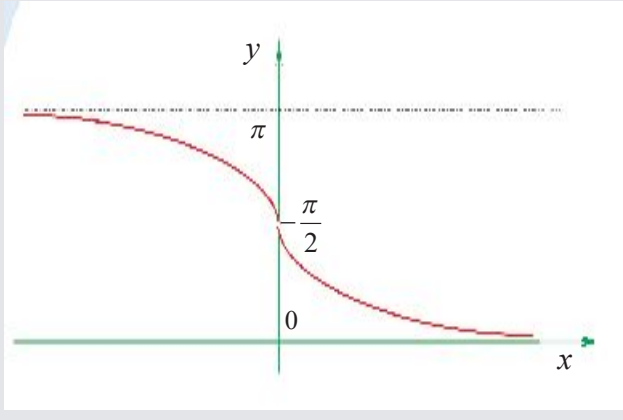
Quyidagi jadvalda $\operatorname{arctg} a$ ifodaning ayrim qiymatilari keltirilgan.

a	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{arctg} a$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$

y = arcctgx funksiya va uning xossalari, grafigi

$y = \operatorname{ctg} x$ tenglik $(0;\pi)$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim $x = \operatorname{arctg} y$ ko‘rinishda yoziladi. Bu tenglik bilan $\mathbb{R} = (-\infty;+\infty)$ to‘plamning har bir y elementiga $(0;\pi)$ to‘plamning yagona x elementini mos qo‘yuvchi arkkotangens funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni $y = \operatorname{arcctg} x$ ko‘rinishda yozamiz (4-rasm).

4-rasm



y=arcctg *x funksiyaning grafigi.*

$y = \operatorname{arcctg} x$ funksiya $y = \operatorname{ctg} x$ funksiyaga teskari funksiya bo‘ladi:

$$\operatorname{ctg}(\operatorname{arcctg} a) = a,\ a \in (-\infty;+\infty),\ \operatorname{arcctg}(\operatorname{ctg}\alpha) = \alpha,\ \alpha \in (0;\pi).$$

y = arcctgx funksiya quyidagi xossalarga ega:

- $D(y) = (-\infty;+\infty)$;
- $E(y) = (0;\pi)$;
- $y = \operatorname{arcctg} x$ – kamayuvchi funksiya;
- $y = \operatorname{arcctg} x$ funksiya eng katta va eng kichik qiymatlarga erishmaydi;
- $y = \operatorname{arcctg} x$ funksiya grafigi Ox o‘qi bilan kesishmaydi, Oy o‘qi bilan esa ordinatasi $y = \frac{\pi}{2}$ bo‘lgan $\left(0;\frac{\pi}{2}\right)$ nuqtada kesishadi;
- $y = \operatorname{arcctg} x$ – toq ham emas, juft ham emas. Bu funksiya uchun $\operatorname{arcctg}(-x) = \pi - \operatorname{arcctg} x$ tenglik bajariladi;
- $y = \operatorname{arcctg} x$ funksiya davriy emas.

4-misol. $\operatorname{arcctg}\sqrt{3}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $\operatorname{arcctg}\sqrt{3} = x$ bo‘lsin. U holda $\operatorname{ctg} x = \sqrt{3}$ tenglikni qanoatlantiruvchi x ning qiymatini topish talab etiladi. Ma’lumki, $\operatorname{ctg}\frac{\pi}{6} = \sqrt{3}$ bo‘ladi. Demak, $\operatorname{arcctg}\sqrt{3} = \frac{\pi}{6}$.

Quyidagi jadvalda $\operatorname{arcctg} a$ ifodaning ayrim qiymatilari keltirilgan.

a	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{arcctg} a$	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

MISOLLAR

1. Quyidagi ifodalar ma'noga egami?

a) $arcsin(\sqrt{3}-1)$ b) $arcsin(4-\sqrt{5})$ c) $arccos\left(-\frac{\pi}{3}\right)$

d) $\arccos(\sqrt{2})$ e) $arctg(\sqrt{2})$ f) $arcctg(-100)$

2. Hisoblang:

a) $arcsin\frac{1}{2}$ b) $arcsin\,(-1)$ c) $arcsin\frac{1}{\sqrt{2}}$ d) $arcsin 0$

e) $arcsin\frac{\sqrt{3}}{2}$ f) $arcsin\left(-\frac{\sqrt{2}}{2}\right)$ g) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ h) $\arccos\frac{1}{2}$

i) $\arccos 0;$ j) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ k) $\arccos\frac{1}{\sqrt{2}};$ l) $arctg 0$

m) $arctg(-1)$ n) $arctg\frac{1}{\sqrt{3}}$ o) $arctg\sqrt{3}$ p) $arctg 1$

q) $arctg(-\sqrt{3})$ r) $arcctg\sqrt{3}$ s) $arcctg(-1)$ t) $arcctg\frac{1}{\sqrt{3}}$

3. Hisoblang.

a) $\arcsin\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}-1}-\frac{\sqrt{2}-\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}\right)$

b) $\arccos\left(\frac{10+5\sqrt{5}}{\frac{5}{2}\left(3+\sqrt{5}\right)}-\frac{\frac{5}{2}\left(3+\sqrt{5}\right)}{10+5\sqrt{5}}\right)$

c) $arcctg\left(\frac{1-2(2+\sqrt{3})}{3+\sqrt{3}}-\frac{1}{1+\sqrt{3}}\right)$

d) $arctg\left(\frac{1+\sqrt{\frac{7}{3}}-\frac{2}{\sqrt{3}}}{1-\sqrt{\frac{7}{3}}+\frac{2}{\sqrt{3}}}-\frac{1-\sqrt{\frac{7}{3}}+\frac{2}{\sqrt{3}}}{1+\sqrt{\frac{7}{3}}-\frac{2}{\sqrt{3}}}\right)$

4. Hisoblang.

a) $cos\left(arctg2\right)$ b) $sin\left(arctg7\right)$ c) $cos\left(arcsin\frac{1}{4}\right)$

d) $ctg\left(arctg5\right)$ e) $sin\left(arctg11\right)$ f) $sin\left(\arccos\frac{1}{5}\right)$

g) $cos\left(arcctg\left(-4\right)\right)$ h) $tg\left(\arccos\left(-0,3\right)\right)$ i) $\sin\left(\arccos\left(-\frac{4}{7}\right)\right)$

j) $ctg\left(arctg\left(-15\right)\right)$ k) $ctg\left(arc\sin\left(-0,9\right)\right)$ l) $ctg\left(\arccos\left(-\frac{\pi}{4}\right)\right)$

5. Hisoblang.

a) $cos\left(2arcsin0,2\right)$ b) $sin(2arccos\left(-\frac{2}{3}\right)$ c) $sin\left(2arcctg\sqrt{26}\right)$

d) $tg\left(2arccos0,6\right)$ e) $tg\left(2arcsin\frac{7}{9}\right)$ f) $cos\left(2arccos\left(-0,8\right)\right)$

g) $tg\left(2arcctg\left(-3\right)\right)$ h) $sin\left(2arcsin\left(-0,1\right)\right)$ i) $tg\left(2arcctg20\right)$

LOYIHA ISHI

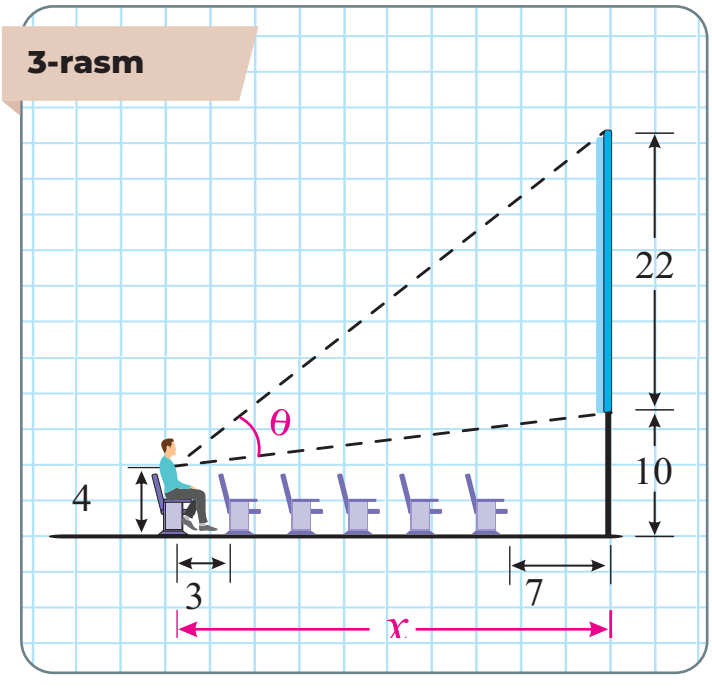
KINOTEATRDA QAYERDA O‘TIRISH KERAK?

Ob’yektning ko‘rinadigan o‘lchami uning tomoshabindan uzoqligiga bog‘liqligini hamma biladi. Ob’ekt qanchalik uzoqda bo‘lsa, uning ko‘rinadigan o‘lchami shunchalik kichik bo‘ladi. Ko‘rinadigan o‘lcham ob’yektning tomoshabinning ko‘ziga qaragan burchagi bilan belgilanadi.

Agar siz devorga osilgan rasmga qarasangiz, maksimal ko‘rinishga ega bo‘lish uchun qancha masofada turishingiz kerak? Agar rasm ko‘z darajasidan yuqorida osilgan bo‘lsa, unda quyidagi rasmlarda ko‘rsatilgandek, agar siz juda yaqin yoki juda uzoq bo‘lsangiz, ko‘z nazari ostidagi burchak kichik ekanligini aniq. Xuddi shu holat kinoteatrda o‘rindiq tanlashda ham sodir bo‘ladi.

1. Kinoteatrdagi ekran 22 fut balandlikda va tekis poldan 10 fut balandlikda joylashgan. O‘rindiqlarning birinchi qatori ekrandan 7 fut, qatorlar esa 3 fut masofada joylashgan. Siz maksimal ko‘rinishga ega bo‘lgan qatorga o‘tirishga qaror qildingiz, ya’ni ekranning ko‘zingiz nazaridagi burchagi θ maksimal bo‘lgan joyda. Aytaylik, ko‘zlaringiz rasmdagi kabi poldan 4 fut balandlikda va siz ekrandan x masofada o‘tiribsiz (1 fut = 0,3048 m)(3-rasm).

3-rasm



Quyidagini isbotlang: $\theta = tan^{-1}28x - tan^{-1}6x$.

Quyidagini keltirib chiqarish uchun tangensni ayirish formulasidan foydalaning:

$$\theta = tan^{-1}\left(\frac{22x}{x^2+168}\right)$$

Geogebra ilovasidan foydalanib, θ ning x ga nisbatan funksiya sifatida grafigini tuzing. x ning qaysi qiymati θ ni maksimal darajada oshiradi? Qaysi qatorda o‘tirish kerak? Ushbu qatordagi ko‘rish burchagi qanday?

Endi faraz qilaylik, birinchi qatordagi o‘rindiqlardan boshlab, o‘tiradigan joyning gorizontaldan tekislikdan yuqorida pollari burchak ostida egilgan va qiyalikda o‘tirgan masofa rasmda ko‘rsatilganidek x ga teng.

1. Quyidagini keltirib chiqarish uchun kosinuslar formulasidan foydalaning:

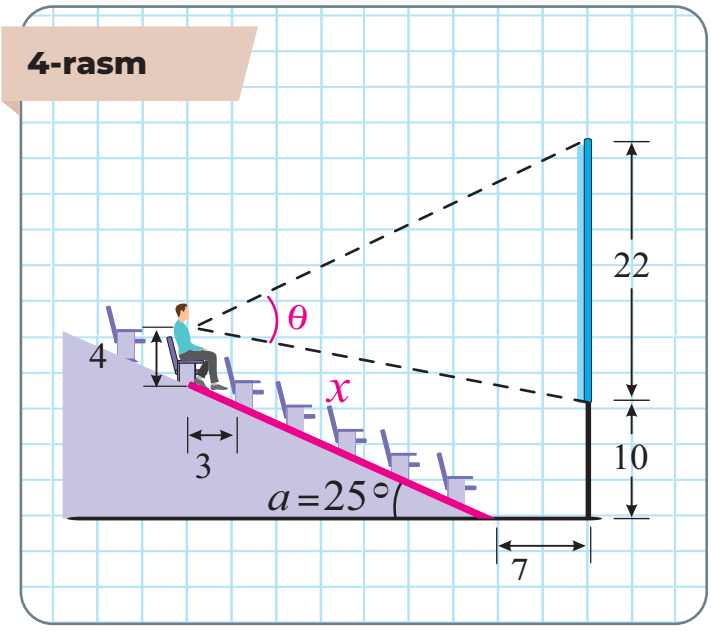
$$\theta = \cos^{-1}\left(\frac{a^2 + b^{2-484}}{2ab}\right).$$ Bu yerda

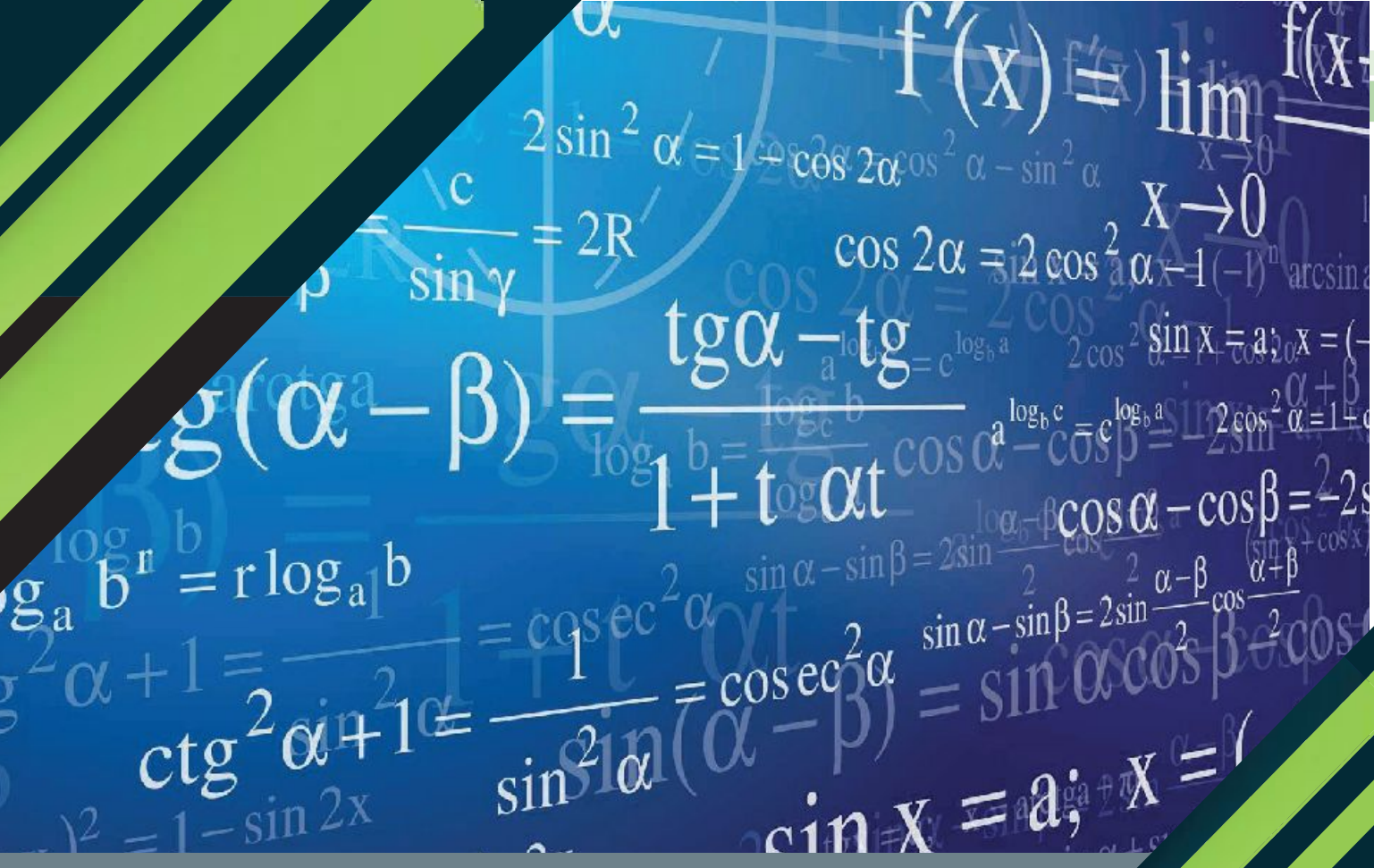
$$a^2 = (7 + x\cos\alpha)^2 + (28 - x\sin\alpha)^2$$

va $$b^2 = (7 + x\cos\alpha)^2 + (x\sin\alpha - 6)^2$$

2. ***Geogebra*** grafik ilovasidan foydalanib θ ning x ning funksiyasi sifatida grafigini tuzing va θ ni maksimallashtiruvchi x ning qiymatini toping. Qaysi qatorda o‘tirish kerak? Bu qatordagi ko‘rish burchagi qanday?

4-rasm





5-BOB TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLAR

- TRIGONOMETRIK TENGLAMALAR
- BA’ZI TRIGONOMETRIK TENGLAMALARNI YECHISH USULLARI
- TRIGONOMETRIK TENGSIZLIKLAR

TRIGONOMETRIK TENGLAMALAR

Eng sodda trigonometrik tenglamalar

Davriy funksiyalar bilan tavsiflanadigan jarayonlar qachon qanday qiymat qabul qilishini bilish muhim ahamiyatga ega. Buning uchun davriy funksiyalar qatnashgan

$$\sin x = a,\ \cos x = a,\ \operatorname{tg} x = a,\ \operatorname{ctg} x = a$$

ko‘rinishdagi eng sodda trigonometrik tenglamalarni yechishni bilish zarur bo‘ladi.

Eng sodda trigonometrik tenglamalarni yechishni o‘rganish uchun:

1) tenglama tushunchasini;

2) tenglamaning ildizi tushunchasini; ildizlar to‘plami yechim deb atalishini;

3) trigonometrik funksiyalar davriy ekanligidan trigonometrik tenglamaning ildizlari cheksiz ko‘p bo‘lishini;

4) topilgan cheksiz ko‘p ildizlarni umumlashtirib qisqa formulalar orqali yoza olishni (bunda har bir k butun son uchun $n = 2k$ ifoda juft sonni, $n = 2k+1$ ifoda esa toq sonni anglatishini) bilish talab etiladi.

1-misol. $\sin x = \frac{1}{2}$ tenglamani yeching.

Yechish. Ma’lumki, $\sin\frac{\pi}{6} = \frac{1}{2}$ bo‘ladi. $\sin x = \frac{1}{2}$ tenglik x ning $\frac{5\pi}{6} = -\frac{\pi}{6} + \pi$ qiymatida ham bajariladi (1-rasm). Sinus davriy funksiya bo‘lganligidan har qanday k butun son uchun

$$x = \frac{\pi}{6} + 2k\pi$$

yoki

$$x = \left(-\frac{\pi}{6} + \pi\right) + 2k\pi = -\frac{\pi}{6} + (2k+1)\pi$$

bo‘lganda ham $\sin x = \frac{1}{2}$ bo‘ladi (2-rasm). Bu ikkita tenglikni quyidagicha umumlashtirish mumkin:

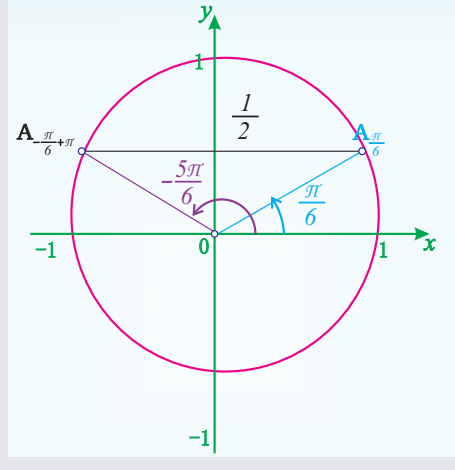
$$x = (-1)^n \frac{\pi}{6} + n\pi,\ n \in \mathbb{Z}.$$

Haqiqatan ham, n juft, ya’ni $n = 2k$ bo‘lsa, u holda $x = \frac{\pi}{6} + 2k\pi$ tenglikka; n toq, ya’ni $n = 2k+1$ bo‘lsa, $x = -\frac{\pi}{6} + (2k+1)\pi$ tenglikka ega bo‘lamiz. Shunday qilib, $\sin x = \frac{1}{2}$ tenglik x ning

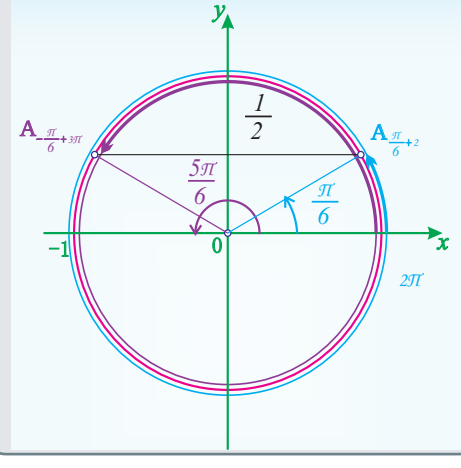
$$x = (-1)^n \frac{\pi}{6} + n\pi,\ n \in \mathbb{Z}.$$

qiymatlarida bajarilar ekan.

1-rasm



2-rasm



$\sin x = a$ ko‘rinishdagi tenglamada agar:

$a > 1$ yoki $a < -1$ bo‘lsa, u holda $\sin x = a$ tenglama ildizga ega bo‘lmaydi. Shuning uchun bunday hollarda $\sin x = a$ tenglamaning yechimi bo‘sh to‘plam $\varnothing$ dan iborat degan javob yoziladi;

$-1 \le a \le 1$ bo‘lsa, u holda $\sin x = a$ tenglamaning yechimi

$$\boxed{x = (-1)^n \arcsin a + \pi n}, \quad n \in \mathbb{Z}$$

ko‘rinishda bo‘ladi.

Ayrim xususiy hollarni keltiramiz.

$\sin x = -1$ tenglamaning yechimi x erkli o‘zgaruvchining

$$x = -\frac{\pi}{2} + 2\pi n, \quad n \in \mathbb{Z}$$

qiymatlaridan iborat.

$\sin x = 0$ tenglamaning yechimi x erkli o‘zgaruvchining

$$x = \pi n, \ n \in \mathbb{Z}$$

qiymatlaridan iborat.

$\sin x = 1$ tenglamaning yechimi x erkli o‘zgaruvchining

$$x = \frac{\pi}{2} + 2\pi n, \quad n \in \mathbb{Z}$$

qiymatlaridan iborat.

$\cos x = a$ ko‘rinishdagi tenglama

2-misol. $\cos x = \frac{\sqrt{2}}{2}$ tenglamani yeching.

Yechish. $\cos\frac{\pi}{4} = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ bo‘lishi ma’lum (3-rasm). Kosinus davriy funksiya bo‘lgaligidan har qanday n butun son uchun

$$x = \frac{\pi}{4} + 2n\pi \text{ yoki } x = -\frac{\pi}{4} + 2n\pi$$

bo‘lganda ham $\cos x = \frac{\sqrt{2}}{2}$ bo‘ladi. Bu ikkita tenglikni quyidagicha umumlashtirish mumkin:

$$x = \pm\frac{\pi}{4} + 2n\pi, \ n \in \mathbb{Z}.$$

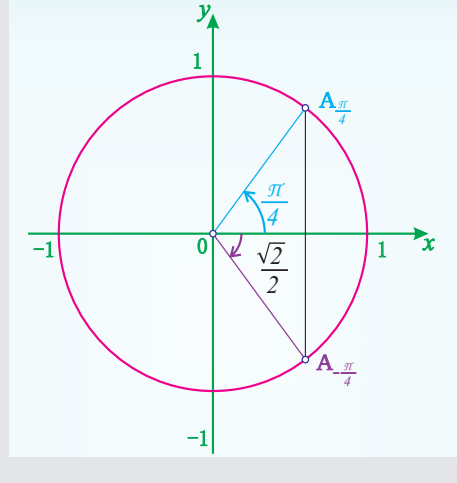
$\cos \boldsymbol{x} = \boldsymbol{a}$ ko‘rinishdagi tenglamada

agar: $a > 1$ yoki $a < -1$ bo‘lsa, u holda $\cos x = a$ tenglama ildizga ega bo‘lmaydi. Bunday hollarda $\cos x = a$ tenglamaning yechimi $\varnothing$ degan javob yoziladi;

$-1 \le a \le 1$ bo‘lsa, u holda

$$\boxed{x = \pm\arccos a + 2\pi n}, \ n \in \mathbb{Z}$$

3-rasm



bo‘ladi.

Quyidagi xususiy hollarga e’tibor qarating:

1) $\cos x = -1$ tenglamaning yechimi

$$x = \pi + 2\pi n,\ n \in \mathbb{Z}$$

dan iborat.

2) $\cos x = 0$ tenglamaning yechimi

$$x = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$$

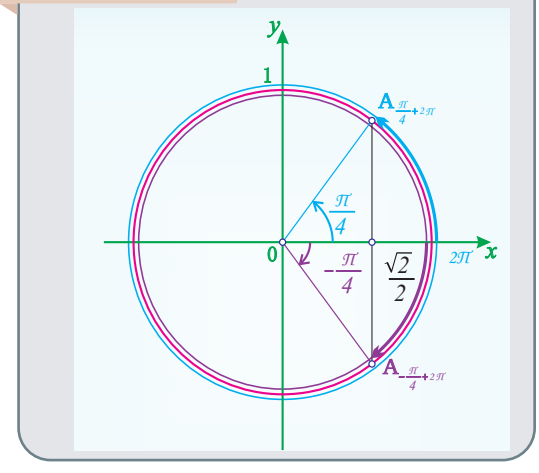
dan iborat.

3) $\cos x = 1$ tenglamaning yechimi argumentning

$$x = 2\pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

4-rasm



tg*x* = *a* ko‘rinishdagi tenglama

Har bir n butun soni uchun x erkli o‘zgaruvchining

$$x = \operatorname{arctg} a + \pi n$$

qiymati $\operatorname{tg} x = a$ tenglamaning ildizi bo‘ladi. Bu holda yechim

$$\boxed{x = \operatorname{arctg} a + \pi n},\ n \in \mathbb{Z}$$

ko‘rinishda bo‘ladi.

Xususiy hollar:

$\operatorname{tg} x = -1$ tenglamaning yechimi x erkli o‘zgaruvchining

$$x = -\frac{\pi}{4} + \pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

$\operatorname{tg} x = 0$ tenglamaning yechimi

$$x = \pi n,\ n \in \mathbb{Z} \text{ bo‘ladi.}$$

$\operatorname{tg} x = 1$ tenglamaning yechimi

$$x = \frac{\pi}{4} + \pi n,\ n \in \mathbb{Z} \text{ dan iborat.}$$

ctg*x* = *a* ko‘rinishdagi tenglamalar

Har bir n butun soni uchun x erkli o‘zgaruvchining

$$x = \operatorname{arcctg} a + \pi n$$

qiymati $\operatorname{ctg} x = a$ tenglamaning ildizi bo‘ladi. Bu holda yechim

$$\boxed{x = \operatorname{arcctg} a + \pi n},\ n \in \mathbb{Z}$$

ko‘rinishda bo‘ladi.

Xususiy hollar:

1) $\operatorname{ctg} x = -1$ tenglamaning yechimi x erkli o‘zgaruvchining

$$x = -\frac{\pi}{4} + \pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

2) $\operatorname{ctg}x = 0$ tenglamaning yechimi argumentning

$$x = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

3) $\operatorname{ctg}x = 1$ tenglamaning yechimi

$$x = \frac{\pi}{4} + \pi n,\ n \in \mathbb{Z}$$

bo'ladi.

MISOLLAR

1. Tenglamani yeching:

a) $\sin 2x = 1$ b) $\sin\frac{x}{3} = -1$ c) $\sin\left(2x - \frac{\pi}{5}\right) = 0$

d) $2\sin 4x = \sqrt{5}$ e) $\sin(4x-1) = -\frac{\pi}{3}$ f) $\sin x = \frac{1}{2}$

h) $\sin x = -\frac{\sqrt{3}}{2}$ i) $\sin 4x = 1$ j) $\sin 2x = \frac{\sqrt{2}}{2}$

k) $\sin\left(x + \frac{\pi}{7}\right) = \frac{\sqrt{3}}{2}$ l) $\sin\frac{2x}{3} = -1$ m) $\sin\left(2x + \frac{\pi}{5}\right) = 0$

n) $\sin(3x+1) = -\frac{\sqrt{2}}{2}$ o) $\sin(-x) = -\frac{1}{2}$ p) $\sin\left(\frac{3\pi}{4} - x\right) = 1$

2. a ning qanday qiymatlarida $\sin x = a + \frac{1}{a}$ $(\text{bunda } a \neq 0)$ tenglik o'rinli bo'lishi mumkin?

3. Tenglamani yeching:

a) $\cos\frac{2x}{5} = 1$ b) $\cos\left(2x - \frac{\pi}{7}\right) = -1$ c) $\cos 8x = 0$

d) $\cos 3x = 1,2$ e) $2\cos(x-1) = \frac{11}{2}$ f) $\cos x = \frac{\sqrt{3}}{2}$

g) $\cos x = -\frac{1}{2}$ h) $\cos x = -1$ i) $\cos\frac{x}{2} = -\frac{\sqrt{2}}{2}$

j) $\cos\left(x - \frac{\pi}{5}\right) = \frac{1}{2}$ k) $\cos\frac{3x}{4} = 0$ l) $\cos 4x = -\frac{\sqrt{3}}{2}$

m) $\sqrt{3} + 2\cos\frac{\pi x}{9} = 0$ n) $1 - 2\cos\frac{3\pi x}{4} = 0$ o) $\cos(\pi(x-3)) = 1$

p) $\sin^2\frac{2}{3}x=\frac{3}{4}$ q) $\cos^2\frac{3}{2}x=\frac{1}{4}$ r) $2\cos\left(2x-\frac{\pi}{6}\right)+\sqrt{3}=0$

4. Tenglamani yeching:

a) $tgx=\frac{1}{\sqrt{3}}$ b) $tgx=-\frac{1}{\sqrt{3}}$ c) $tgx=-1$

d) $tgx=\sqrt{3}$ e) $tg\frac{2x}{5}=-\sqrt{3}$ f) $tg\left(x+\frac{7\pi}{3}\right)=1$

g) $tg(\frac{\pi}{4}(x-1)=0$ h) $1-\sqrt{3}tg\frac{2\pi x}{7}=0$ i) $tg9x=tg45°$

j) $tg6x=tg\frac{2\pi}{3}$ k) $3tg\left(x+\frac{5\pi}{36}\right)+\sqrt{3}=0$

5. a ning qanday qiymatlarida, $tgx=\frac{a+1}{a-1}$ tenglik o‘rinli bo‘lishi mumkin?

6. Tenglamani yeching:

a) $ctgx=\sqrt{3}$ b) $ctgx=-\frac{1}{\sqrt{3}}$ c) $ctg4x=sin0°$

d) $ctg(\pi(2x+3)=cos0°$ e) $\sqrt{3}+ctg\frac{\pi x}{5}=0$ f) $ctgx=-1$

g) $ctg\frac{3x}{2}=1$ h) $ctg3x=\sqrt{3}$ i) $\text{ctg}\left(2x-\frac{\pi}{5}\right)=0$

j) $ctg7x=-\sqrt{3}$

BA'ZI TRIGONOMETRIK TENGLAMALARNI YECHISH USULLARI

1-misol. Tenglamani yeching: $2\cos^2 x - 5\sin x + 1 = 0$.

Yechish:

1. $\cos^2 x$ ni $1-\sin^2 x$ bilan almashtirib, $2(1-\sin^2 x) - 5\sin x + 1 = 0$, yoki $2\sin^2 x + 5\sin x - 3 = 0.$

$\sin x = y$ belgilash kiritib, $2y^2 + 5y - 3 = 0$ ni hosil qilamiz.

$y_{1,2} = \dfrac{-5 \pm \sqrt{25 + 4 \cdot 2 \cdot 3}}{4}$. Bundan, $y_1 = -3; y_2 = \dfrac{1}{2}.$

2. $\sin x = -3$ tenglama yechimga ega emas, chunki $|-3| > 1$.

3. $\sin x = \dfrac{1}{2}$ tenglamani yechamiz. Bundan, $x = (-1)^n \arcsin\dfrac{1}{2} + \pi n = (-1)^n \dfrac{\pi}{6} + \pi n, n \in Z$

Javob: $x = (-1)^n \dfrac{\pi}{6} + \pi n, \quad n \in Z$

2-misol. Tenglamani yeching: $(tgx + 1)(2\cos\dfrac{x}{2} - \sqrt{3}) = 0$.

Yechish:

1) $tgx + 1 = 0 \Rightarrow tgx = -1 \Rightarrow x = -\dfrac{\pi}{4} + \pi n,\ n \in Z.$ x ning ushbu qiymatlarida ikkinchi ko'paytuvchi ma'noga ega bo'lgani uchun, x ning bu qiymatlari dastlabki tenglamaning ildizlari bo'ladi.

2) $2\cos\dfrac{x}{2} - \sqrt{3} = 0$, $\cos\dfrac{x}{2} = \dfrac{\sqrt{3}}{2}$, $\dfrac{x}{2} = \pm\dfrac{\pi}{6} + 2\pi n$, $x = \pm\dfrac{\pi}{3} + 4\pi n.$ x ning ushbu qiymatlarida birinchi ko'paytuvchi ma'noga ega, demak, x ning bu qiymatlari ham dastlabki tenglamaning ildizlari bo'ladi.

Javob: $x = \pm\dfrac{\pi}{3} + \pi n,\ n \in Z.$ $x = -\dfrac{\pi}{4} + \pi n,\ n \in Z$

3-misol. Tenglamani yeching: $3\cos x - 2\sin x = 0.$

Yechish:

1. Tenglamaning ikkala tarafini $\cos x$ ga bo'lib, $3tgx - 2 = 0$ tenglamani hosil qilamiz.

2. $3tgx - 2 = 0$, $tgx = \dfrac{2}{3}$, $x = arctg\dfrac{2}{3} + \pi n,\ n \in Z$.

Ushbu tenglamani yechishda, tenglamaning ikki tomoni $\cos x$ ga bo'lindi. Tenglamani noma'lum qatnashgan ifodaga bo'lganda ildiz yo'qolishi mumkin, shuning uchun $\cos x = 0$ tenglamaning ildizlari berilgan tenglamaning ildizlari bo'lishini tekshiramiz. Agar $\cos x = 0$ bo'lsa, u holda $3\sin x - 2\cos x = 0$ tenglamada $\sin x = 0$ bo'lib qoladi. Ammo, $\sin x$ va $\cos x$ bir vaqtda 0 ga teng bo'la olmaydi ($\sin^2 x + \cos^2 x = 1$). Xulosa qilib aytganda, $a\sin x + b\cos x = c$ tenglamani $\cos x$ (yoki $\sin x$) ga bo'lganda, berilgan tenglamaga teng kuchli tenglama hosil bo'ladi.

Javob: $x = arctg\dfrac{2}{3} + \pi n,\ n \in Z$.

MISOLLAR

Tenglamalarni yeching.

1. $sin^2x+2sinx-3=0$

2. $2sin^2x+sinx-1=0$

3. $2cos^2x-5cosx+2=0$

4. $cos^2x-cosx-2=0$

5. $cos^2 2x+cos2x-6=0$

6. $tg^2x-2tgx=3$

7. $3tg^2 2x+2tg2x-5=0$

8. $2ctg^2 3x-3ctg3x+1=0$

9. $2\cos x=1-\sqrt{cosx}$

10. $2sin^2x-7cosx-5=0$

11. $2\cos^2 x+\sin x-1=0$

12. $3sin^2 2x+7cos2x-3=0$

13. $sin5x=\frac{2}{3}cos^2 5x$

14. $cos^4\frac{x}{5}+sin^2\frac{x}{5}=1$

15. $3tg2x-2ctg2x-1=0$

16. $2tgx-2ctgx=3$

17. $\sqrt{3}sinx-cosx=0$

18. $sin2x+cos2x=0$

19. $sin\left(x+\frac{\pi}{6}\right)+cos\left(x+\frac{\pi}{6}\right)=0$

20. $\sqrt{3}sin\left(x+\frac{\pi}{12}\right)+cos\left(x+\frac{\pi}{12}\right)=0$

21. $3cosx+sinxcosx+3sinx+sin^2x=0$

22. $sin2x=cos^4x-sin^4x$

23. $sinx+sin3x=4cos^3x$

24. $sin^2x-sinx=0$

25. $cos^2x-cosx=0$

26. $tg^5x-tg^2x=0$

27. $2\sin^2 3\pi x-sin3\pi x=0$

28. $ctg^2x-4ctgx=0$

29. $sinxcosx-\sqrt{3}cos^2x=0$

30. $cos^2x+sinxcosx=0$

31. $cos^2x-3sinxcosx+2sin^2x=2$

32. $cos2x+3sinx=2$

33. $5sin\frac{x}{6}-cos\frac{x}{3}+1=-2$

34. $2cos2x-4cosx=1$

35. $cos2x+4sin^3x=1$

36. $2sin^2x-5\sin x\cos x+3cos^2x=0$

37. $6sin^2x+\frac{1}{2}sin2x-cos^2x=2$

38. $4sin^2x+2sinxcosx=3$

39. $3cos^2x-5sin^2x-sin2x=0$

40. $3sin^2x\cos x-7\sin xcos^2x+4cos^3x=0$

41. $cosx-sinx=1$

42. $sinx+cosx=\sqrt{2}$

43. $sin\frac{x}{2}+cos\frac{x}{2}=-1$

44. $\sqrt{3}sinx+cosx=\sqrt{2}$

45. $3sinx+4cosx=3$

46. $sin4x+cos4x=4$

47. $sin2x=cos^4\frac{x}{2}-sin^4$

48. $cos2x=\sqrt{2}\left(cosx-sinx\right)$

49. $cos3xcos2x=sin3xsin2x$

50. $sin5xcos4x-cos5xsin4x=\frac{\sqrt{3}}{2}$

51. $cos3xcos4x=cos7x$

52. $\frac{tg6x-tg4x}{1+tg6xtg4x}=1$

55. $cos9x-cos7x+cos3x-cosx=0$

56. $cos7x+sin8x=cos3x-sin2x$

57. $sin3x+sin5x=sin4x$

58. $sin2xsin6x=cosxcos3x$

59. $cos3xcos6x=cos4xcos7x$

60. $sin7xcos13x=sinxcos19x$

61. $(2\cos x-3)\cdot ctgx=0$

62. $(tgx-3)(cosx-\frac{1}{2})=0$

63. $tg3xcosx=0$

64. $sin2xtgx=0$

65. $1-2\sin x\cos x+\sin x+\cos x=0$

66. $2\sin x\cos x+5\sin x+5\cos x+1=0$

67. $10\sin x\cos x-11\cos x-11\sin x+7=0$

68. $3sin2x+5sinx-5cosx-5=0$

69. $2sin^2\frac{x}{2}+cos2x=0$

70. $6sin^2x+2sin^2 2x=5$

71. $sin^2 2x+sin^2 3x+\ sin^2 4x+\ sin^2 5x=2$

72. $\frac{cos2x}{1+tgx}=0$

73. $\frac{1-2cos2x}{cos2x-2}=0$

74. $\frac{tgx}{sin5x}=0$

75. $\frac{cosx}{1-cos4x}=0$

76. $sin=\frac{1}{2}x$ tenglama nechta yechimga ega

77. $|cos2x-1|-2|cos2x+2|=0$

78. $sin^3x+cos^4x=1$

79. $sin^6x+cos^7x=1$

80. $sin^{12}x+cos^{12}x=1$

81. $sin^{13}x+cos^{13}x=1$

82. $sinx\sqrt{cosx}=0$

83. $cosx\sqrt{sinx}=0$

84. $cos3x+2cosx=0$

85. $sin9x\ =\ 2sin3x$

86. $3sin\frac{x}{3}=sinx$

87. $sin6x+2=2cos4x$

88. $2sin2x+3tgx=5$

89. $1+cosx+\ tg\frac{x}{2}=0$

90. $\begin{cases} x-y=\frac{\pi}{6} \\ cosx\cdot siny=\frac{1}{4} \end{cases}$

91. $\begin{cases} x+y=\frac{2\pi}{3} \\ sinx-2siny=0 \end{cases}$

92. $\begin{cases} tgx+tgy=2 \\ 2cosx\cdot cosy=1 \end{cases}$

TRIGONOMETRIK TENGSIZLIKLAR

Eng sodda trigonometrik tengsizliklarni yechishda:

1) *Oy* o‘qi **sinuslar o‘qi** deb atalishini;
2) *Ox* o‘qi **kosinuslar o‘qi** deb atalishini;
3) burchaklar orasidagi <, ≤, >, ≥ munosabatlarning geometrik talqinini;
4) x o‘zgaruvchining har bir qiymatida $-1\le \sin x \le 1$ bo‘lishini;
5) x o‘zgaruvchining har bir qiymatida $-1\le \cos x \le 1$ bo‘lishini

bilish talab etiladi.

Aytaylik, $f(x)$ yozuv $\sin x$, $\cos x$, $\operatorname{tg} x$ *yoki* $\operatorname{ctg} x$ trigonometrik funksiyalardan birini anglatsin, ya’ni

$f(x)=\sin x$, $f(x)=\cos x$, $f(x)=\operatorname{tg} x$, yoki $f(x)=\operatorname{ctg} x$

bo‘lsin. U holda biror a soni uchun

$f(x)<a$, $f(x)\le a$, $f(x)>a$, $f(x)\ge a$

ko‘rinishdagi tengsizliklar eng sodda trigonometrik tengsizliklar deb yuritiladi.

1) $\sin x < a$ tengsizlikni yechish

Bu tengsizlikni yechishda a ning qiymatlarining quyidagi hollarini ko‘rib o‘tamiz.

1-hol. $a \le -1$ bo‘lsa, $\sin x<a$ tengsizlikning yechimi $\varnothing$ bo‘ladi (1-rasm, 1-hol).

2-hol. $a > 1$ bo‘lsa, $\sin x<a$ tengsizlikning yechimi $(-\infty,+\infty)$ bo‘ladi (1-rasm, 2-hol).

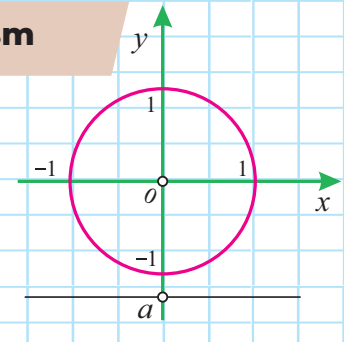
3-hol. $a = 1$ bo‘lsa, $\sin x<a$ tengsizlik $(-\infty,+\infty)$ oraliqning

$$\frac{\pi}{2}+2\pi n,\quad n\in\mathbb{Z}$$

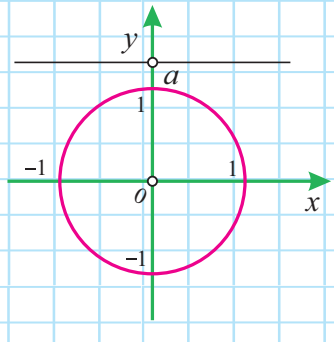
nuqtalaridan boshqa barcha nuqtalarida bajariladi (1-rasm, 3-hol). Bu holat quyidagicha yozilishi mumkin:

$$x\in \bigcup_{n\in Z}\left(\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi(n+1)\right).$$

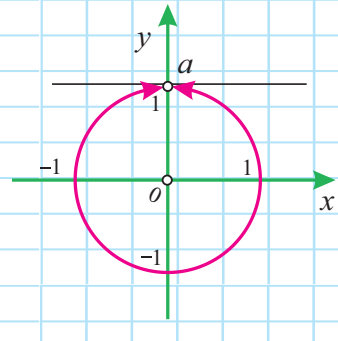
1-rasm



1-hol. a ≤ –1 bo‘lsa, $\sin x < a$ tengsizlikning yechimi ∅ bo‘ladi.



2-hol. a > 1 bo‘lsa, $\sin x < a$ tengsizlikning yechimi $(-\infty,+\infty)$ bo‘ladi.



3-hol. $a = 1$ bo‘lsa, $\sin x < a$ tengsizlikning yechimi

$$x\in \bigcup_{n\in Z}\left(\frac{\pi}{2}+2\pi n;\frac{\pi}{2}+2\pi(n+1)\right)$$

bo‘ladi.

4-hol. $-1 < a < 1$ bo‘lganda $\sin x < a$ tengsizlikning yechimi

$$-\arcsin a+(2n-1)\pi < x < \arcsin a+2n\pi,\ n\in Z$$

bo‘ladi (2-, 3-rasmlar).

$\sin x \le a$ tengsizlikni yechish

1-hol. $a < -1$ bo‘lsa, $\sin x \le a$ tengsizlikning yechimi $\emptyset$ bo‘ladi.
2-hol. $a = -1$ bo‘lsa, $\sin x \le a$ tengsizlikning yechimi

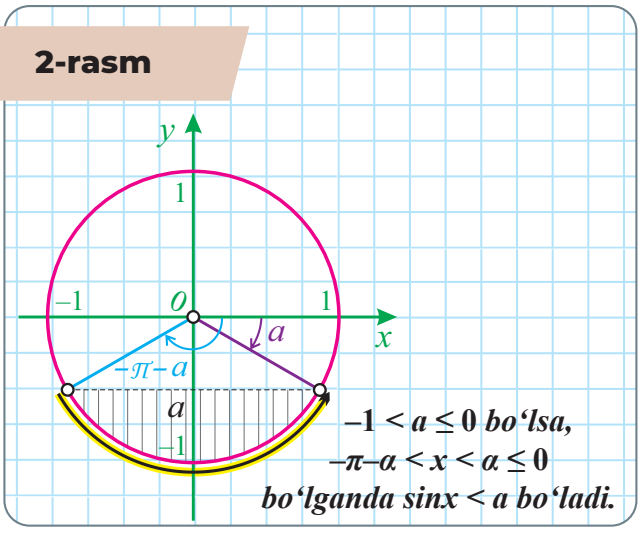
$$x = -\frac{\pi}{2} + 2\pi n,\ n \in Z.$$

bo‘ladi.
3-hol. $a \ge 1$ bo‘lsa, $\sin x \le a$ tengsizlikning yechimi $(-\infty,+\infty)$ bo‘ladi.
4-hol. $-1 < a < 1$ bo‘lsa, u holda $\sin x \le a$ tengsizlikning yechimi

$$-arcsin a + (2n-1)\pi < x < arcsin a + 2n\pi,\ n \in Z$$

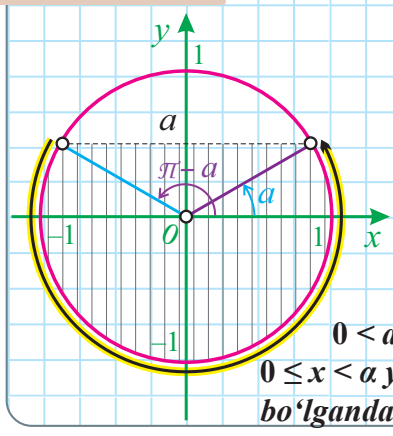
bo‘ladi

2-rasm



–1 < a ≤ 0 bo‘lsa, –π–α < x < α ≤ 0 bo‘lganda sinx < a bo‘ladi.

3-rasm



0 < a < 1 bo‘lsa, 0 ≤ x < α yoki π–α < x ≤ 2π bo‘lganda sinx < a bo‘ladi.

Topshiriq. *sinx>a, sinx≥a* ko‘rinishdagi tengsizliklarni yechishni tushuntiring.

$\cos x < a$ tengsizlikni yechish

1-hol. $a \le -1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $\emptyset$ bo‘ladi (4-rasm, 1-hol).

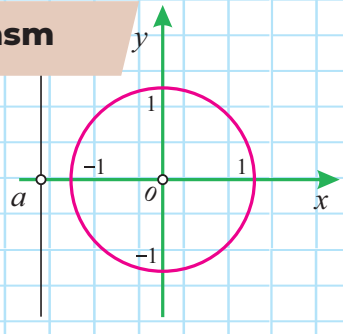
2-hol. $a > 1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $(-\infty, +\infty)$ bo‘ladi (4-rasm, 2-hol).

3-hol. $a = 1$ bo‘lsa, $\cos x < 1$ tengsizlikning yechimi

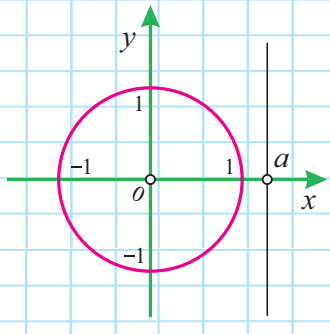
$$2\pi n,\ n \in Z$$

nuqtalar bo‘ladi. (4-rasm, 3-hol). Buni quyidagicha yozilishi mumkin: $x \in (2\pi n;\ 2\pi(n+1)),\ n \in Z$

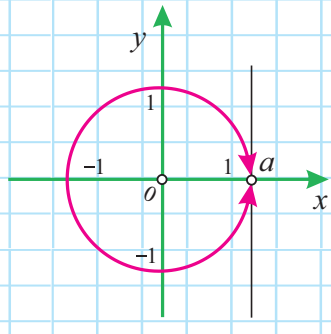
4-rasm



1-hol. $a \le -1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $\emptyset$ bo‘ladi.



2-hol. $a > 1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $(-\infty,+\infty)$ bo‘ladi.



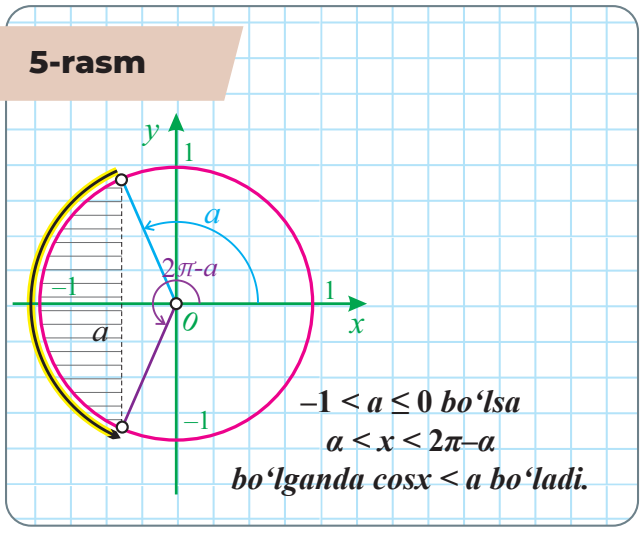
3-hol. $a = 1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $x \in \cup\ (2\pi n;\ 2\pi(n+1))$ bo‘ladi.

4-hol. $-1 < a < 1$ bo'lsa, $\cos x < a$ tengsizlikning yechimi

$$\boldsymbol{\arcsin a + (2n-1) < x < -\arccos a + 2n\pi(n+1),\ n \in Z}$$

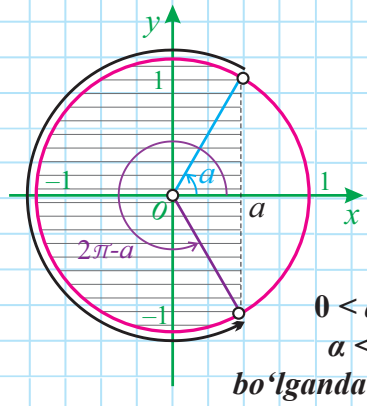
bo'ladi.

5-rasm



–1 < a ≤ 0 bo'lsa
a < x < 2π–a
bo'lganda cosx < a bo'ladi.

6-rasm



0 < a < 1 bo'lsa
a < x ≤ 2π–a
bo'lganda cosx < a bo'ladi.

cosx ≤ a tengsizlikni yechish

1-hol. $a < -1$ bo'lsa, $\cos x \le a$ tengsizlikning yechimi $\emptyset$ bo'ladi.
2-hol. $a = -1$ bo'lsa, $\cos x \le -1$ tengsizlikning yechimi

$$\pi + 2\pi n,\ n \in Z.$$

nuqtalardan iborat. Bu holat quyidagicha yozilishi mumkin:

$$x = 2\pi(n+1),\ n \in Z.$$

3-hol. $a \ge 1$ bo'lsa $\cos x \le 1$ tengsizlikning yechimi $(-\infty,+\infty)$ bo'ladi.
4-hol. $-1 < a < 1$ bo'lsa, $\cos x \le a$ tengsizlikning yechimi

$$\boldsymbol{\arccos a + 2\pi n \le x \le -\arccos a + 2\pi(n+1),\ n \in Z.}$$

bo'ladi.

Topshiriq. $\cos x > a$, $\cos x \ge a$ ko'rinishdagi tengsizliklarni yechishni tushuntiring.

tgx<a tengsizlikni yechish

$tgx<a$ tengsizlikni yechishda $y = tgx$ funksiya grafidan foydalanish maqsadga muvofiq (7-rasm).

7-rasmdan ravshanki, $tgx < a$ tengsizlik x o'zgaruvchining $-\dfrac{\pi}{2} < x < arctga$ qo'shtengsizlikni qanoatlantiruvchi qiymatlarida bajariladi. $y = tgx$ funksiya davriy ekanligidan

$tgx < a$ tengsizlikning yechimi

$$-\frac{\pi}{2} < x < arctga + \pi n, \quad n \in \mathbb{Z}$$

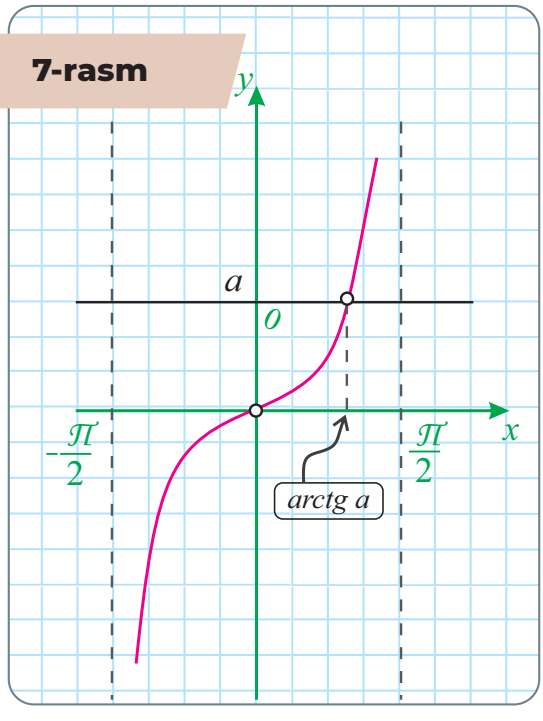
bo'lishi kelib chiqadi. Xuddi shuningdek:

6) $tgx \le a$ tengsizlikning yechimi

$$-\frac{\pi}{2} < x \le arctga + \pi n, \quad n \in \mathbb{Z}$$

bo'ladi.

7-rasm



$tgx > a$ tengsizlikning yechimi

$$arctga + \pi n < x < \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

bo‘ladi;

$tgx \geq a$ tengsizlikning yechimi

$$arctga + \pi n \leq x < \frac{\pi}{2} + \pi n, \quad n \in \mathbb{Z}$$

bo‘ladi;

$ctgx < a$ tengsizlikni yechish

$ctgx < a$ tengsizlikni yechishda $y=ctgx$ funksiya grafidan foydalanish maqsadga muvofiq (8-rasm).

8-rasmdan ko‘rinadiki, $ctgx<a$ tengsizlik x o‘zgaruvchining

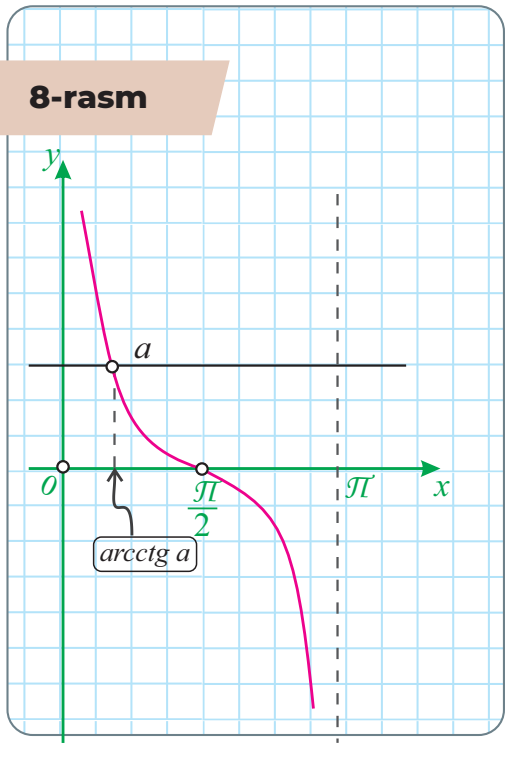
$$arcctga < x < \pi$$

qo‘shtengsizlikni qanoatlantiruvchi qiymatlarida bajariladi. $y=ctgx$ funksiya davriy ekanligidan $ctgx < a$ tengsizlikning yechimi

$$arcctga + \pi n < x < \pi(n+1), \; n \in Z$$

bo‘lishi kelib chiqadi.

8-rasm



Xuddi shuningdek:

$ctgx \leq a$ tengsizlikning yechimi

$$arcctga + \pi n < x < \pi(n+1), \; n \in Z$$

$ctgx > a$ tengsizlikning yechimi

$$\pi n < x < arctga + \pi n, \; n \in Z$$

$ctgx \geq a$ tengsizlikning yechimi

$$\pi n < x \leq arctga + \pi n, \; n \in Z$$

MISOLLAR

1. Tengsizlikni yeching.

a) $\sin x > 1$ b) $\sin x \geq 1$; c) $\sin x < 1$ d) $\sin x \leq 1$

e) $\sin x > -1$ f) $\sin x \geq -1$; g) $\sin x < -1$ h) $\sin x \leq -1$

i) $\sin x > 1,5$ j) $\sin x \geq -1,2$; k) $\sin x < 1,1$ l) $\sin x \leq -2$

2. Tengsizlikni yeching.

a) $\cos x > 1$ b) $\cos x \geq 1$ c) $\cos x < 1$ d) $\cos x \leq 1$

e) $\cos x > -1$ f) $\cos x \geq -1$ g) $\cos x < -1$ h) $\cos x \leq -1$

i) $\cos x > 2$ j) $\cos x \geq -1,6$ k) $\cos x < 1,4$ l) $\cos x \leq -1,7$

3. Tengsizlikni yeching.

a) $\sin 2x \geq 0$ b) $\cos 3x \leq 0$ c) $\cos x < \frac{1}{2}$ d) $\sin x > -\frac{\sqrt{3}}{2}$

e) $\cos 2x > \frac{\sqrt{2}}{2}$ f) $\sin 3x < \frac{\sqrt{3}}{2}$ g) $\sqrt{2} - 2\sin x > 0$ h) $2\cos x + \sqrt{3} \leq 0$

4. Tengsizlikni yeching.

a) $\sin\left(3x - \frac{\pi}{4}\right) < 0$ b) $\cos\left(2x + \frac{\pi}{6}\right) \leq 0$ c) $tg\left(2x - \frac{5\pi}{6}\right) > 0$

d) $tg\left(3x + \frac{\pi}{4}\right) \leq 0$ e) $ctg\left(2x + \frac{\pi}{3}\right) \geq 0$ f) $ctg\left(3x - \frac{\pi}{3}\right) < 0$

5. Tengsizlikni yeching.

a) $tg\left(2x + \frac{\pi}{3}\right) \geq \sqrt{3}$ b) $\cos\left(2x + \frac{\pi}{6}\right) < -\frac{\sqrt{3}}{2}$

c) $\sin\left(3x - \frac{\pi}{3}\right) \leq -\frac{\sqrt{2}}{2}$ d) $ctg\left(2x - \frac{\pi}{4}\right) > -\frac{\sqrt{3}}{3}$

6. Tengsizlikni yeching.

a) $\sin\left(2x + \frac{\pi}{3}\right) > -\frac{1}{2}$ tengsizlikni $[0; \pi]$ oraliqdagi yechimlarini toping.

b) $tg\left(2x - \frac{\pi}{6}\right) < -\sqrt{3}$ tengsizlikni $\left[-\frac{3}{8}; \frac{21}{8}\right]$ oraliqdagi yechimlarini toping.

7. Tengsizlikni yeching.

a) $\cos^2 x - 3\cos x < 0$ b) $2\sin^2 x - 5\sin x + 3 \geq 0$

c) $3\cos^2 x + 7\cos x + 4 \leq 0$ d) $tg^2 x - 4tgx + 3 < 0$

8. Tengsizlikni yeching.

a) $\cos\left(3\sin\left(x - \frac{\pi}{6}\right)\right) < -\frac{\sqrt{3}}{2}$ b) $\sin\left(\cos\left(x + \frac{\pi}{4}\right)\right) > -\frac{1}{2}$



6-BOB EHTIMOLLAR NAZARIYASI

- TASODIFIY HODISALAR. TAJRIBA VA NATIJA. MUQARRAR HODISALAR. TASODIFIY HODISALAR. BIRGALIKDA VA BIRGALIKDA BOʻLMAGAN HODISALAR. BOGʻLIQ VA BOGʻLIQ BOʻLMAGAN HODISALAR. TASODIFIY HODISA EHTIMOLLIGI. QARAMA-QARSHI HODISA VA UNING EHTIMOLLIGI
- EHTIMOLLIKNI HISOBLASHDA KOMBINATORIKA FORMULALARIDAN FOYDALANISH
- TENG EHTIMOLLI HODISALAR. TENG EHTIMOLLI HODISALAR VA ULARNING EHTIMOLLIGINI HISOBLASH

TASODIFIY HODISALAR

Ehtimollar nazariyasi – matematik fan bo‘lib, hozirgi zamon matematikasining asosiy tarmoqlaridan biridir. Ehtimollar nazariyasi predmeti tasodifiy hodisalar bilan boshqariladigan qonuniyatlarni o‘rganishdan iborat. Uning asosiy tushunchalari tajriba va hodisa hisoblanadi.

Tajriba deyilgan aniq shartlar majmuyini amalga oshirish tushuniladi. Tajriba natijasi hodisa deyiladi.

Hodisalar uch xil, ya’ni: mumkin bo‘lmagan (hech qachon bajarilmaydi), muqarrar (har doim bajariladi) va tasodifiy *(*bajarilishi ham mumkin, bajarilmasligi ham mumkin) bo‘lib, bulardan eng asosiysi tasodifiy hodisa ehtimolliklarini hisoblashni o‘rganishdir.

Tabiat va jamiyat qonunlarida uchraydigan har qanday hodisalar tasodifiyatga bog‘liqdir. Masalan, ulardan ayrimlarini oldindan aytish mumkin, ayrimlarini esa taqribiy bashorat qilinadi: ob - havo, narx - navo, hosilning mo‘l bo‘lishi va bo‘lmasligi va hokazolarni oldindan aniq aytish qiyin.

Ehtimollar nazariyasi, ma'lum bir kompleks shartlar bajarilganda, ko‘p marta takrorlanadigan ommaviy tasodifiy hodisaning asosiy xossasi esa ehtimollik deb ataluvchi kattalik bilan ifodalanadi.

Ehtimollar nazariyasi XVII asr o‘rtalarida qimor o‘yinlarda kuzatilayotgan hodisalarning ba'zi qonuniyatlarini o‘rganishda Paskal, Ferma, Bernulli kabi olimlar jiddiy e'tibor berib, jarayonlarni o‘rganganlar va natijada bo‘lg‘usi ehtimollar nazariyasi deb ataluvchi fanning vujudga kelishiga, ya’ni yaratilishiga ulkan hissa qo‘shishgan. Ehtimollar nazariyasi turli tarmoqlarda, jumladan iqtisodiyotda, biologik, tibbiyot, qishloq xo‘jaligi, texnika va boshqa saholarida keng ko‘lamda qo‘llaniladi.

Har qanday hodisani kuzatish yoki tajriba tariqasida o‘rganish ma’lum sinovlarni o‘tkazish orqali amalga oshiriladi.

Hodisalar haqida tushuncha

Ta’rif.

Tajriba sinovlarning har qanday natijasi (yoki oqibati)ga ***hodisa*** deyiladi. Hodisalarni lotin alifbosining bosh harflari bilan ***A, B, C, ...*** kabi belgilanadi.

Odatiy turmushda, amaliy faoliyatda hamda ilmiy tekshirishlarda natijalarni to‘la ishonch bilan oldindan aytish mumkin bo‘lmagan tajribalar va sinovlar tez-tez uchrab turadi.

Masalan, tangani tashlaganda u yoki bu tomonining tushishini to‘la ishonch bilan aytish mumkin emas, nishonga o‘q uzganda tegish yoki tegmasligi aniq emas, shoshqoltosh (kubik) tashlandi. Bunda 6 ochkoning tushishi oldindan ma’lum emas, biror nomerli lotereya biletiga yutuq chiqishini ham oldindan aytib bo‘lmaydi.

Ta’rif.

Tajriba natijasida albatta ro‘y beradigan hodisalar ***muqarrar hodisa*** deyiladi va u odatda Ω harfi bilan belgilanadi.

Masalan, o‘yin soqqasi tashlanganda 1 dan 6 gacha bo‘lgan butun sonlarning tushishi,

tavakkaliga tanlangan so‘zda 45 dan ortiq bo‘lmagan harfning bo‘lishi, kun ketidan tun kelishi va hokazolar muqarrar hodisalardir.

Ta’rif.

Tajriba natijasida hech qachon ro‘y bermaydigan hodisaga esa ***mumkin bo‘lmagan hodisa*** deyiladi va odatda $\emptyset$ belgisi bilan belgilanadi. Masalan, bitta lotereyaga ikkita yutuq chiqishi, kosmik kemaning quyoshga qo‘nib qaytib kelishi, samolyotning 20 km balandlikda uchishi va hokazolar mumkin bo‘lmagan hodisalardir.

Ta’rif.

Tajriba natijasida hodisaning ro‘y berishi ham ro‘y bermasligi ham mumkin bo‘lgan hodisaga ***tasodifiy hodisa*** deyiladi.

Masalan, tanga tashlaganda gerbli tomonning tushishi, o‘q uzilganda nishonga tegishi, lotereya biletiga yutuq chiqishi, shoshqoltosh (kubik) tashlaganda 6 ochkoning tushishi va hokazolar tasodifiy hodisalarga misol bo‘ladi.

Ta’rif.

Agar bir hodisaning ro‘y berishi, boshqa bir hodisani keltirib chiqarmasa yoki boshqa hodisalarning ro‘y berishini yo‘qqa chiqarsa, bunday hodisalar ***birgalikdamas (birgalikda bo‘lmagan)*** deyiladi. Bunda har gal bitta hodisa sodir bo‘ladi.

1-misol. Detallar solingan qutidan tavakkaliga bitta detal olindi. Bunda sifatli detal chiqishi sifatsiz detal chiqishini yo‘qqa chiqaradi yoki aksincha. “Sifatli detal chiqdi” va “sifatsiz detal chiqdi” hodisalari birgalikda emas.

2-misol. Tanga tashlashda, gerbli tomoni tushishi raqamli tomon tushishini yo‘qqa chiqardi. “Gerbli tomon tushdi” va “Raqamli tomon tushdi” hodisalari birgalikda emas.

Ta’rif.

Agar bir hodisaning ro‘y berishi boshqa bir hodisani keltirib chiqarsa, bunday hodisalar ***birgalikda bo‘lgan hodisalar*** deyiladi.

Masalan “quyosh chiqdi” va “kun sovuq” bu hodisalar birgalikda bo‘lishi mumkin bo‘lgan hodisa bo‘ladi.

Ta’rif.

Tajribaning har bir natijasini ifodalovchi hodisa ***elementar hodisa*** deyiladi.

Ta’rif.

Elementar hodisalarga ajratish mumkin bo‘lgan hodisa ***murakkab hodisa*** deyiladi.

Ta’rif.

Agar bir necha hodisalardan istalgan birini tajriba natijasida ro‘y berishi boshqalariga qaraganda kattaroq imkoniyatga ega deyishga asos bo‘lmasa, bunday hodisalar ***teng imkoniyatli hodisalar*** deyiladi.

Masalan. tanga tashlanganda gerbli yoki raqamli tomoni tushishi yoki shoshqoltosh tashlanganda bir ochkoning tushishi, ikki ochkoning tushishi,..., olti ochkoning tushishi – bularning barchasi teng imkoniyatli hodisalar bo‘ladi.

Ta’rif.

A hodisaga ***qarama-qarshi hodisa*** deb, *A* hodisaning ro‘y bermasligidan iborat $\overline{A}$ hodisaga aytiladi. *A* va $\overline{A}$ hodisalar birgalikda bo‘lmasligi o‘z-o‘zidan ravshan.

Bog‘liq va bog‘liqmas hodisalar haqida tushuncha

Ehtimolliklarni ko‘paytirish qoidalarini, bog‘liq va bog‘liqmas hodisalar uchun alohida-alohida keltiramiz.

Ta’rif:

Agar ikkita hodisadan birining ro‘y berishi ikkinchi hodisaning ro‘y berish yoki ro‘y bermasligiga bog‘liq bo‘lmasa bu hodisalar ***erkli hodisalar*** deyiladi.

3-misol. Tanga ikki marta tashlangan. Birinchi tashlashda gerbli tomon tushish *(A* hodisa) ehtimolligi ikkinchi tashlashda gerbli tomon tushishi yoki tushmasligiga *(B* hodisa) bog‘liq emas. O‘z navbatida, ikkinchi tajribada gerbli tomon tushish ehtimolligi birinchi tajriba natijasiga bog‘liq emas. Shunday qilib, *A* va *B* hodisalar erkli.

Ta’rif:

Bir nechta hodisaning ixtiyoriy ikkitasi bog‘liq bo‘lmasa, ularga ***juft-juft erkli*** deyiladi.

4-misol. Tanga 3 marta tashlangan. *A,B,*C mos ravishda birinchi, ikkinchi va uchinchi tajribalarda gerbli tomon tushish hodisasi bo‘lsin. Ravshanki, ko‘rilayotgan hodisalardan har ikkitasi (ya’ni *A* va *B, A* va *C*, *B* va *C*) bog‘liq emas. Shunday qilib, *A, B* va *C* juft –juft erkli.

Ta’rif:

Agar ikki hodisadan birining ro‘y berish ehtimolligi ikkinchi hodisaning ro‘y berishi yoki ro‘y bermasligiga bog‘liq bo‘lsa, bu ***hodisalar bog‘liq*** deyiladi.

5-misol. Qutida 100 ta detal bor, shulardan 90 tasi standart 10 tasi nostandart. Tavakkaliga bitta detal olinib, qaytarib qo‘yilmaydi. Agar birinchi olishda standart detal *(A* hodisa) olingan bo‘lsa, u holda ikkinchi urinishda ham standart detal chiqish *(B* hodisa) ehtimolligi $P(B)=\frac{89}{99}$ ga teng; agar birinchi urinishda nostandart detal olingan bo‘lsa, u holda $P(B)=\frac{90}{99}$. Shunday qilib, *B* hodisaning ro‘y berish ehtimolligi *A* hodisaning ro‘y berishi yoki ro‘y bermasligiga bog‘liq. *A* va *B* hodisalar bog‘liq.

EHTIMOLLIK TA'RIFLARI

Ehtimol tushunchasi ehtimollar nazariyasining asosiy tushunchalaridan biridir. Misolda ko‘raylik:

Aytaylik, idishda yaxshilab aralashtirligan bir xil 12 ta shar bo‘lib, ulardan 5 tasi qizil, 4 tasi qora va 3 tasi oq rangli bo‘lsin. Haqiqattan ham, idishdan olingan sharning qizil yoki qora rangli sharlar bo‘lish imkoniyati oq rangli bo‘lish imkoniyatidan ko‘proq bu imkoniyatni son bilan xarakterlash mumkinmi? Ha, mumkin ekan. Mana shu son ***hodisaning ehtimolligi*** deb ataladi.

Shunday qilib, ehtimol hodisaning ro‘y berish imkoniyatini xarakterlovchi sondir.

Biz o‘z oldimizga tavakkaliga olingan sharning qizil yoki qora rangli bo‘lish imkoniyatini miqdoriy baholash vazifasini qo‘yaylik. Qizil yoki qora rangli shar chiqishini *A* hodisa sifatida qaraymiz. Tajribaning (tajriba idishdan shar olishdan iborat) mumkin bo‘lgan natijalarning har birini, ya’ni tajribada ro‘y berishi mumkin bo‘lgan har bir hodisani elementar hodisa deb ataymiz. Elementar hodisalarni $E_1, E_2, E_3, \ldots$ bilan belgilaymiz. Bizning misolda quyidagi 12 ta elementar hodisa bo‘lishi mumkin: E_1, E_2, E_3, E_4, E_5 – qizil shar chiqdi; E_6, E_7, E_8, E_9 – qora shar chiqdi; E_{10}, E_{11}, E_{12} – oq shar chiqdi.

Osongina ko‘rish mumkinki, bu natijalar yagona mumkin bo‘lgan (bitta shar albatta chiqadi) va teng imkoniyatli (shar tavakkaliga olinadi, sharlar bir xil va yaxshilab aralashtirilgan) hodisadir.

Bizni qiziqtirayotgan hodisaning ro‘y berishiga olib keladigan elementar hodisalarni bu hodisaning ro‘y berishiga qulaylik tug‘diruvchi deymiz. Bizning misolda A (qizil yoki qora rangli shar chiqishi) hodisaning ro‘y berishiga quyidagi 9 ta elementar hodisa qulaylik tug‘diradi: $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9$. A hodisaning ro‘y berishiga qulaylik tug‘diruvchi elementar hodisalar sonining ularning umumiy soniga nisbati *A* hodisaning ehtimolligi deyiladi va $P(A)$ bilan belgilanadi. Ko‘rilayotgan misolda elementar hodisalar jami 12 ta, ulardan 9 tasi *A* hodisaga qulaylik tug‘diradi. Demak, olingan sharning qizil yoki qora bo‘lish ehtimolligi: $P(A)=\frac{9}{12}=\frac{3}{4}$. Topilgan son (ehtimol) biz oldimizga qo‘ygan masaladagi qizil yoki qora shar chiqishi mumkinligining miqdoriy bahosini beradi.

Ehtimollikning turli ta’riflari mavjud. Bular klassik, statistik va geometrik ta’riflardir.

Ehtimollikning klassik ta’rifi

Ta’rif.

A hodisaning ehtimolligi deb, tajribaning bu hodisa ro‘y berishiga qulaylik tug‘diruvchi natijalari soni *m* ning tajribaning mumkin bo‘lgan barcha elementar hodisalari soni *n* ga nisbatiga aytiladi, va quyidagi ko‘rinishda belgilanadi:

$$P(A)=\frac{m}{n}$$

Ehtimollik ta'rifidan quyidagi xossalar kelib chiqadi:

1) Muqarrar hodisaning ehtimolligi 1 ga teng. Ya'ni $P(\Omega)=1$.

Haqiqattan ham, agar hodisa muqarrar bo'lsa, u holda tajribaning har qanday elementar natijasi shu hodisaning ro'y berishiga qulaylik tug'diradi. Bu holda, $m=n$ va demak,

$$P(\Omega)=\frac{m}{n}=\frac{n}{n}=1.$$

1-misol. Idishda 20 ta shar bo'lib, ular 1 dan 20 gacha raqamlangan. Idishdan tavakkaliga bitta shar olindi. Bu sharning tartib raqami 20 dan katta bo'lmaslik (A hodisa) ehtimoli qanday?

Yechish: Yashikdagi sharlarning istalganining tartib raqami 20 dan oshmaydi. Shuning uchun bu hodisaning ro'y berishiga qulaylik tug'diruvchi hodisalar soni va barcha mumkin bo'lgan hollar soni o'zaro teng: $m = n = 20$ va $P(A)=\frac{m}{n}=1$. Bu holda A hodisa muqarrar hodisadir.

2) Mumkin bo'lmagan hodisaning ehtimolligi nolga teng.

Haqiqattan ham, agar hodisa ro'y bermaydigan bo'lsa, u holda tajribaning hech bir elementar natijasi bu hodisaning ro'y berishiga qulaylik tug'dirmaydi. Bu holda, $m = 0$ va demak,

$$P(V)=\frac{m}{n}=\frac{0}{n}=0$$

2-misol. Qutida 10 ta shar bo'lib, ulardan 4 tasi oq, qolganlari qora rangda. Shu qutidan tavakkaliga bitta shar olindi. Uning qizil shar bo'lish (A hodisa) ehtimoli qanday?

Yechish: Qutida qizil shar yo'q, ya'ni $m = 0$, lekin, $n=10$. Demak, $P(A)=\frac{m}{n}=\frac{0}{n}=0$. Bu holda A hodisa mutlaqo yuz bermaydigan, ya'ni mumkin bo'lmagan hodisadir.

3) Tasodifiy hodisaning ehtimolligi musbat son bo'lib, u 0 va 1 oralig'ida bo'ladi.

Haqiqattan ham, tasodifiy hodisaning ro'y berishiga tajribaning barcha elementar hodisalarining bir qismigina qulaylik tug'diradi. Bu holda, $0<m<n$. Shuning uchun, $0<\frac{m}{n}<1$ va demak, $0<P(A)<1$.

Shunday qilib, istalgan hodisaning ehtimolligi quyidagi qo'sh tengsizlikni qanoatlantiradi:

$$0\le P(A)\le 1.$$

Endi quyidagi misollarni yechishdan oldin bitta formulani keltirib o'tamiz.

Idishda n ta shar bo'lib, ulardan n_1 -tasi oq, n_2 -tasi qora, n_3 -tasi qizil va hokazo n_k -tasi sariq. Shu idishdan tavakkaliga m ta shar olinganda, ulardan m_1 -tasi oq, m_2 -tasi qora, m_3 -tasi qizil va hokazo m_k -tasi sariq bo'lish A hodisasining ehtimolligi topish formulasi:

$$P(A)=\frac{C_{n_1}^{m_1}\cdot C_{n_2}^{m_2}\cdot C_{n_3}^{m_3}\cdot\ldots\cdot C_{n_k}^{m_k}}{C_n^m},$$

bu yerda, $0 \le m_i \le n_i$, $i = \overline{1,k}$, $\sum_{i=1}^{k} m_i = m$, $\sum_{i=1}^{k} n_i = n$.

3-misol. Qopda 12 ta shar mavjud, ular: 3 ta oq, 4 ta qora va 5 ta qizil. Tavakkaliga bitta shar olindi. Uning qora shar chiqishi (A hodisa) ehtimolini toping.

Yechish: Bizga qulaylik tug'diruvchi elementar hodisalar soni $m = 4$, hamda jami elementar hodisalar soni $n = 12$, demak, A hodisaning ehtimolligi:

$$P(A) = \frac{m}{n} = \frac{4}{12} = \frac{1}{3}.$$

4-misol. Yashikda 10 ta shar bor: 6 ta oq va 4 ta qora. Tavakkaliga 2 ta shar olindi. Ikkala shar ham oq bo'lishi A hodisaning ehtimolini toping.

Yechish: Bu masalada mumkin bo'lgan barcha holatlar soni, $n = C_{10}^2 = \frac{10 \cdot 9}{1 \cdot 2} = 45$ ga teng.

A hodisaga qulaylik tug'diruvchi hollar soni esa, $m = C_6^2 = \frac{5 \cdot 6}{1 \cdot 2} = 15$ ga teng. Bundan esa, $P(A) = \frac{m}{n} = \frac{15}{45} = \frac{1}{3}$.

5-misol. 2000 lotereya bileti sotilgan. Bunda 1 ta biletga 100000 so'm, 4 ta biletga 50000 so'm, 10 ta biletga 20000 so'm, 20 ta biletga 10000 so'm, 165 ta biletga 5000 so'm, 400 ta biletga 1000 so'mdan yutuq chiqishi belgilangan, qolgan biletlar yutuqsiz. Bitta biletga 10000 so'mdan kam bo'lmagan yutuq chiqish ehtimoli qanday?

Yechish: Bu yerda $m = 1 + 4 + 10 + 20 = 35$, $n = 2000$. Chunki, 35 ta biletga 10000 so'mdan yuqori yutuqlar belgilangan. Shuning uchun,

$$P(A) = \frac{m}{n} = \frac{35}{2000} = 0{,}0175.$$

6-misol. Do'konda 6 erkak va 4 ayol kishi ishlaydi. Tabeldagi tartib raqami bo'yicha tasodifiy ravishda 7 kishi tanlab olindi. Tanlab olinganlar orasida 3 ayol kishi bo'lish ehtimolligini toping.

Yechish: Umumiy ro'y berishlar soni ya'ni 10 kishidan 7 kishini necha xil usulda tanlash mumkinligi. Bu esa, $n = C_{10}^7 = \frac{8 \cdot 9 \cdot 10}{1 \cdot 2 \cdot 3} = 120$ ga teng va endi bizga qulaylik tug'diruvchi elementar hodisalar sonini topish kerak. Buning uchun 7 kishilik jamoani quyidagi ko'rinishda tuzamiz. 4 ta ayoldan 3 tasini va 6 ta erkakdan 4 tasini olishimiz kerak. Ya'ni $m = C_4^3 \cdot C_6^4 = \frac{4}{1} \cdot \frac{5 \cdot 6}{1 \cdot 2} = 60$. Demak, bu hodisaning ehtimolligi $P(A) = \frac{m}{n} = \frac{60}{120} = \frac{1}{2}$ ga teng.

7-misol. Turli 2 ta matematika, 2 ta fizika va 2 ta kimyo kitobi javonning bir tokchasiga qo'yilmoqda. Kimyo kitoblarining yonma-yon kelish ehtimolligi nimaga teng?

Yechish: Barcha o'rin almashishlar sonini topib olamiz, ya'ni 6 ta kitobning o'rin almashishlar soni $n = P_6 = 6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$ ta. Endi kimyo kitoblari yonma-yon kelishi uchun kimyo kitoblarini 1 ta kitob deb qarab barcha o'rin almashishlar sonini

$P_5 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$ topamiz va kimyo kitoblarini ham o‘rin almashishlar sonini hisobga olishimiz shart, yani $P_2 = 2! = 1 \cdot 2 = 2$. Bundan esa, $m = P_5 \cdot P_2 = 120 \cdot 2 = 240$. Demak, bu, ehtimollikning klassik ta’rifi bo‘yicha $P(A) = \frac{m}{n} = \frac{240}{720} = \frac{1}{3}$ ga teng ekan.

8-misol. Abonent telefon raqamini terayotganda oxirgi uchta raqamni eslay olmadi. Lekin, raqamlar turli ekanligini biladi. Barcha terishlardan to‘g‘ri nomerni terish ehtimolligi nimaga teng bo‘ladi?

Yechish: To‘g‘ri raqamni terish hodisasini *A* bilan, uning ehtimolligini esa $P(A)$ bilan belgilaymiz.

Oxirgi uchta raqamni A_{10}^3 usul bilan terish mumkin. Shunda jami sinovlar soni $n = A_{10}^3 = \frac{10!}{(10-3)!} = 8 \cdot 9 \cdot 10 = 720$ ga teng bo‘ladi. Izlanalayotgan telefon raqami shu 720 tadan bittasi bo‘ladi. Ya’ni, $m = 1$. Ehtimollikning klassik ta’rifi bo‘yicha $P(A) = \frac{m}{n} = \frac{1}{720}$ bo‘ladi.

MASALALAR

1. Lotereyada 1000 ta bilet bor. Ulardan 500 tasi yutuqli, 500 tasi yutuqsiz. Ikkita bilet sotib olindi. Ikkala biletning ham yutuqli bo‘lish ehtimoli qanday?
2. Tanga ikki marta tashlandi. Ikki marta ham gerb tushish ehtimoli qanday?
3. 20 ta kitob javonlarga tavakkaliga taxlandi. 20 ta kitobdan aniq 5 tasining yonma-yon turishi (A hodisa) ehtimoli nimaga teng?
4. Ikkita shoshqoltosh tashlangan. Ularning yoqlarida chiqqan ochkolar yig‘indisi juft son. Shu bilan birga tashlangan soqqalarning kamida bittasida har doim 6 ochko tushish ehtimolini toping.
5. Ikkita shoshqoltosh tashlangan. Ularning yoqlarida chiqqan ochkolar yig‘indisi 5 ga, ko‘paytmasi esa 4 ga teng bo‘lish ehtimolini toping.
6. Ikkita shoshqoltosh tashlangan bo‘lib, ularning yoqlarida chiqqan ochkolar yig‘indisi 7 ga teng bo‘lish ehtimolini toping.
7. Raqamlari har xil ikki xonali son o‘ylangan. O‘ylangan son tasodifan aytilgan ikki xonali son bo‘lish ehtimolini toping.
8. Qutida 10 ta shar bor: 7 ta qora va 3 ta oq. Qutidan tasodifiy ravishda bir shar olindi. Bu shar: a) oq; b) qora shar bo‘lish ehtimolligini toping.
9. “DAFTAR” so‘zidan tasodifiy ravishda bitta harf tanlandi. Bu “U” harfi bo‘lish ehtimolligi nimaga teng? Bu unli harf bo‘lish ehtimolligi-chi?
10. Bir xaltada 5 ta ko‘k va 3 ta qizil shar bor. Qaytarib qo‘ymaslik sharti bilan ketma-ket olingan ikkita sharning biri ko‘k ikkinchisi qizil bo‘lish ehtimolligi qancha?
11. 3 ta o‘g‘il bola va 3 ta qiz bola ixtiyoriy ravishda bir qatorga o‘tirishdi. O‘g‘il bolalarning

yonma-yon, qiz bolalarning yonma-yon o‘tirishlari ehtimolligini aniqlang.

12. Bir savatdagi 12 tuxumdan faqat 4 tasi oq rangda. Ixtiyoriy ravishda olingan 3 ta tuxumning uchalasi ham oq rangda bo‘lmasligi ehtimolligini toping.

Ehtimollikning statistik ta’rifi

Ehtimolning “klassik” ta’rifida tajribaning elementar hodisalar soni chekli deb faraz qilinadi. Amalyotda esa mumkin bo‘lgan natijalar soni cheksiz bo‘lgan tajribalar ancha ko‘p uchrab turadi. Bunday hollarda klassik ta’rifni qo‘llab bo‘lmaydi. Shu hollarning o‘zi ham klassik ta’rifning cheklangan ekanligini ko‘rsatadi. To‘g‘ri, bu kamchilikni ehtimol ta’rifini tegishlicha umumlashtirish yo‘li bilan bartaraf qilish mumkin. Shu sababli klassik ta’rif bilan bir qatorda hodisaning ehtimolligi sifatida nisbiy chastota yoki unga yaqin sonni olib statistik ta’rifdan ham foydalaniladi.

Nisbiy chastota ehtimol bilan bir qatorda ehtimollar nazariyasining asosiy tushunchalaridan biri hisoblanadi.

Ta’rif:

Hodisaning nisbiy chastotasi deb, hodisa ro‘y bergan tajribalar sonining aslida o‘tkazilgan jami tajribalar soniga nisbatiga aytiladi. Shunday qilib, A hodisaning nisbiy chastotasi quyidagi formula bilan aniqlanadi:

$$W(A)=\frac{m}{n},$$

bu yerda m – hodisaning ro‘y berish soni, n – tajribalarning jami soni.

Ehtimol va nisbiy chastota ta’riflarini solishtirib, quyidagi xulosaga kelamiz: ehtimolning ta’rifida tajribalarning haqiqattan o‘tkazilganligi talab qilinmaydi, nisbiy chastotaning ta’rifida esa tajribalarning aslida o‘tkazilganligi talab qilinadi. Soddaroq aytganda, ehtimol tajribadan oldin (ilgari), nisbiy chastota esa tajribadan keyin (so‘ng) hisoblanadi.

Agar $m=n$ bo‘lsa, ya’ni o‘tkazilgan tajribalar soni hodisaning ro‘y berishlar soniga teng bo‘lsa, bu hodisa muqarrar hodisa bo‘ladi.

Agar $m=0$ bo‘lsa, ya’ni o‘tkazilgan tajriba natijasida hodisa biror marta ham sodir bo‘lmasa, u holda bu hodisa mumkin bo‘lmagan hodisa bo‘ladi.

1-misol. Mergan nishonga qarata 30 ta o‘q uzdi. Bunda ulardan 23 tasi nishonga tekkanligi ma’lum bo‘lsa, mergan o‘qlarining nishonga tegishining nisbiy chastotasini toping.

Yechish: Mergan o‘qlarining 23 tasi nishonga tegdi, demak, hodisaning ro‘y berishlar soni $m=23$ va jami uzilgan o‘qlar soni $n=30$, demak, bu hodisaning nisbiy chastotasi $W(A)=\frac{23}{30}$ bo‘ladi.

2-misol. Dastlabki 1000 ta natural sonlar ichidan olingan sonning 5 ga karrali bo‘lishining nisbiy chastotasini toping.

Yechish: Bu yerda sonning 5 ga karrali chiqish hodisasini A bilan, uning nisbiy chastotasini esa $W(A)$ bilan belgilaymiz. O‘tkazilgan jami sinovlar soni

$n = 1000$ ga, dastlabki 1000 ta natural sonlar ichida 5 ga karrali 200 ta natural son bor, demak, $m = 200$, nisbiy chastota esa $W(A) = \frac{200}{1000} = \frac{1}{5}$.

3-misol. Bir mamlakatga xorijdan kelgan sayyohlar va shu mamlakat hududida sayohat qilgan fuqarolar (ichki sayyohlar) haqida quyidagi ma'lumotlar berilgan bo'lsin.

Yillar	Xorijiy sayyohlar soni	Ichki sayyohlar	Sayyohlar soni
2014	610623	403989	1014612
2015	746224	348953	1095177
2016	822558	316897	1139455
2017	774262	346103	1120365
2018	811314	351028	1162342
$\sum$	3764981	1766970	5531951

Qaralayotgan yillarda mamlakat ichida sayohat qilgan mamlakat fuqarolari sonining nisbiy chastotasini toping.

Mamlakat ichida sayohat qilgan mamlakat fuqarolari soni: ***M*** = 1766970

Xorijiy sayyohlar soni: ***K*** = 3764981

Umumiy sayyohlar soni: $N = 1766970 + 3764981 = 5531951$. U holda,

$$W = \frac{M}{N} = \frac{1766970}{5531951} \approx 0{,}3194.$$

MASALALAR

1. Texnik nazorat bo'limi tasodifan ajratib olingan 100 ta kitobdan iborat partiyada 5 ta yaroqsiz kitob topdi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.

2. Nishonga 20 ta o'q uzilgan. Shundan, 18 ta o'q nishonga tekkani qayd qilindi. Nishonga tegishning nisbiy chastotasini toping.

3. Buyumlar partiyasini sinashda yaroqli buyumlar nisbiy chastotasi 0,9 ga teng bo'ldi. Agar hammasi bo'lib, 200 ta buyum tekshirilgan bo'lsa, yaroqli buyumlar sonini toping.

4. Bir shaharda 920 ta odamdan ishga qanday yetib borishlarini so'rashganda ularning: 350 tasi mashinada, 420 tasi jamoat transportida, 80 tasi velosipedda, 70 tasi piyoda borishlari ma'lum bo'ldi.

1) Mashinada; 2) jamoat transportida; 3) velosipedda; 4) piyoda boruvchilar sonining nisbiy chastotasini toping.

5. Jadvalning oxirgi ustunini to'ldiring.

Tartib raqam	Tajriba	Tajribalar soni(N)	*A* hodisa	*A* hodisaning chastotasi	*A* hodisaning nisbiy chastotasi $\left(W(A)=\frac{M}{N}\right)$
1	Tanga tashlash	150	raqamli tomonining tushishi	78	
2	Sportchi kamondan nishonga otyapti	200	nishonga tegishi	182	
3	O‘yin kubogi tashlanyapti	400	4 tushishi	67	

Ehtimollikning geometrik ta’rifi.

Barcha nuqtalari teng imkoniyatga ega bo‘lgan biror soha (chiziq, yuza yoki hajm) berilgan bo‘lib, bu sohaga tashlangan nuqtaning unga tushishi muqarrar bo‘lsin. Shu berilgan sohadan kichkina sohacha (chiziqcha yoki yuzacha yoki hajmcha) ajrataylik. Sohaga tashlangan nuqtaning ajratilgan sohachaga tushish ehtimolligi so‘ralgan bo‘lsin. Ajratilgan sohacha qancha katta bo‘lsa, tushish ehtimolligi ham kattalashib boradi, sohacha sohaga tenglashganda esa tushish ehtimolligi muqarrar hodisaga aylanadi. Demak, tashlangan nuqtaning sohachaga tushish ehtimolligi sohacha kattaligiga to‘g‘ri proporsional bo‘lib, uni geometrik nuqtai nazardan talqin qilish kerak bo‘ladi. Bu joyda ehtimollikning klassik yoki statistik ta’riflaridan foydalanish unchalik to‘g‘ri emas. Bunday hollarda ehtimollikning geometrik ta’rifidan foydalanish qulaydir.

Agar tashlangan nuqtaning Ω sohaga tushishi muqarrar bo‘lsa, u holda bu nuqtaning shu sohadan ajratilgan ω sohachaga tushish ehtimolligi ω sohacha o‘lchovining Ω soha o‘lchoviga nisbatiga teng bo‘ladi:

$$P(A)=\frac{m(\omega)}{m(\Omega)},$$

$m(\omega)$ – bu erda ω – sohaning o‘lchovi, ya’ni bir o‘lchovli holda uzunlik, ikki o‘lchovlida yuza, uch o‘lchovlida hajm va hokazo.

Agar Ω sohani o‘lchovi L chiziq va ω sohachani l chiziqcha deb olsak, L chiziqqa tashlangan nuqtaning l chiziqchaga tushish ehtimolligi quyidagicha bo‘ladi:

$$P(A)=\frac{l}{L}.$$

Agar Ω sohani S yuza va ω sohachani s yuzacha deb olsak, S yuzaga tashlangan nuqtaning s yuzachaga tushish ehtimolligi quyidagicha bo‘ladi:

$$P(A)=\frac{s}{S}.$$

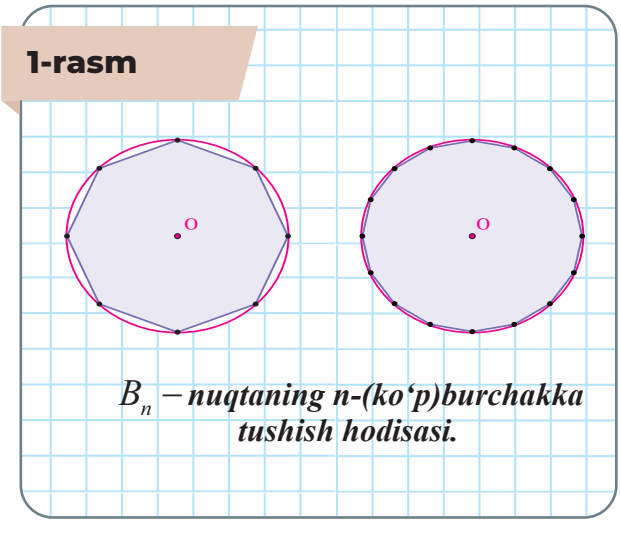
Agar Ω sohani V hajm va ω sohachani v hajmcha deb olsak, V hajmga tashlangan nuqtaning v hajmchaga tushish ehtimolligi quyidagicha bo‘ladi:

$$P(A)=\frac{v}{V}.$$

Geometrik ta’rifdan vaqtga nisbatan ham foydalanish mumkin. Agar voqea T vaqt ichida sodir bo‘lishi muqarrar bo‘lsa, bu voqeaning t vaqt ichida sodir bo‘lish ehtimolligi quyidagicha bo‘ladi:

$$P(A)=\frac{t}{T}.$$

1-rasm



B_n – nuqtaning n-(ko‘p)burchakka tushish hodisasi.

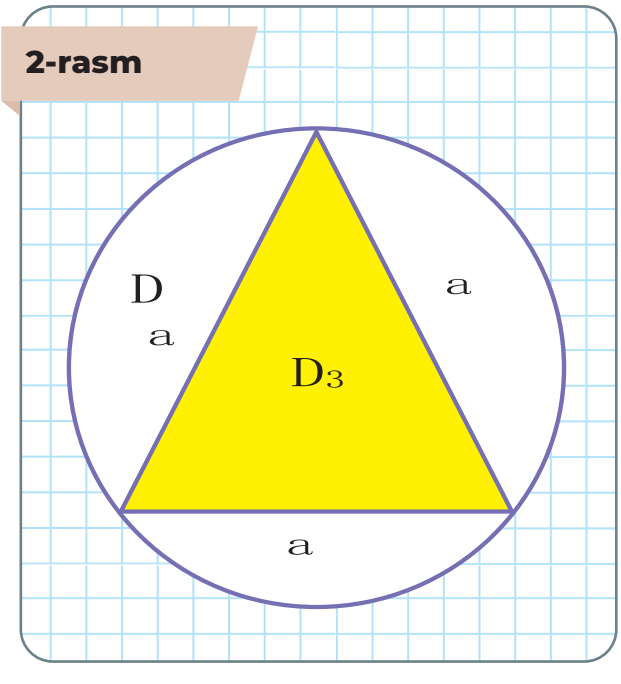
1-misol. R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam n-(ko‘p)burchak ichiga tushish ehtimolligini toping.

Yechish: $S(D_n)$ – n(ko‘p)burchakning yuzi, $S(D)$ – doiraning yuzi bo‘lsin. (1-rasm)

U holda: $$P(B_n)=\frac{S(D_n)}{S(D)}=\frac{n\cdot\frac{R^2}{2}\cdot\sin\frac{2\pi}{n}}{\pi R^2}=\frac{n\cdot\sin\frac{2\pi}{n}}{2\pi}=\frac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}}$$

a) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam uchburchak ichiga tushish ehtimolligini toping.

2-rasm



Yechish: $S(D_3)$ – uchburchakning yuzi, $S(D)$ – doiraning yuzi bo‘lsin (2-rasm).

B_3 – nuqtaning uchburchakga tushish hodisasi. U holda:

$$P(B_3)=\frac{S(D_3)}{S(D)}=\frac{3\sqrt{3}R^2}{4\pi R^2}=\frac{3\sqrt{3}}{4\pi}\approx 0,4137$$

b) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan kvadrat ichiga tushish ehtimolligini toping.

Yechish: $S(D_4)$ – kvadratning yuzi, $S(D)$ – doiraning yuzi bo‘lsin (3-rasm).

B_4 – nuqtaning kvadratga tushish hodisasi.

U holda:
$$P(B_4)=\frac{S(D_4)}{S(D)}=\frac{2R^2}{\pi R^2}=\frac{2}{\pi}\approx 0,637.$$

c) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam oltiburchak ichiga tushish ehtimolligini toping.

Yechish: $S(D_6)$ – oltiburchakning yuzi, $S(D)$ – doiraning yuzi bo‘lsin. (4-rasm)

B_6 – nuqtaning uchburchakka tushish hodisasi.

U holda,

$$P(B_6)=\frac{S(D_6)}{S(D)}=\frac{3\sqrt{3}R^2}{2\pi R^2}=\frac{3\sqrt{3}}{2\pi}\approx 0,8274;$$

d) $P(B_8)=\frac{S(D_8)}{S(D)}=\frac{2\sqrt{2}R^2}{\pi R^2}=\frac{2\sqrt{2}}{\pi}\approx 0,9045;$

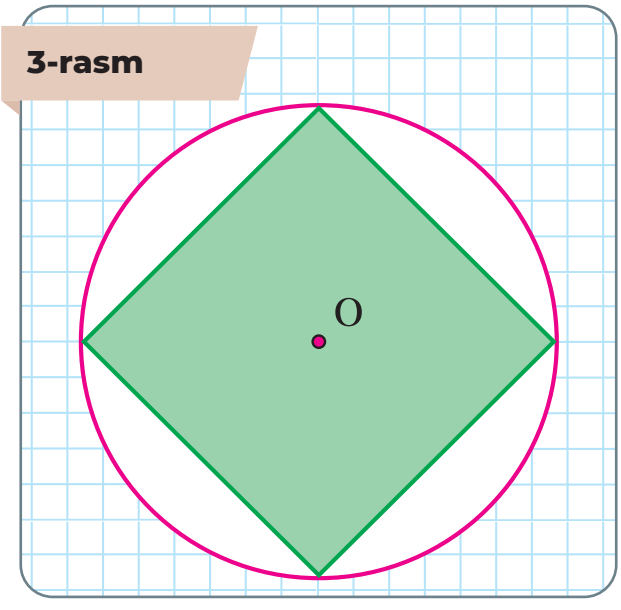
e) $P(B_{12})=\frac{S(D_{12})}{S(D)}=\frac{3R^2}{\pi R^2}=\frac{3}{\pi}\approx 0,955;$

2-misol. Uzunligi 30 sm bo‘lgan L kesma uzunligi 12 cm bo‘lgan l kesma joylashtirilgan. Katta kesmaga tavakkaliga qo‘yilgan nuqtaning kichik kesmaga ham tushish ehtimolligini toping. Nuqtaning kesmaga tushish ehtimolligi kesmaning uzunligiga to‘g‘ri proporsional bo‘lib, uning joylashishiga bog‘liq emas deb faraz qilinadi.

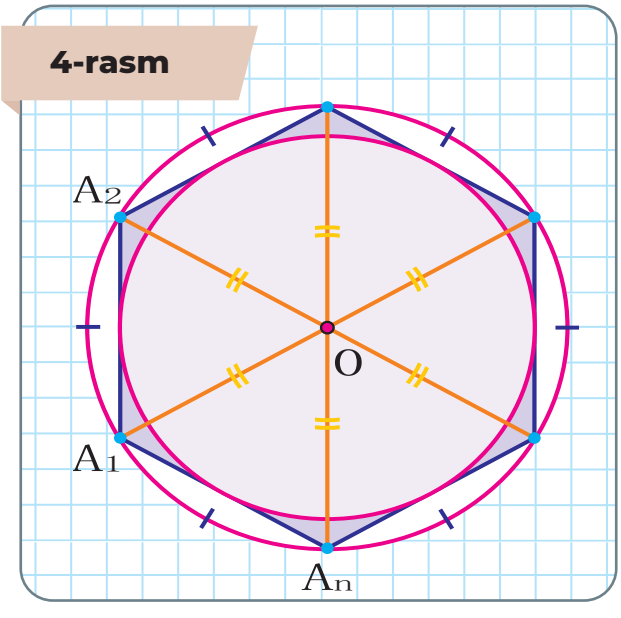
Yechish: Tashlangan nuqtaning L kesmaga tushishi muqarrar. $P(E)$ – bu L kesmada joylashgan l kesmaga tushish ehtimolligini topamiz (5-rasm). Rasmda faqatgina uch holati ko‘rsatilgan. Lekin l kesma L ning istalgan qismida joylashgan bo‘lishi mumkin.

$$P(E)=\frac{l}{L}=\frac{12}{30}=\frac{2}{5}.$$

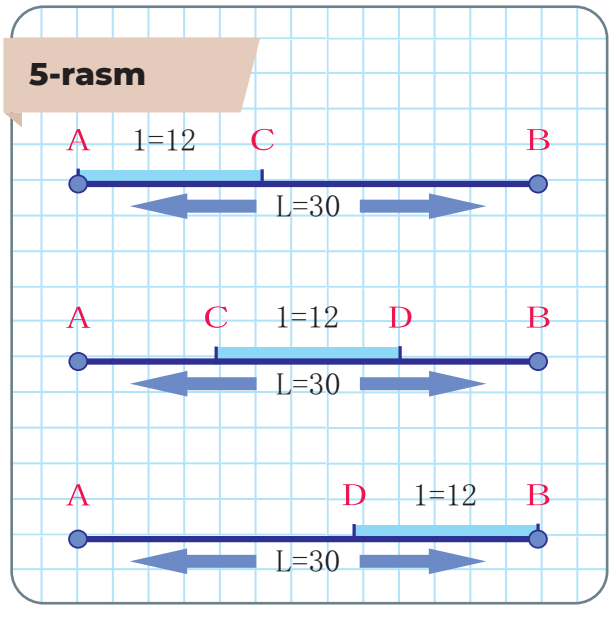
3-rasm



4-rasm

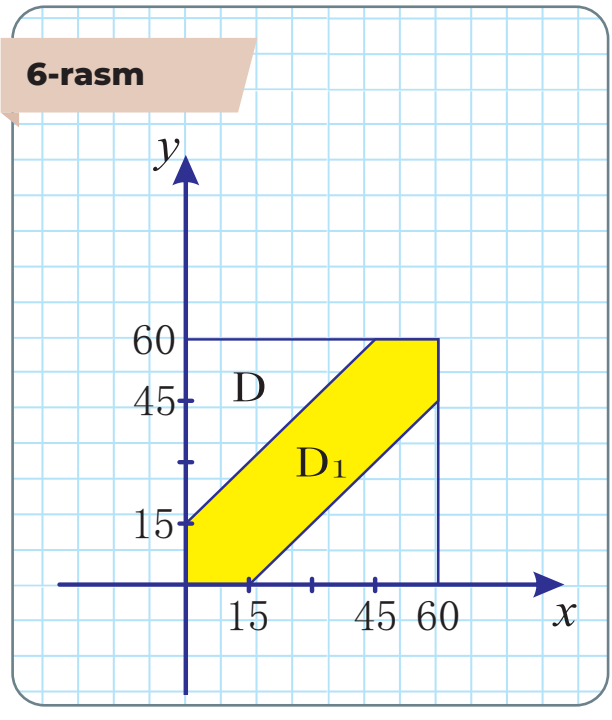


5-rasm



3-misol. Ikki do‘st soat 9 bilan 10 orasida uchrashmoqchi bo‘lishdi. Birinchi kelgan kishi do‘stini 15 minut davomida kutishi avvaldan shartlashib olindi. Agar bu vaqt mobaynida do‘sti kelmasa, u ketishi mumkin. Agar ular soat 9 bilan 10 orasidagi ixtiyoriy paytda kelishlari mumkin bo‘lib, kelish paytlari ko‘rsatilgan vaqt mobaynida tasodifiy bo‘lsa, va o‘zaro kelishib olingan bo‘lmasa, bu ikki do‘stning uchrashish ehtimolligi nimaga teng?

6-rasm



Yechish: Birinchi kishining kelish vaqt momenti x, ikkinchisiniki esa y bo‘lsin. Ularning uchrashishlari uchun $|x-y|\le 15$ tengsizlikning bajarilishi zarur va yetarlidir. x va y larni tekislikdagi Dekart koordinatalari sifatida tasvirlaymiz va masshtab birligi deb minutlarni olamiz. Ro‘y berishi mumkin bo‘lgan barcha imkoniyatlar tomonlari 60 bo‘lgan kvadrat nuqtalaridan va uchrashishga qulaylik to‘g‘diruvchi imkoniyatlar bo‘yalgan soha nuqtalaridan iborat (6-rasm).

Demak, ehtimollikning geometrik ta’rifiga ko‘ra, izlanayotgan ehtimollik bo‘yalgan soha yuzasini kvadrat yuzasiga bo‘lgan nisbatga teng:

$S(D_1)$ – bo‘yalgan sohaning yuzi, $S(D)$ – kvadratning yuzi bo‘lsin (6-rasm).

A–do‘stlarning uchrashish hodisasi.

$$S(D_1)=60\cdot 60-2\cdot\frac{45\cdot 45}{2}=1575;\ S(D)=60\cdot 60=3600.$$

Izlanayotgan ehtimollik:

$$P(A)=\frac{S(D_1)}{S(D)}=\frac{1575}{3600}=\frac{7}{16};\qquad P(A)=\frac{7}{16}.$$

MASALALAR

1. Radiusi 20 sm bo‘lgan doira ichida bir-biri bilan kesishmaydigan va birining radiusi 5 sm, ikkinchisiniki 10 sm bo‘lgan ikkita aylana o‘tkazilgan. Katta doira ichida tavakkaliga olingan nuqta kichik aylanalardan birining ichida bo‘lish ehtimolligini toping.

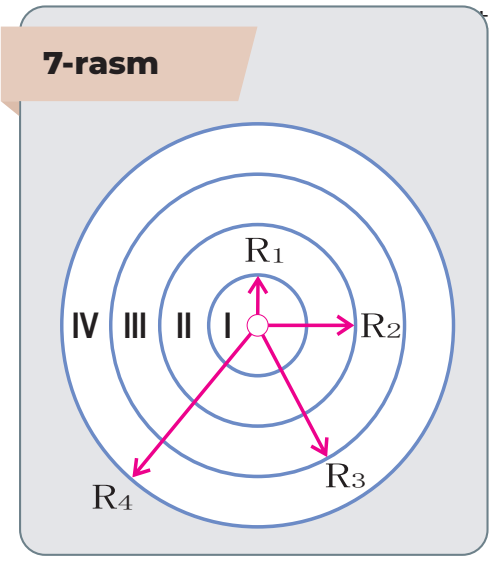
2. Ikki do‘st ma’lum joyda soat 10 bilan 11 orasida uchrashishga kelishishdi. Birinchi kelgan ikkinchini 20 daqiqa davomida kutadi, shundan so‘ng ketadi. Agar ko‘rsatilgan vaqt oralig‘ida do‘stlarning kelish momentlari teng imkoniyatli bo‘lsa, ularning uchrashish ehtimolligini toping.

3. Qattiq bo‘ron natijasida 40- va 70- kilometrlar oralig‘ida telefon simi uzilgan. Uzilish 50- va 55- kilometrlar orasida sodir bo‘lish ehtimolligini toping.

4. Doiraga kvadrat ichki chizilgan. Doira ichiga tavakkasiga tashlangan nuqta kvadrat ichida bo‘lib qolish ehtimolligi qancha?

Nishon radiuslari $R_1 = r,\ R_2 = 2r,\ R_3 = 3r,\ R_4 = 4r$ bo‘lgan konsentrik doiradan iborat. Agar nishonga otilgan nayzaning doiraga tegishi muqarrar bo‘lsa, u holda nayzaning har bir sohaga tushish ehtimolligini toping (7-rasm).

7-rasm



5. Bir tomoni ko‘k, besh tomoni sariq bo‘lgan kub ikki marta tashlanganda pastda qolgan tomonining bir xil rangda bo‘lish ehtimolligi qancha?

6. Ikkita shoshqoltosh baravar tashlandi. Tushgan sonlar yig‘indisining beshga teng bo‘lish ehtimolligini toping.

7. Yashikda 9 ta oq va 12 ta qora shar bor. Tavakkaliga 4 ta shar olindi. Ularning ichida aqalli bittasi oq bo‘lish ehtimolligini toping.

8. Guruhda 30 ta talaba bo‘lib, ulardan 10 tasi matematika to‘garagiga qatnashadi. Guruh ichida tavakkaliga 6 ta talaba tanlab olindi. Ularning ichidan hech bo‘lmaganda bittasi matematika to‘garagiga qatnashadigan talaba bo‘lish ehtimolligini toping.

9. Biror yashikda 14 ta qizil va 6 ta ko‘k tugma bor. Tavakkaliga 2 ta tugma olindi. Olingan ikkala tugmaning bir xil rangli bo‘lish ehtimolligini toping.

10. Uchta mergan bir vaqtda va bir-birlaridan mustaqil ravishda bitta nishonga qaratib faqat bittadan o‘q uzishadi. Agar merganlardan aqalli bittasi nishonga o‘qni tekkizgan bo‘lsa, nishon otilgan bo‘ladi. Har qaysi ovchining nishonga o‘q tekkizish ehtimolligi 0,4 ga teng. Nishonning otilish ehtimolligini toping.

11. 3 ta ko‘k va 4 ta yashil sharlardan ixtiyoriy tanlangan 3 ta sharning 2 tasi ko‘k 1 tasi yashil rangda bo‘lishi ehtimolini topping.

12. Shoshqol tosh bir marta tashlangan, juft ochko tushish ehtimolligini toping.

13. Tashish vaqtida 10 000 ta tarvuzdan 26 tasi yorilgan. Yorilgan tarvuzning nisbiy chastotasini toping.

14. Qutida 7 ta oq, 3 ta qora shar bor. Undan tavakkaliga olingan sharning oq bo‘lish ehtimolligini toping.

15. Telefonda nomer terayotgan abonent oxirida ikkita raqamini esdan chiqarib qo‘ydi va faqat bu raqamlar har xil ekanligini bilgan holda ularni tavakkal terdi. Kerakli raqamlar terilganligi ehtimolligini toping.

16. Qurilma 5 ta elementdan iborat bo‘lib, ularning 2 tasi eskirgan. Qurilma ishga tushirilganda tasodifiy ravishda 2 ta element ulandi. Ishga tushirishda eskirmagan elementlar

ulangan bo‘lish ehtimolligini toping.

17. Texnik nazorat bo‘limi tasodifan ajratib olingan 100 ta kitobdan iborat partiyada 5 ta yaroqsiz kitob topdi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.

18. Nishonga 20 ta o‘q uzilgan. Shundan 18 ta o‘q nishonga tekkani qayd qilindi. Nishonga tegishning nisbiy chastotasini toping.

19. Buyumlar partiyasini sinashda yaroqli buyumlar nisbiy chastotasi 0,9 ga teng bo‘ldi. Agar hammasi bo‘lib, 200 ta buyum tekshirilgan bo‘lsa, yaroqli buyumlar sonini toping.

20. Qutida m ta oq va n ta qora sharlar bor. Qutidan tavakkal bitta shar olingan. Olingan sharning oq bo‘lishi ehtimolligini toping.

21. Qutida 10,11,12,13,...,19 gacha nomerlangan kartochkalar bor, tavakkaliga bitta kartoch ka olingan:

a) olingan kartochkada 1 raqamining bo‘lishi;

b) olingan kartochkada 23 soni bo‘lishi;

c) olingan kartochkaning 3 ga karrali bo‘lishi ehtimolligini toping.

22. Qutida 5 ta bir xil buyum bo‘lib, ularning 3 tasi bo‘yalgan. Tavakkaliga olingan 2 ta buyum orasida:

a) bitta bo‘yalgan bo‘lishi;

b) ikkita bo‘yalgan bo‘lishi;

c) hech bo‘lmaganda bitta bo‘yalgan bo‘lishi ehtimolligini toping.

23. Tavakkaliga 20 dan katta bo‘lmagan natural son tanlanganda, uning 5 ga karrali bo‘lish ehtimolligini toping.

Ikkita o‘yin soqqasi barobar tashlanganda quyidagi hodisalarning ro‘y berish ehtimolligini toping:

a) tushgan ochkolar yig‘indisi 8 ga teng;

b) tushgan ochkolar ko‘paytmasi 8 ga teng.

24. Bitta shoshqol tosh (kubik, o‘yin soqqasi) tashlangan. Quyidagi ehtimollarni toping.

a) juft ochko tushishi;

b) 5 ochkodan kam bo‘lmagan ochko tushishi.

25. Qutichada 6 ta bir xil (nomerlangan) kubik bor. Tavakkaliga bitta-bittadan barcha kubiklar olinganda kubiklarning nomerlari o‘sib borish tartibida chiqishi ehtimolligini toping.

26. Qutida 12 ta oq, 8 ta qizil shar bor. Tavakkaliga

a) bitta shar olinganida uning oq bo‘lishi;

b) bitta shar olinganda uning qizil bo‘lishi;

c) 2 ta shar olinganda ularning turli rangda bo‘lishi;

d) 8 ta shar olinganda ularning 3 tasi qizil rangli bo‘lishi ehtimolligini toping.

27. Qutida 100 ta lampochka bo‘lib, ularning 10 tasi yaqoqsiz. Tavakkaliga 4 ta lampochka olinadi. Olingan lampochkalar ichida:

a) yaroqsizi bo‘lmasligi;

b) yaroqlilari bo‘lmasligi ehtimolligini toping.

TAKRORLASH

FUNKSIYA VA UNING XOSSALARI

1. Funksiyalarning aniqlanish sohasini toping.

a) $f(x)=\dfrac{x-3}{x^2-4}$ b) $y=\sqrt{3x-x^3}$ c) $y=\dfrac{1}{\sqrt{x-5}-\sqrt{9-x}}$

d) $y=\sqrt{\dfrac{(x-1)(3-x)}{x(4-x)}}$ e) $y=\sqrt{\dfrac{x(x+1)}{(x-2)(4-x)}}$ f) $y=\sqrt{25-x^2}+\dfrac{2x-3}{x+1}$

2. Agar $f(x)=x^2$ va $g(x)=2x-1$ bo‘lsa, x ning nechta qiymatida $f(g(x))=g(f(x))$ bo‘ladi?

3. Agar $f(x+1)=x^2-3x+2$ bo‘lsa, $f(x)=?$

4. Agar $f(x)=\sqrt{x^3-1}$ bo‘lsa, $f\left(\sqrt[3]{x^2+1}\right)$ nimaga teng?

5. Agar $f(x)=\dfrac{1-x}{1+x}$ bo‘lsa, $f\left(\dfrac{1}{x}\right)+\dfrac{1}{f(x)}$ nimaga teng?

6. Funksiyalar qanday qiymatlarni qabul qiladi?

a) $f(x)=\dfrac{3}{x-4}$ b) $f(x)=\dfrac{2x}{1+x^2}$ c) $f(x)=\dfrac{|x-2|}{x-2}+2$

d) $y=-x^4+2x^2+5$ e) $y=\dfrac{x^2-4x+9}{x^2-4x+5}$ f) $y=\sqrt{x^2-6x+11}$

7. Berilgan funksiyalardan qaysi biri juft funksiya?

a) $y=\dfrac{5x^2}{(x-3)^2}$ b) $y=\dfrac{x(x-2)(x-4)}{x^2-6x+8}$ c) $y=x^2+|x+1|$

d) $f(x)=x^3-\dfrac{2}{x^3}$ e) $y=\begin{cases}-x^2, x<0\\ x^2, x\geq 0\end{cases}$ f) $y=\sqrt{x^2-6x+11}$

8. Berilgan funksiyalardan qaysi biri toq funksiya?

a) $y=3x^5+x^3$ b) $y=(0,25)^x+(0,25)^{-x}$ c) $y=\begin{cases}x, x<0\\ -x, x\geq 0\end{cases}$

d) $y=|x|-1$ e) $y=\dfrac{x^4-2x^2}{3x}$ f) $y=\sqrt{3-x^2-2x}$

RATSIONAL TENGLAMALAR

1. t ning qanday qiymatlarida $18x+7=5$ va $18x+7+t=5+t$ tenglamalar teng kuchli bo‘ladi?

2. $\dfrac{4x^2-7x-2}{x^2-5x+6}=0$

3. $2+\dfrac{4}{x^2}=\dfrac{9}{x}$

4. $1-\dfrac{15}{x}=\dfrac{16}{x^2}$

5. $\dfrac{9}{x}+\dfrac{13}{2x}=2$

6. $\dfrac{2}{x-3}=\dfrac{x}{x+3}$

7. $\dfrac{x^3-3x^2}{x+2}\cdot\dfrac{x^2-4}{x^2}=0$

8. $\dfrac{1-x}{(2-x)(x-3)}+1=\dfrac{1}{2-x}$

9. $\dfrac{1}{x^2-9}+\dfrac{1}{3x-x^2}=\dfrac{3}{2x+6}$

10. $\left(\dfrac{x-1}{x+1}\right)^2-\left(\dfrac{x+2}{x-2}\right)^2=0$

11. $\dfrac{1}{x}+\dfrac{36}{9x-x^2}-\dfrac{x-5}{9-x}=0$

12. $\dfrac{x-49}{50}+\dfrac{x-50}{49}=\dfrac{49}{x-50}+\dfrac{50}{x-49}$

13. $5-\dfrac{x^2-14x-51}{x^2-x-12}=\dfrac{3}{x-4}$

14. $\dfrac{2}{x^2-4}+\dfrac{x-4}{x^2+2x}=\dfrac{1}{x^2-2x}$

15. $\dfrac{30}{x^2-1}-\dfrac{13}{x^2+x+1}=\dfrac{18x+7}{x^3-1}$

16. $\dfrac{x^2-3x}{x-2}+\dfrac{x-2}{x^2-3x}=2,5$

17. $\dfrac{4}{x^2-3x+2}-\dfrac{3}{2x^2-6x+1}+1=0$

18. $\dfrac{x-2}{x+1}+\dfrac{4(x+1)}{x-2}=5$

19. $\dfrac{x^2-x}{x^2-x+1}-\dfrac{x^2-x+2}{x^2-x-2}=1$

20. $\dfrac{1}{x^2+2x-3}+\dfrac{18}{x^2+2x+2}=\dfrac{18}{x^2+2x+1}$

21. $x^2+\dfrac{x^2}{(x+1)^2}=\dfrac{40}{9}$

22. Poyezd yo‘lda 30 minut to‘xtab qoldi. Poyezd jadval bo‘yicha yetib kelishi uchun haydovchi 80 km masofada tezlikni $8\dfrac{km}{h}$ ga oshirdi. Poyezd jadval bo‘yicha qanday tezlik bilan yurishi kerak edi?

23. Daryo oqimi bo‘yicha motorli qayiqda 28 km va oqimga qarshi 25 km o‘tildi. Bunda butun yo‘lga sarflangan vaqt turg‘un suvda 54 kmni o‘tish uchun ketgan vaqtga teng. Agar daryo oqimining tezligi $2\dfrac{km}{h}$ bo‘lsa, motorli qayiqning turg‘un suvdagi tezligini toping.

24. $\dfrac{x}{2}+\dfrac{2}{x}=\dfrac{x}{3}+\dfrac{3}{x}$

25. $\dfrac{1+x}{6}-\dfrac{6}{1+x}\quad\dfrac{4}{x+1}-\dfrac{x+1}{4}$

26. $\dfrac{x-2}{x+2}+\dfrac{x+2}{x-2}=3\dfrac{1}{3}$

27. $\dfrac{2x+1}{2x-1}\quad\dfrac{2x-1}{2x+1}=5,2$

28. $\dfrac{x^2-2x}{x-1}-\dfrac{2x-1}{1-x}=3$

29. $\dfrac{2}{x-4}+\dfrac{4}{x^2-4x}=0,625$

30. $\dfrac{(x^2+1)x}{(x^2-x+1)^2}=\dfrac{10}{9}$

31. $\dfrac{1}{x-1}+\dfrac{2}{x-2}+\dfrac{3}{x-3}=\dfrac{6}{x+6}$

32. $x^2+\dfrac{1}{x^2}-3\left(x+\dfrac{1}{x}\right)+4=0$

33. $x^4-6xv^3+10x^2-6x+1=0$

34. $31\left(\dfrac{24-5x}{x+1}+\dfrac{5-6x}{x+4}\right)+370=29\left(\dfrac{17-7x}{x+2}+\dfrac{8x+55}{x+3}\right)$

35. $\dfrac{x+3}{4x^2-9}-\dfrac{3-x}{4x^2+12x+9}=\dfrac{2}{2x-3}$

36. $\dfrac{30}{x^2-1}+\dfrac{7-18x}{x^3+1}=\dfrac{13}{x^2-x+1}$

37. $\dfrac{2x+7}{x^2+5x-6}+\dfrac{3}{x^2+9x+18}=\dfrac{1}{x+3}$

38. $2x^4+x^3-x^2+x+2=0$

RATSIONAL TENGLAMALAR SISTEMASI

1. $\begin{cases}\dfrac{x}{y}+\dfrac{y}{x}=2,5\\ x^2-y^2=3\end{cases}$

2. $\begin{cases}x+y+\dfrac{x}{y}=9\\ \dfrac{(x+y)x}{y}=20\end{cases}$

3. $\begin{cases}2xy-\dfrac{3x}{y}=15\\ xy+\dfrac{x}{y}=15\end{cases}$

4. $\begin{cases}\dfrac{1}{x+y}+\dfrac{2}{x-y}=3\\ \dfrac{3}{x+y}+\dfrac{4}{x-y}=7\end{cases}$

5. $\begin{cases}\dfrac{x}{y}-\dfrac{3y}{x}=\dfrac{1}{2}\\ x^3-\dfrac{y^3}{8}=-28\end{cases}$

6. $\begin{cases}\dfrac{x}{y}+\dfrac{y}{x}=\dfrac{25}{12}\\ x^2-y^2=7\end{cases}$

7. $\begin{cases}\dfrac{4y}{x}+\dfrac{x}{y}=5\\ xy=4\end{cases}$

8. $\begin{cases}x+y=5\\ \dfrac{x}{y}+\dfrac{y}{x}=-\dfrac{13}{6}\end{cases}$

9. Agar tenglamalar sistemasining yechimlari uchun $xy<0$ bo'lsa, $x-y$ ni toping:

$$\begin{cases}\dfrac{2}{2x-y}+\dfrac{3}{x-2y}=\dfrac{1}{2}\\ \dfrac{2}{2x-y}-\dfrac{1}{x-2y}=\dfrac{1}{18}\end{cases}$$

10. $\begin{cases}\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}=\dfrac{10}{3}\\ 2x^2+y^2=27\end{cases}$

11. $\begin{cases}\dfrac{6}{x+y}+\dfrac{5}{x-y}=7\\ \dfrac{3}{x+y}-\dfrac{2}{x-y}=-1\end{cases}$

12. $\begin{cases} \dfrac{11}{2x-3y}+\dfrac{18}{3x-2y}=13 \\ \dfrac{27}{3x-2y}-\dfrac{2}{2x-3y}=1 \end{cases}$

13. $\begin{cases} \dfrac{2}{x}+\dfrac{y}{3}=3 \\ \dfrac{x}{2}+\dfrac{3}{y}=\dfrac{3}{2} \end{cases}$

RATSIONAL TENGSIZLIKLAR

1. $(-4x+3)(-5x+4)>0$

2. $(x^2-16)^3(x+7)<0$

3. $(x-2)^2(x-1)(x+7)(x-5)\geq 0$

4. $\dfrac{(x+6)^3(x-4)}{(7-x)^5}<0$

5. $(x^2-1)(x^2+5x+6)(x^2-5x+6)\leq 0$

6. $\left(2x+\dfrac{1}{x}\right)^2+2x+\dfrac{1}{x}-12<0$

7. $\dfrac{5x+4}{x-2}<1$

8. $\dfrac{3x+2}{x-3}>1$

9. $\dfrac{x-4}{x^2-9x+14}>0$

10. $\dfrac{x^4-10x^2+9}{6-2x}<0$

11. $\dfrac{x^2+1}{x-3}$

12. $\dfrac{x+3}{x^2+7}<0$

13. $(3-\sqrt{10})(2x-7)<0$

14. $\dfrac{(x^2-x-2)^2}{x^2+7x-8}\geq 0$

15. $\dfrac{3x-1}{x^2+x+1}\leq 0$

16. $\dfrac{x^2+2x-15}{3x^2+5x-8}\leq 0$

17. $\dfrac{2}{x+2}<\dfrac{1}{x-3}$

18. $\dfrac{3}{2-x}>\dfrac{1}{x+3}$

19. $\dfrac{2}{x+3}<\dfrac{1}{2x-1}$

20. $\dfrac{x+1}{x-2}>\dfrac{3}{x-2}-\dfrac{1}{2}$

21. $\dfrac{x^3-5x^2+8x-4}{x-3}\leq 0$

22. $\dfrac{6}{x-1}\leq\dfrac{3}{x+1}+\dfrac{7}{x+2}$

23. $\dfrac{14x(2x+3)}{x+1}<\dfrac{(9x-30)(2x+3)}{x-4}$

24. $\dfrac{(5x+4)(3x-2)}{x+3}\leq\dfrac{(3x-2)(x+2)}{1-x}$

25. $(x-3)^2+\dfrac{1}{x^2-6x+9}>2$

26. $\dfrac{2x-3}{4\sqrt{6}-10}>5+2\sqrt{6}$

RATSIONAL TENGSIZLIKLAR SISTEMASI

1. $\begin{cases} 2x-14<0 \\ -3x+9<0 \end{cases}$

2. $\begin{cases} 6x-1>9-4x \\ 3-2x<x+16 \end{cases}$

3. $\begin{cases} 3(2-3x)+2(3-2x)>x \\ 6<x^2-x(x-8) \end{cases}$

4. $\begin{cases} 5\left(1-\dfrac{x-4}{4}\right)-7(2x-3)>0 \\ \dfrac{3x-14}{5}-\dfrac{3x-10}{20}-0,7(x+8)<0 \end{cases}$

5. $\begin{cases} x^2 \le 9 \\ x+1>0 \end{cases}$

6. $\begin{cases} 4x+\dfrac{2x-3}{2}>\dfrac{7x-5}{2} \\ \dfrac{7x-2}{3}-2x>\dfrac{5(x-2)}{4} \end{cases}$

7. $\begin{cases} \dfrac{5x-4}{4}-\dfrac{4x+1}{3}\ge\dfrac{x+2}{4}-7 \\ \dfrac{4x}{3}-1-\dfrac{6x+2}{2}>x+\dfrac{6}{5} \end{cases}$

8. $\begin{cases} x^2+5x-6<0 \\ x+3\ge 0 \end{cases}$

9. $\begin{cases} 13-\dfrac{3-7x}{10}+\dfrac{x+1}{2}<14-\dfrac{7-8x}{2} \\ 7(3x-5)+4(17-x)>18-\dfrac{5(2x-6)}{2} \end{cases}$

10. $\begin{cases} \dfrac{x}{3}-\dfrac{3x-1}{6}<\dfrac{2-x}{12}-\dfrac{x+1}{2}+3 \\ x>\dfrac{5x-4}{10}-\dfrac{3x-1}{5}-2,5 \end{cases}$

11. $\begin{cases} \dfrac{3}{4}(x-1)+\dfrac{7}{8}<\dfrac{1}{4}(x-1)+\dfrac{5}{2} \\ \dfrac{x}{4}-\dfrac{2x-3}{3}<2 \end{cases}$

12. $\begin{cases} x^2+x+8<0 \\ x^2+6x+5\ge 0 \end{cases}$

13. $\begin{cases} 5x\ge 2 \\ -0,3x^2+4,8<0 \\ -2x^2+17x+19\ge 0 \end{cases}$

14. $\begin{cases} 2-5x\le 0 \\ x-x^2\ge 0 \\ -4x^2-5x+21\ge 0 \end{cases}$

15. $\begin{cases} \dfrac{x-4}{4}-x+1<\dfrac{x-2}{2}-\dfrac{x-3}{3} \\ 3-x>2x-10 \end{cases}$

16. $\begin{cases} \dfrac{2x}{3}-1<3-2(1-2x) \\ 3x-5>1-2(1-x) \\ 1-2x<3(2x-1) \end{cases}$

17. $\begin{cases} 7<2x+1<11 \\ \dfrac{x+2}{x-5}<\dfrac{x-6}{x-3} \end{cases}$

18. $\begin{cases} -2<2-x<1 \\ \dfrac{x+3}{1-x}\le\dfrac{8-x}{x-4} \end{cases}$

19. $\begin{cases} \dfrac{2}{7}<2^n<3 \\ 3^n>2 \end{cases}$

20. $\begin{cases} 1<\left(\dfrac{2}{3}\right)^n<3 \\ \left(\dfrac{3}{4}\right)^n<1,5 \end{cases}$

21. $\begin{cases} \dfrac{1}{5}<3^n<4 \\ 2<\left(\dfrac{1}{3}\right)^n<10 \end{cases}$

IRRATSIONAL TENGLAMALAR

1. $\sqrt{x-1}=-4$
2. $\sqrt{x}=8$
3. $\sqrt{x}=-16$
4. $\sqrt[3]{x+1}=2$
5. $\sqrt[4]{x-7}=-3$
6. $\sqrt{x^2+2x-6}\cdot\sqrt{x-9}=0$
7. $1+\sqrt{x+3}=0$
8. $\sqrt[3]{2x-1}+\sqrt[3]{x-1}=1$
9. $\sqrt{x-4}+\sqrt{x^2-3}=0$
10. $\sqrt{1+4x-x^2}=x-1$
11. $\sqrt{x-3}+\sqrt{2x+4}=-11$
12. $\sqrt{2x^2+8x+7}-2=x$
13. $\sqrt{x^2-7x+12}=2x-6$
14. $\sqrt{4-x}=\sqrt{x-7}$
15. $\sqrt{3+\sqrt{5-x}}=\sqrt{x}$
16. $\sqrt{4-x}+\sqrt{5+x}=3$
17. $2\sqrt{x+18}+\sqrt{4x-3}=15$
18. $\sqrt{x+20}-\sqrt{x-1}=3$
19. $\sqrt{x-5}+\sqrt{1-x}=7$
20. $(x^2-5x+6)\cdot\sqrt{2-x}=0$
21. $(2-x)\cdot\sqrt{x^2-x-20}=12-6x$
22. $(x-1)\cdot\sqrt{\dfrac{x-2}{x^2-1}}=0$
23. $(4x-x^2-3)\cdot\sqrt{x^2-2x}=0$
24. $\sqrt[3]{9x+1}=1+3x$
25. $\sqrt{x+5}+\sqrt[4]{x+5}=12$
26. $x^2+11+\sqrt{x^2+4}=42$
27. $x^2+5x+\sqrt{x^2+5x-5}=17$
28. $\sqrt{x^2-x}+\sqrt{2-x-x^2}=\sqrt{x}$
29. $\sqrt{4-x}+\sqrt{x-4}=0$
30. $\sqrt{7-5x}+\sqrt{5x-7}=29$
31. $\sqrt{\dfrac{x-1}{2x+1}}+\sqrt{\dfrac{2x+1}{x-1}}=\dfrac{10}{3}$
32. $\sqrt{5+2x}=10-3\sqrt[4]{5+2x}$
33. $\sqrt{3-x}+\sqrt{x-2}=(x-7)^2\cdot(x-5)$
34. $2\sqrt{x-1}-5=\dfrac{3}{\sqrt{x-1}}$
35. $\dfrac{\sqrt{x^2-3x-4}}{x+2}=\dfrac{\sqrt{x^2-3x-4}}{4-x}$
36. $x^2+\sqrt{x^2+20}=22$
37. $\sqrt{x^3+4x-1-8\sqrt{x^4-x}}=\sqrt{x^3-1}\quad\sqrt{x}$
38. $6x^2+7x\sqrt{1+x}=24(1+x)$
39. $\sqrt{(x^2+8x)^2}=x^2+8x$
40. $\sqrt{(4x^2-5x)^2}=5x-4x^2$
41. $\sqrt{x-4\sqrt{x-4}}=2-\sqrt{x-4}$
42. $\sqrt{x^2+\dfrac{1}{x^2}-2}=x-\dfrac{1}{x}$

43. $\sqrt{5-x}+\sqrt{x-6}=x^2+2x$

44. $\sqrt{x+6\sqrt{x-9}}+\sqrt{x-6\sqrt{x-9}}=6$

45. $\sqrt{x+8\sqrt{x-16}}+\sqrt{x-8\sqrt{x-16}}=2\sqrt{x-16}$

46. $\dfrac{\sqrt[4]{x^4-16}+\sqrt[6]{x^3-8}}{3x-x^2-2}=0$

IRRATSIONAL TENGLAMALAR SISTEMASI

1. $\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{x}\cdot\sqrt{y}=15\end{cases}$

2. $\begin{cases}\sqrt{xy}=12\\ \sqrt{x}+\sqrt{y}=7\end{cases}$

3. $\begin{cases}\sqrt{x}+\sqrt{y}=6\\ x-y=12\end{cases}$

4. $\begin{cases}\sqrt{x+3y+6}=2\\ \sqrt{2x-y+2}=1\end{cases}$

5. $\begin{cases}x\sqrt{y}+y\sqrt{x}=30\\ \sqrt{x}+\sqrt{y}=5\end{cases}$

6. $\begin{cases}3\sqrt{x}-\sqrt{y}=8\\ \sqrt{x}+2\sqrt{y}=19\end{cases}$

8. $\begin{cases}25y+x=100-10\sqrt{xy},\\ \sqrt{x}-\sqrt{y}=4,\end{cases}$

9. $\begin{cases}xy=64\\ x-y+\sqrt{xy}=20\end{cases}$

10. $\begin{cases}\sqrt{x+y-1}=1\\ \sqrt{x-y+2}=2y-2\end{cases}$

11. $\begin{cases}x+y-\sqrt{xy}=7\\ xy=9\end{cases}$

12. $\begin{cases}\sqrt{x}+\sqrt{y}=26\\ \sqrt[4]{x}+\sqrt[4]{y}=6\end{cases}$

13. $\begin{cases}\sqrt[3]{x}+\sqrt[3]{y}=-3\\ xy=8\end{cases}$

14. $\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\dfrac{3}{4}\\ xy=1\end{cases}$

15. $\begin{cases}\sqrt{x}-\sqrt{y}=5\\ \sqrt[4]{x}-\sqrt[4]{y}=1\end{cases}$

18. *a* $\begin{cases}5x+3\sqrt{xy}+4y=12\\ 3x+2\sqrt{xy}+3y=8\end{cases}$ *b* $\begin{cases}2x-6\sqrt{xy}+7y=9\\ x-4\sqrt{xy}+5y=6\end{cases}$

19. *a* $\begin{cases}\sqrt{x}+\sqrt{y}=5\\ x+y+4\sqrt{xy}=37\end{cases}$ *b* $\begin{cases}\sqrt{x}+\sqrt{y}=4\\ x+y-3\sqrt{xy}=1\end{cases}$

20. *a*) $\begin{cases}x\sqrt{x}+12y\sqrt{x}=28\\ 8y\sqrt{y}+6x\sqrt{y}=36\end{cases}$ *b*) $\begin{cases}x\sqrt{x}+27y\sqrt{x}=36\\ 27y\sqrt{y}+9x\sqrt{y}=28\end{cases}$

TRIGONOMETRIK VA TESKARI TRIGONOMETRIK FUNKSIYALAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y=\frac{1}{\sin x}$ b) $y=\frac{1}{\cos x}$ c) $y=\frac{\cos x}{\sin x-2\sin^2 x}$ d) $y=\frac{3x}{2\cos x-1}$

e) $y=\cos x+\sin x$ f) $y=\cos x+ctgx$

2. Funksiyaning qiymatlar to‘plamini toping.

a) $y=3\cos x-1$ b) $y=2-\sin x$ c) $y=1-2\sin^2 x$ d) $y=2\cos^2 x-1$

3. Berilgan funksiyaning juft yoki toqligini aniqlang.

a) $y=\frac{\sin x}{x}$ b) $y=x\cos x$ c) $y=\sin x+x^2$ d) $y=\cos x-x^3$.

4. Funksiyaning eng kichik musbat davrini toping.

a) $y=\sin\frac{x}{2}$ b) $y=\cos(3x-1)$ c) $y=tg2x$ d) $y=\cos\frac{x}{3}$.

5. Funksiyaning eng katta va eng kichik qiymatini toping.

a) $y=\cos^4 x-\sin^4 x$ b) $y=\cos(x+\frac{\pi}{4})\cos(x-\frac{\pi}{4})$ c) $y=1-2|\sin 3x|$

6. Funksiya nollarini toping.

a) $y=\sin x-2$ b) $y=2\cos x+1$ c) $y=x\cos x$ d) $y=\cos(x+\frac{\pi}{6})$.

7. Funksiyaning aniqlanish sohasini toping.

a) $y=\arccos\frac{4-x}{3}$ b) $y=\arcsin(2+3x)$

c) $y=\arcsin(3\sqrt{x}+2)$ d) $y=\arccos\frac{2x+3}{4}$

8. Taqqoslang.

a) $\arccos\frac{\sqrt{3}}{2}$ va $\arcsin(-\frac{1}{2})$ b) $arctg(-1)$ va $\arccos(-\frac{1}{2})$

c) $arctg\sqrt{3}$ va $\arcsin 1$ d) $\arccos(-\frac{\sqrt{3}}{2})$ va $\arcsin\frac{1}{2}$

9. Ifodalar qiymatini toping.

a) $2\arcsin(-\frac{\sqrt{3}}{2})+arctg(-1)+\arccos\frac{\sqrt{2}}{2}$ b) $3\arcsin\frac{1}{2}+4\arccos(-\frac{1}{\sqrt{2}})-arctg(-\sqrt{3})$

c) $arctg(-\sqrt{3})+\arccos(-\frac{\sqrt{3}}{2})+\arccos 1$ d) $3\arcsin 1-\frac{2}{3}\arccos(-\frac{1}{2})+6arctg\sqrt{3}$

TRIGONOMETRIK VA TESKARI TRIGONOMETRIK TENGLAMALAR

1. $0 \le x < 360^\circ$ oraliqda tenglamalarni yeching.

a) $\sin x = -0.3$ b) $\sin x = 0{,}15$ c) $\cos x = 0{,}6$ d) $\cos x = -0{,}43$

2. Oraliqlarni hisobga olgan holda x ning qiymatini toping.

a) $4\sin x + 2 = 0, \quad 0 \le x < 2\pi$ b) $ctgx - \sqrt{3} = 0, \quad 0 \le x < 2\pi$

c) $2\sin^2 x + 5\sin x = 3, \quad 0 \le x < 2\pi$ d) $\cos 2x = -\frac{1}{\sqrt{2}}, \ 0 \le x < 2\pi$

3. $0 \le x < 360^\circ$ oraliqda tenglamalarni yeching.

a) $7 - 6\cos^2 x = 5\sin x$ b) $7 + 2\cos x = 8\sin^2 x$ c) $2\sin x - 3\cos x = 0$

4. Tenglamani yeching.

a) $\sin 10x = -\frac{\sqrt{3}}{2}$ b) $\cos 10x = \frac{\sqrt{3}}{2}$ c) $tg10x = \sqrt{3}$ d) $ctg10x = \frac{\sqrt{3}}{3}$

5. Tenglmani yeching.

a) $\sin 4x \cos 3x tg8x = 0$ b) $\cos 4x = -\cos 5x$ c) $tg5x = -tg\frac{x}{3}$

6. Tenglamani yeching.

a) $2\sin^2 x + \cos^2 x - 2 = 0$ b) $2\sin^2 x + \cos x = 0$ c) $\sin x \cos x = 0$

7. Tenglamani yeching.

a) $\sin^2 x - 2\sin x \cos x + \cos^2 x = 0$ b) $7\cos^2 x - 3\sin^2 x = 0$

c) $\cos^2 2x - 10\sin 2x \cos 2x + 21\sin^2 2x = 0$ d) $8\sin^2 x - \cos^2 x = 0$

8. O‘rniga qo‘yish usulidan foydalanib yeching.

a) $\cos^2 2x + 1 = 2\cos x$ b) $3\cos^2 x \sin x + 1 = 3\cos^2 x + \sin x$

c) $6\cos^3 x + 6\sin^2 x - 3\cos x - 3 = 0$ d) $5\sin^2 x \cos x + 6\cos^2 x - 10\cos x + 6 = 0$

9. Tenglamni yeching.

a) $\cos 2x + \cos x = 0$ b) $\cos 3x = 2\cos 2x - 1$

c) $2\cos^2 x = 4\sin x \cos x - 1$ d) $\cos^2 x - 3\sin x \cos x = -1$

10. Tenglamani $\sin x + \cos x = t$ almashtirish yordamida yeching.

a) $2(\sin x + \cos x) + \sin 2x + 1 = 0$ b) $\sin x + \cos x = 1 + \frac{\sin 2x}{2}$

11. Tenglamani baholash usuli bilan yeching.

a) $2\sin^8 x + 3\cos^8 x = 5$ b) $(\cos 2x - \cos 4x)^2 = 4 + \cos^2 3x$

12. Tenglamani yordamchi burchak kiritish usuli bilan yeching.

a) $12\cos x - 5\sin x = -13$ b) $\sin x + \cos x = \sqrt{2}$

www.idum.uz saytidan yuklab olindi

O‘quv nashri

ALGEBRA

Umumiy o‘rta ta’lim maktablarining
10-sinfi uchun darslik

Muharrir Orifjon Madvaliyev
Badiiy muharrir Sarvar Farmonov
Texnik muharrir Akmal Sulaymonov
Rassom Behzod Zufarov
Dizayner Ilhom Boltayev
Sahifalovchi Rustam Xudayberganov
Musahhih Xurshid

Bosishga 00.00.2022-yilda ruxsat etildi. Bichimi 60x84 1/8.
“Cambria” garniturasi. Kegli 12. Ofset bosma.
Shartli bosma tabog‘i 20,46. Nashriyot-hisob tabog‘i 21,59.
Adadi nusxa. Buyurtma №

www.idum.uz saytidan yuklab olindi

Ijaraga berilgan darslik holatini ko‘rsatuvchi jadval

№	O‘quvchining ismi va familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshirilgan-dagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqova chizilgan, yirtilgan, asosiy qismidan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.